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DESIGNING A DIFFRACTIVE OPTICAL ELEMENT FOR CONTROLLING THE BEAM PROFILE IN A THREE-DIMENSIONAL SPACE USING THE SIMULATED ANNEALING ALGORITHM*

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We have designed a spatially quantized diffractive optical element (DOE) for controlling the beam profile in a three-dimensional space with the help of the simulated annealing (SA) algorithm. In this paper, we investigate the annealing schedule and the neighbourhood which are the deterministic parameters of the process that warrant the quality of the SA algorithm. The algorithm is employed to solve the discrete stochastic optimization problem of the design of a DOE. The objective function which constrains the optimization is also studied. The computed results demonstrate that the procedure of the algorithm converges stably to an optimal solution close to the global optimum with an acceptable computing time. The results meet the design requirement well and are applicable.

Keywords: diffractive optical element, beam profile, simulated annealing algorithm
PACC: 4230, 4283

I. INTRODUCTION

Various applications such as communication, fabrication and detection require the control of the beam profile in three-dimensional (3D) space while the incident light beam is transmitted. Thus, the approaches to achieving 3D beam shaping have been widely studied. The diffractive optical elements (DOEs) based on the Fresnel diffractive theory have the ability to control the beam profile, so the use of the DOEs to control the beam profile along a 3D region has been developed.[1−3] Compared with other optical elements, the design of the DOE is flexible and DOEs can be integrated with other devices during fabrication.[4] Thus DOEs provide a novel tool to obtain the desired 3D beam profile in a large variety of applications. The design of the DOE with discrete phase modulation is a stochastic combination optimization problem. The solution of this problem requires an effective algorithm to obtain its optimum, especially when the computing scale is large. Some optimization algorithms such as the G-S algorithm,[5] Y-G algorithm[6] and genetic algorithm[3] may be successfully applied to the optimal design of DOEs.

The simulated annealing algorithm[7] can find the global optimal solution and is widely suitable for solving the discrete stochastic optimization problems in physics and engineering. The proper parameters that determine the process and the quality of the SA algorithm must be obtained by experiments when the SA algorithm is employed to solve a practical problem. In recent years, the SA algorithm has been successfully employed to design DOEs for modulating the intensity distribution in two-dimensional space.[8−10] In this paper, a quantized DOE for controlling the beam profile in a 3D region is designed with the attempt to employ the SA algorithm. The annealing schedule and the neighbourhood structure are investigated and the output objective function that constrains the beam profiles in the multi-output planes is studied. The computed results demonstrate that the parameters and the objective function obtained are suitable for achieving the design of a DOE for 3D beam shaping with the SA algorithm. The designed DOE can control the output intensity distribution in the 3D region of interest, and the beam profile in each output section fulfils the requirements. The results are discussed in comparison with other optical elements.

II. THEORETICAL DESIGN OF THE DOE FOR 3D SHAPING

An optical diagram of the DOE for controlling the beam profile along a 3D region in space is illustrated

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in Fig.1, where the input plane p_0 is located at z_0, and H_1 is the discrete phase distribution of the DOE which modulates the incident light beam $f(x,y)$. The multi-output planes $p_1, p_2 \cdots p_N$ are located at points $z_1,\ z_2 \cdots\ z_N$ in succession in the region of interest, and the distributions of the output beam intensity in these planes are constrained by the given beam profiles. To obtain the phase distribution H_1 of the DOE is a phase retrieval problem.[11,12] When the distance between the input plane p_0 and the output plane p_α at distance z_α fulfils the Fresnel transform relation, the complex amplitude $g(\xi,\eta,z_\alpha)$ in the output plane is given by

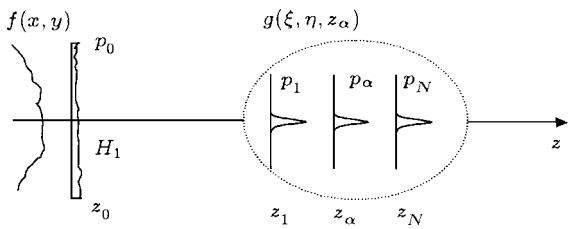


Fig.1. A diagram of the optical system of the DOE for controlling the beam profile in a 3D region.

