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SENSITIVE ERROR ANALYSIS OF CHAOS SYNCHRONIZATION*

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We study the synchronizing sensitive errors of chaotic systems for adding other signals to the synchronizing signal. Based on the model of the Henon map masking, we examine the cause of the sensitive errors of chaos synchronization. The modulation ratio and the mean square error are defined to measure the synchronizing sensitive errors by quality. Numerical simulation results of the synchronizing sensitive errors are given for masking direct current, sinusoidal and speech signals, separately. Finally, we give the mean square error curves of chaos synchronizing sensitivity and three-dimensional phase plots of the drive system and the response system for masking the three kinds of signals.

Keywords: chaos synchronization, sensitive error
PACC: 0545, 0540

I. INTRODUCTION

Since Pecora and Carroll[1−3] proposed the scheme of chaotic self-synchronization in 1990 and Cuomo, Oppenheim and Strogatz[4] realized secure communication based on Pecora and Carroll's scheme by the circuit experiment, chaotic secure communication has become an important topic in chaotic application research. One considers that a message is recovered accurately at the receiver by synchronization as long as the message is simply added at the transmitter to the chaotic synchronizing signal. Until now, the scheme of chaos masking has been widely accepted in science and technology. Oppenheim's chaos masking scheme is regarded as a perfection of chaotic secure communication. So some schemes of chaotic secure communication are presently developed based on Oppeheim's scheme.[5−11] Assuming that there is no other signal on the synchronizing signal, Pecora and Carroll's scheme may realize synchronization exactly between the response and the drive chaotic systems. When other signals are added at the transmitter to the chaotic synchronizing signal, can the chaotic synchronization be realized by Oppenheim's scheme? Until now, nobody has studied the problem because one looks at the original and the recovered waveforms in Fig.7 of Ref.[4] at a large scale and is misled by two similar waveforms. This paper presents an analytical version of the sensitive errors of chaos synchronization for masking and is organized as follows. The model and the analysis of the cause of chaos synchronizing errors are given in section II. In section III, we simulate the synchronizing sensitive errors for masking direct current, sinusoidal and speech signals. The main conclusion is presented in section IV.

II. MODEL AND ERROR ANALYSIS

We take the Henon map as the chaotic system in chaotic masking.[12] The block diagram of chaos masking is shown in Fig.1. The dynamics of the Henon map is described by the following equation

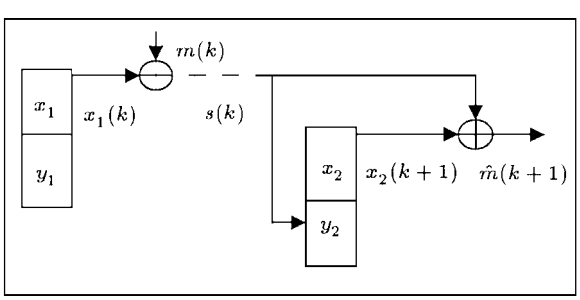


Fig.1. The block diagram of chaos masking based on the Henon map.

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$$\begin{cases} x(k) = 1 - \alpha x^2(k-1) + y(k-1), \\ y(k) = \beta x(k-1), \end{cases} \quad (1)$$

where α=1.4 and β=0.3. In Fig.1, the variables with subscript 1 denote the state variables of the drive chaotic system and the variables with subscript 2 denote those of the response chaotic system. At the transmitter, the message $m(k)$ modulates the state variables $x_1(k)$ of the drive chaotic system. The modulated chaotic signal $s(k)$ is transmitted to the receiver and is divided into two identical signals. One signal is fed to the input port of the response chaotic system which is forced to synchronize with the drive chaotic system, and its output signal is added to the adder. The other is directly transmitted to the adder. According to the usual consideration, a perfect synchronization is achieved and maintained between the receiver and the transmitter during the communication, so that the transmitted message is exactly extracted from the modulated chaotic signal at the receiver. Even if the recovered signal $\hat{m}(k)$ is different from the original signal $m(k)$, we can consider that the errors are brought about by the noise or the interference in the transmission process. A further discussion of the cause of the error is given below in the context of the synchronization process. The analysis process of the synchronizing sensitive error of the Henon map is shown in Fig.2.

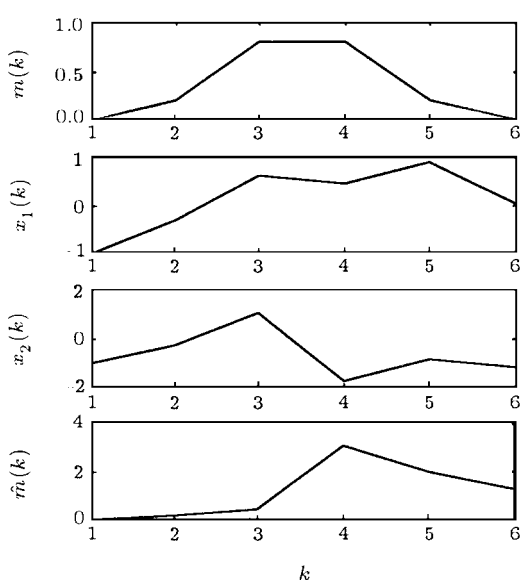


Fig.2. The curves of the synchronizing sensitive error for the Henon map.