$$\begin{aligned} g(\xi,\eta,z_\alpha) =& \mathrm{Fr}T\{f(x,y)\cdot H(x,y)\}z_\alpha \\ =& \frac{\exp(jkz_\alpha)}{j\lambda z_\alpha}\int\!\!\int_{\Sigma_0} f(x,y)\cdot H(x,y) \\ & \cdot\exp\left\{j\frac{k}{2z_\alpha}[(x-\xi)^2+(y-\eta)^2]\right\}\mathrm{d}x\mathrm{d}y, \end{aligned} \tag{1}$$

where Σ_0 is the aperture of the input plane p_0, λ is the wavelength of the incident light beam, and $k=2\pi/\lambda$ is the wave vector. $\mathrm{Fr}T\{\}_{z_\alpha}$ denotes the Fresnel transform from z_0 to z_α. The input beam is a plane wave, the required output intensity of our design is uniform along the z-axis in a circular segment, so the DOE can be constrained in rotational symmetry[13] and the description of the problem is simplified. Then the complex amplitude of the input beam $f(r)$ is given by

$$f(r) = |f(r)|\mathrm{e}^{j\varphi(r)}. \tag{2}$$

The DOE is designed in pure phase structure to improve the diffraction efficiency,[14,15] and its complex amplitude $H(r)$ is given by

$$H(r) = \mathrm{e}^{j\,\Psi(r)}. \tag{3}$$

Equation (1) can be described by

$$\begin{aligned} g(\rho,z_\alpha) =& 2\pi\frac{\exp(j2\pi z_\alpha/\lambda)}{j\lambda z_\alpha}\exp\left(j\frac{\pi\rho^2}{\lambda z_\alpha}\right)\int_0^{R_0}|f(r)| \\ & \times\exp\left\{j\left[\phi(r)+\frac{\pi r^2}{\lambda z_\alpha}\right]\right\} \\ & \times J_0\left(\frac{2\pi r\rho}{\lambda z_\alpha}\right)\cdot r\mathrm{d}r, \end{aligned} \tag{4}$$

where R_0 is the radius of the beam in the input plane z_0, and $\phi(r)=\varphi(r)+\Psi(r)$, $J_0()$ is the zero-order Bessel function of the first kind.

To ensure that the output energy is focused uniformly in the 3D region in question, the intensity distribution in each output plane must fulfil the constraint of the corresponding output objective profile. Considering the actual application, the output intensity of each cross section in the 3D region is designed to focus on a special segment. The deviation between the output field of the true design and that of the desired DOE is measured by the error function $Err(g)$ given by

$$Err(g) = \sum_{\alpha=1}^{N}\sum_{i}(|g_i(\rho,z_\alpha)| - |obj_i(\rho)|)^2, \tag{5}$$

where i is the number of pixels of the output plane, and $obj_i(\rho)$ is the objective function describing the output intensity. The error function $Err(g)$ is the criterion for estimating the optimal solution in the algorithm. The object of the design is to obtain the optimum phase distribution $\phi(r)$ which minimizes the error function.

Ⅲ. PARAMETERS OF THE SA ALGORITHM

To obtain the optimum phase distribution of the DOE, the approach of the design must avoid local optimal solutions during the optimization procedure. The SA algorithm is derived from the simulation of the process of the solid annealing, in which the worst state will be accepted with some probability controlled by the Metropolis model during the algorithm procedure. So it has the hill-climbing feature and can avoid stagnation in local optima.[7,16] The SA algorithm performs well on solving discrete stochastic optimization problems.[17]

We designed the DOE with the SA algorithm, in which the parameters of the algorithm in our case must be selected so that the optimization procedure can ensure the achievement of a solution very close

to the global optimum with an acceptable computing time. These parameters turn out to be the neighbourhood structure and the annealing schedule.

A. Neighbourhood structure

The neighbourhood $\{N(x) : x \in \Omega\}$, where Ω denotes the solution space which determines the searching space when a new solution is generated. The searching space will reach the whole solution space during optimization with a proper neighbourhood structure. Alrefaei *et al.*[17] even tried a neighbourhood structure corresponding to pure random search essentially. Both the quality of the new state and the procedure of the optimization respond to the neighbourhood structure obviously. The size of the solution space of a DOE with 128 pixels and eight phase levels is Ω=8^{128}, and a proper neighbourhood structure leads to a significant reduction of the required computational effort. The neighbourhood structure in our case is given by

$$N(\phi) = \phi \pm \Delta, \tag{6}$$

where $\Delta \in \Omega$, $0 \leq \Delta \leq \pi/4$. When a new state is generated, the phase value of each pixel of the DOE will be modulated by a quantized phase value Δ, which is generated by the algorithm.

B. Constructing the annealing schedule

The annealing schedule is constructed by the initial temperature t_0, the decreasing sequence of the temperature t, the length of the Markov chain and the criterion of convergence. The annealing schedule must ensure that the solutions fulfil the quasi-equilibrium condition for each t value.[7]

The parameter t is a fictitious temperature which controls the algorithm procedure. Aarts *et al.*[7] presented a method to obtain the appropriate initial temperature t_0. We tried m trial experiments, calculating the error function $Err(g)$ of each phase distribution we obtained, then we computed t_0 with

$$t_0 = \frac{\overline{\Delta Err(g)^-}}{\ln \dfrac{m_2}{m_2\chi_0 - m_1(1-\chi_0)}}, \tag{7}$$

where $\chi_0 \in$(0,1) is the initial probability. The proper value of χ_0 is chosen experimentally. $m = m_1+m_2$, m_2 are the times of trials in which the $Err(g)$ decreased, and $\overline{\Delta Err(g)^-}$ is the average value of the decreases.

The ideal t decreasing sequence can ensure that the SA algorithm achieves a global optimum solution, but it will require unacceptable computing time. In our design, in computing the Fresnel transform from the input plane to the multi-output planes, much computer time is spent in calculating the $Err(g)$, so we used the t decreasing sequence given by[9]

$$t_{k+1} = \delta \cdot t_k, \tag{8}$$

where $\delta \in$(0,1) is a constant very close to 1, which controls the decreasing rate of t. The rapid decreasing rate of this t sequence reduces the computer time, and the decreasing rate slowing down along with the optimization procedure is conducive to the convergence of the algorithm to a solution close to the global optimum.

The parameter that denotes how many new states are generated by the algorithm for each temperature t_k is termed the Markov chain, which is given by[7]

$$\overline{L} = n \cdot U, \tag{9}$$

where n is a constant, and U corresponds to the number of pixels and phase levels of the DOE. The Markov chain determines the computer time of the optimization procedure directly. The quality of the final solution obtained by the algorithm relies on the value of n, but larger n values make the algorithm take a longer time to converge.

The t sequence is terminated by the criterion of convergence. We use the rule presented by Kirkpatrick *et al.*[7] to avoid too long a computer time because of the large computing scale involved in the design of our DOE for 3D beam shaping. The optimization procedure will be terminated if there is no change of the optimum solution for a Markov chain.

IV. DESIGN APPROACH AND RESULT

A. Design approach

The objective output function that constrains the output beam profile in the design plays an important role in the quality of the computing result. The intensity distribution of the ideal objective function shown in Fig.2 is not achievable physically because the error function $Err(g)$ is difficult to reduce during computing. We performed many trial experiments, then chose a preferable beam profile in the output sections to be the objective function in the design. The objective output function actually used in the design is shown in Fig.3. The output intensity is required to focus on segment $\rho \in$[0, 0.04mm].

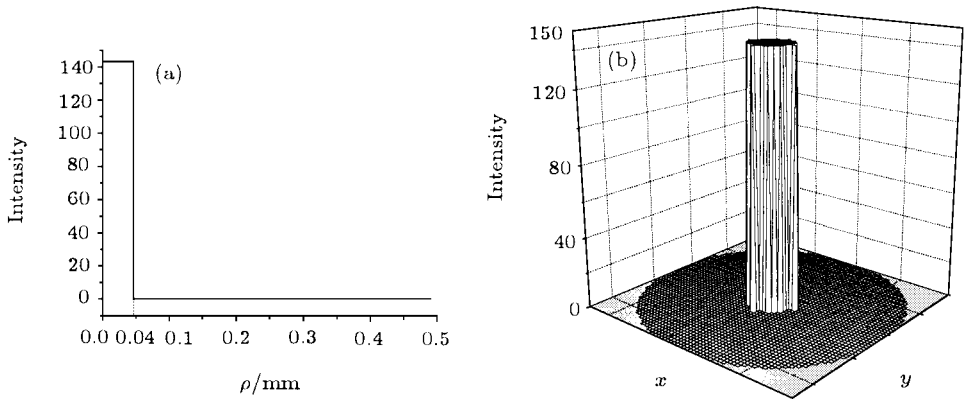


Fig.2. The ideal intensity distribution of each cross section in the 3D region: (a) intensity profile along the ρ-axis; (b) intensity distribution in a cross section.