We assume that the delay of the output signal of a chaotic system is a time step as compared with its input signal, and that when $k = 1$ the response chaotic system has synchronized with the drive chaotic system ($x_1(1) = x_2(1) = -1$, $y_1(1) = y_2(1) = 0.1$). When the value of the message signal is $m(1) = 0$, the modulated signal value will be $s(1) = x_1(1) + m(1) = -1$. The adder outputs a value $\hat{m}(1) = s(1) - x_2(1) = 0$, so that the value of the message signal is recovered accurately at the receiver. When $k = 2$, the drive system evolves freely as the initial value $x_1(1) = -1$, $y_1(1) = 0.1$ and its output value $x_1(2) = -0.3$. When the value of the message signal is $m(2)$=0.2, the value of the modulated signal is $s(2) = x_1(2) + m(2) = -0.1$. At $k = 2$, the response chaotic system is forced to evolve at the input value $s(1) = x_1(1) + m(1) = -1$ and its output value $x_2(2) = -0.3$. The adder output value is $\hat{m}(2) = s(2) - x_2(2) = 0.2$. The value of the message signal is correctly recovered at the receiver. When $k = 3$, the drive chaotic system evolves freely and its output value is $x_1(3) = 0.574$. When the value of the message signal is $m(3)$=0.8, the value of the modulated signal is $s(3) = x_1(3) + m(3)$=1.374. The response chaotic system is forced to evolve at the input value $s(2) = x_1(2) + m(2) = -0.1$ and its output value is $x_2(3)$=0.896. The adder outputs a value $\hat{m}(3) = s(3) - x_2(3)$=0.478. At the moment, the value of the message signal is not recovered correctly at the receiver. When $k = 4$ the drive chaotic system outputs a value $x_1(4) = 0.448$ and the response chaotic system outputs the value $x_2(4) = -1.733$ at the input value $s(3) = x_1(3) + m(3)$=1.374. For the value of the message signal $m(4)$=0.8, the adder outputs the value $\hat{m}(4) = s(4) - x_2(4)$=2.981, and the value of the message signal is not recovered at the receiver either. According to the above analysis, we have $x_1(5)$=0.89, $x_2(5) = -0.887$, $m(5)$=0.2 and $\hat{m}(5)$=1.977 at $k = 5$. When, $k = 6$ we have $x_1(6)$=0.249, $x_2(6) = -1.184$, $m(6)$=0.0 and $\hat{m}(6)$=1.433. At two time points, none of the values of the message signal are recovered correctly at the receiver. After $k = 6$, when the values of the message signal continue to be zero, the response chaotic system will soon be forced to synchronize with the drive chaotic system, and the zero values of the message signal reproduce only at the adder output port. Figure 2 shows the result of a computer simulation of the Henon map masking. Since the response chaotic system is sensitive to other signals on the synchronizing signal, the response chaotic system does not synchronize with the drive chaotic system, and an error is caused at the adder.

To analyse the synchronizing sensitive error of a chaotic system by quality, we give the definitions of

the modulation ratio and the mean square error. The modulation ratio is given by

$$SNR = 10\log_{10}\frac{\sum X_1^2(k)}{\sum m^2(k)}\mathrm{d}B, \tag{2}$$

and the mean square error between the modulation signal $s(k)$ and the state variables $x_2(k)$ is defined as

$$\begin{aligned} E &= \sqrt{\frac{1}{K}\sum_k (s(k)-x_2(k))^2} \\ &= \sqrt{\frac{1}{K}\sum_k (\hat{m}(k))^2}. \end{aligned} \tag{3}$$

By defining the modulation ratio and the mean square error, it is straightforward to show that the sensitive error of the chaos synchronization is related to some certain factors.

Ⅲ. NUMERICAL MODULATION OF THREE TYPICAL SIGNALS

A. Input direct current signal

We take the Lorenz system as the chaotic masking system, which is given by

$$\begin{cases} \dot{x} = \sigma(y-x), \\ \dot{y} = \gamma x - y - xz, \\ \dot{z} = xy - bz, \end{cases} \tag{4}$$

where $\sigma = 10$, $\gamma = 28$, $b = 8/3$. To simulate the mean square error for the masking direct current signal, we used a total of 5 000 data points and employed a fourth-order Runge–Kutta integrator, changing the time step h from 0.001 to 0.1. The three different direct current voltages are added separately to the output signal $x_1(k)$ of the drive chaotic system. The three curves in Fig.3 illustrate two characteristics: the magnitude of the time step almost does not influence the mean square error, and the magnitude of the direct current voltage will influence the mean square error directly. We simulate the three-dimensional (3D) phase plots of sensitive errors of the chaotic synchronization by using a total of 500 data points and time step 0.03, as shown in Figs.4(a)–(c). Figure 4(c) shows the distorted 3D phase plot of the response chaotic system because it is sensitive to the direct current voltage on the synchronizing signal. In real communication, we must filter the direct current signal on the synchronizing signal before the synchronizing signal is sent to the input port of the response chaotic system.