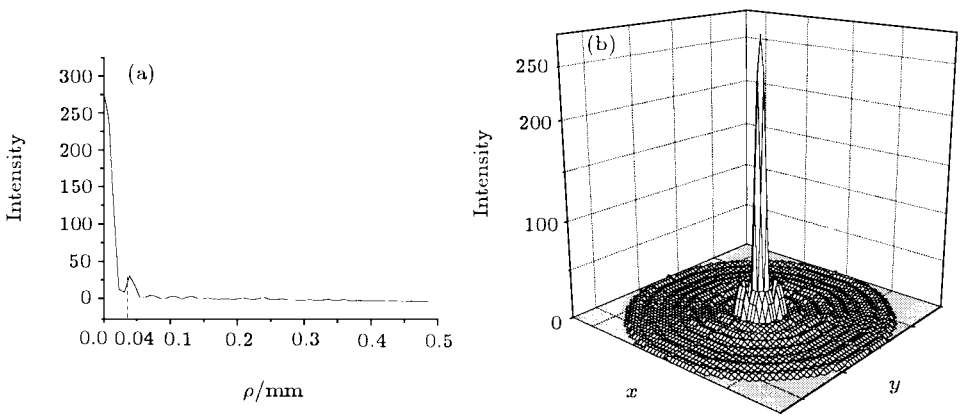


Fig.3. The objective output we actually used in the design to constrain the output beam profile, obtained by the trial experiments: (a) intensity profile along the ρ-axis; (b) intensity distribution in a cross section.

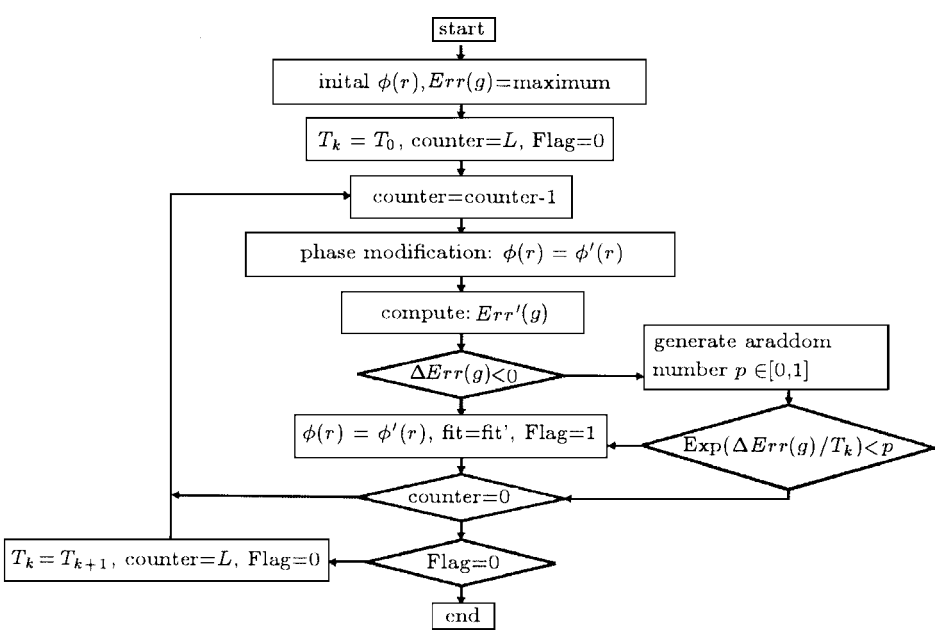


Fig.4. Flow chart of our design with the SA algorithm.

The designed DOE has r=1.5mm, and the illuminating wavelength is λ=780nm. The 3D region we designed to control is at a distance z=76~83.8mm from the DOE with 40 output sections distributed in the region. The DOE has 128 pixels and we sampled 64 signal spots in each output section during the computing.

The steps of our design employing the SA algorithm are shown by Fig.4. The optimization goal during computing is to minimize the cost function $Err(g)$ of the algorithm. The Boolean variable Flag denotes whether the new optimal solution is yielded.

We obtained several optimum solutions of the designed DOE with 128 pixels and eight quantized phase levels. The performance of one of our designs is illustrated by Fig.5, which shows the output intensity distribution in the 3D region along the z-axis. We can see that the main part of the output intensity in each section was focused on the designed segment $\rho \in$[0, 0.04mm]. The phase distribution of the designed DOE is illustrated by Fig.6. The profile in section z=78mm is shown in Fig.7 and the profile in section z=80mm is shown in Fig.8. It is shown that they are close to the objective function of Fig.3.

B. Results and discussion

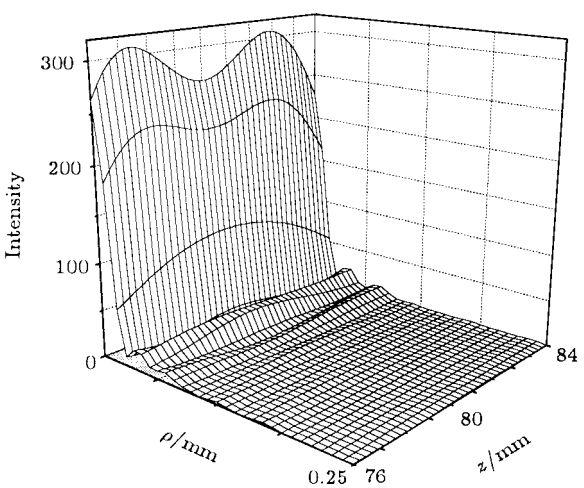


Fig.5. Intensity distribution along the z-axis generated by the designed DOE.

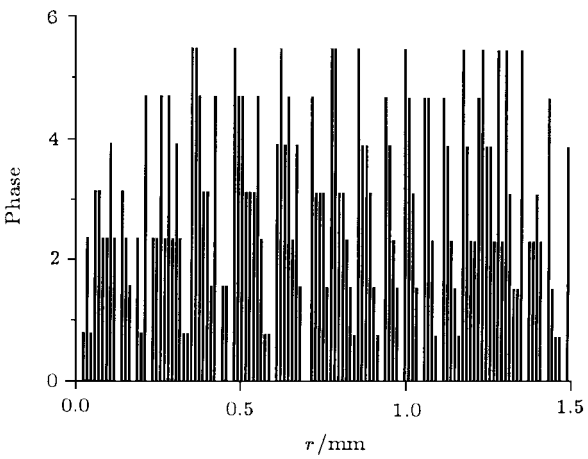


Fig.6. Quantized phase value distribution of the designed DOE.

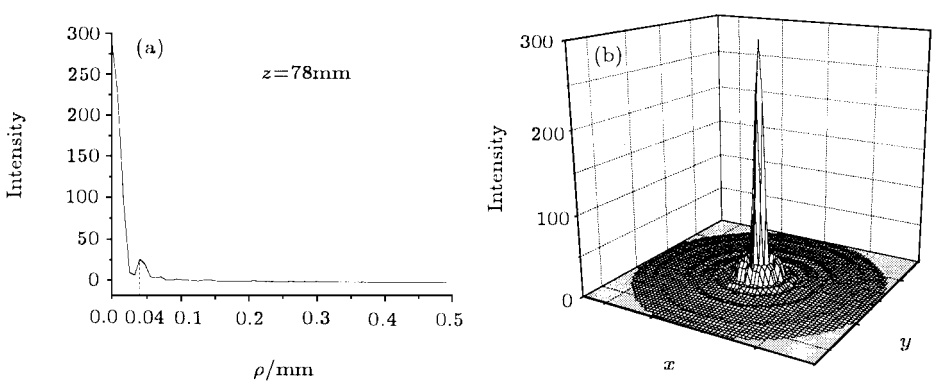


Fig.7. Intensity profile in cross section z=78mm: (a) intensity profile along the ρ-axis; (b) intensity distribution in this cross section.

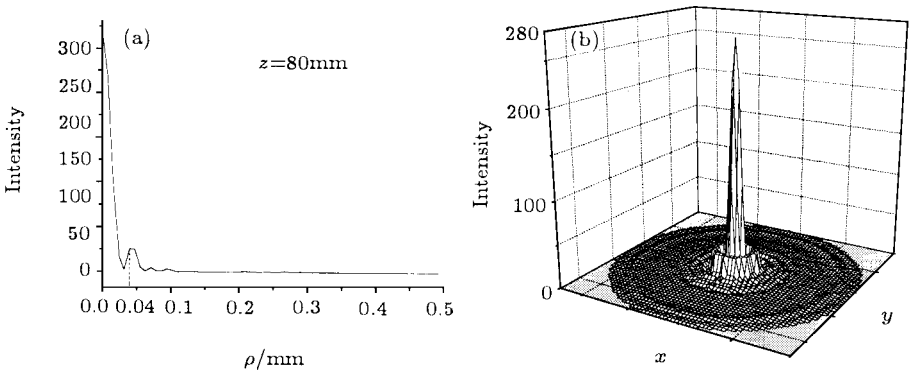


Fig.8. Intensity profile in cross section z=80mm: (a) intensity profile along the ρ-axis; (b) intensity distribution in this cross section.

We used Eq.(10) to evaluate the uniformity of the output intensity distributions

$$E_\alpha = \sum_i \left| g_i(\rho, z_\alpha) \right|^2 S_i(\rho) \Big|_{\rho \in [0, 0.04]} \qquad (10)$$

where E_α is the energy of each output section z_α in the segment $\rho \in$[0, 0.04mm], and $S_i(\rho)$ is the area corresponding to ρ_i in that output plane. The result is shown in Fig.9.

In region $z \in$[78.4mm, 80.8mm], the deviation between each E_α evaluated by Eq.(10) and the average value is less than 0.1%, so we can consider this region as the focus spot of the DOE. The deviation of E_0(z=76.0mm) and E_{39}(z=83.8mm) is 17.6% and 19.4% respectively. The focal tolerance of a conventional optical lens with the same parameters as our DOE (f=79.6mm, a=1.5mm, α=780nm) is Δz=±1.12mm, as evaluated by[18]

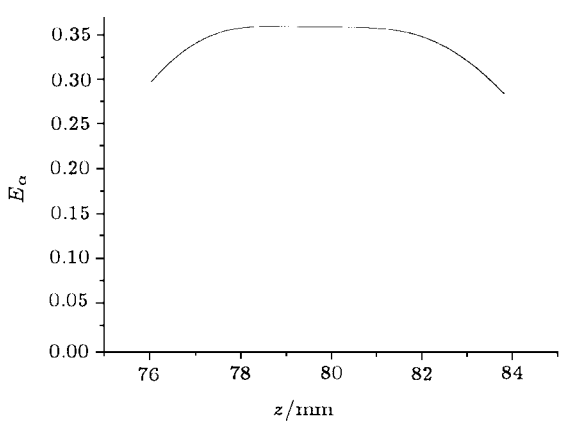


Fig.9. Output energy in the desired region along the z-axis evaluated by Eq.(10).

$$\Delta z \approx \pm 3.2 \frac{\lambda}{2\pi} \left(\frac{f}{a} \right)^2 . \qquad (11)$$

According to Eq.(11), the region $z \in$[76mm, 83.8mm] we designed to control is in the range of the focal tolerance where the intensity decreased by less than 20%. This means that the DOE we designed produces a much longer focal depth than does the conventional optical lens. The focal segment generated by the DOE is uniform along the z-axis, and the intensity of the focal segment is focused on the desired region. For comparison, one may take the axicon based on the solution of the Fresnel integral employing the stationary-phase technique, which is also an optical device for producing focal segments within a specified range. The focal segment generated by the generalized axicon exhibits a rapid oscillation around the average uniform focal segment. Some approaches have been proposed by Sedukhin[19] and Pu *et al.*[20] to solve this oscillation problem.

V. CONCLUSION

The SA algorithm is successfully employed to obtain an optimum design of DOEs for controlling the beam profile in a 3D region. The output intensity distribution along the z-axis is modulated by the designed DOE and the profile in each cross section is constrained by the objective profile so that the intensity is focused on the desired region. The results show that the SA algorithm is suitable to design this kind of DOE and the use of the DOEs is a novel approach to achieving 3D beam shaping. This design will help to develop the design of more complex beam shaping of DOEs in 3D space.

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