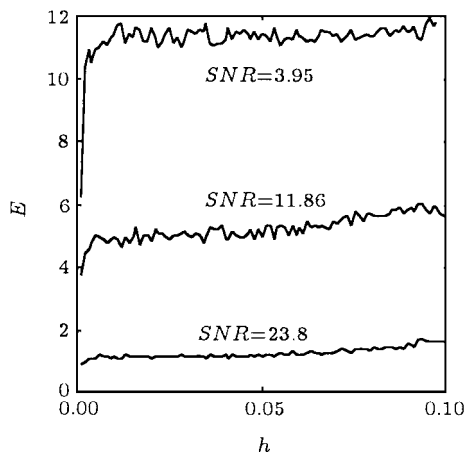


Fig.3. The curves of the synchronizing sensitive mean square error for adding the direct current signal to the output signal $x_1(k)$ of the drive chaotic system.

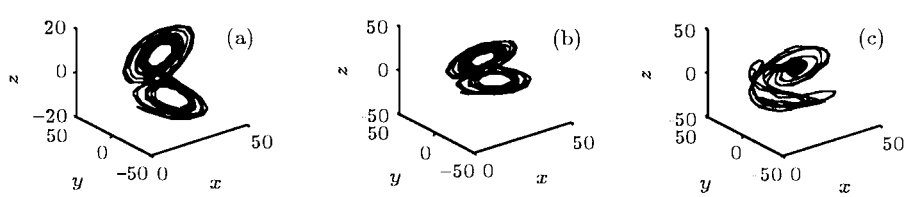


Fig.4. Three-dimensional phase plots of the sensitive error of chaotic synchronization for adding the direct current signal to the output signal $x_1(k)$ of the drive chaotic system: (a) 3D phase plot of the drive chaotic system; (b) 3D phase plot where a direct current voltage of 5V is added to the output signal $x_1(k)$ of the drive chaotic system; (c) 3D phase plot of the response chaotic system.

B. Input sinusoidal signal

Figure 5 shows the numerical modulation of the sensitive error of the chaotic synchronization for masking the sinusoidal signal. In Fig.5 it is easy to see that there are three characteristics. One characteristic is that the amplitude of the sinusoidal signal will influence the mean square error. Under the condition of identical frequency, the larger the amplitude of the sinusoidal signal, the larger the mean square error. This is illustrated by the curves of the same frequency f=10Hz in Fig.5. Another characteristic is that the frequency of the sinusoidal signal will also influence the mean square error, as can be seen in the curves of the same modulation ratio SNR=0.89 in Fig.5. The third characteristic is that the time step will directly influence the mean square error. Given the modulation ratio and the frequency of the sinusoidal signal, the larger the time step, the larger the mean square error. From Fig.6 we can see that the distortion of the 3D phase plot of the response chaotic system is small because the response chaotic system can counteract a greater part of sensitive errors for positive and negative amplitudes of the sinusoidal signal.

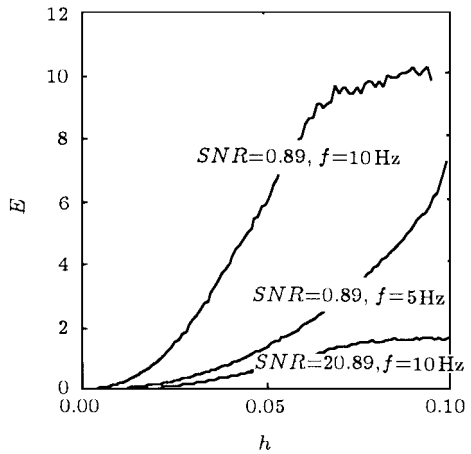


Fig.5. The curves of the synchronizing sensitive mean square error for adding the sinusoidal signal to the output signal $x_1(k)$ of the drive chaotic system.

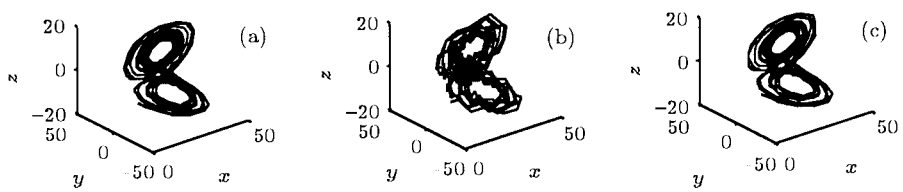


Fig.6. Three-dimensional phase plots of sensitive error of chaotic synchronization for adding the sinusoidal signal to the output signal $x_1(k)$ of the drive chaotic system: (a) 3D phase plot of the drive chaotic system; (b) 3D phase plot where the sine signal $m(k)$=2sin10k is added to the output signal $x_1(k)$ of the drive chaotic system; (c) 3D phase plot of the response chaotic system.

C. Input speech signal

In Fig.7 we show the numerical modulation of the sensitive error of chaotic synchronization for the masking speech signal. This illustrates that both the modulation ratio and the time step will influence the mean square error of the chaotic synchronization. Since a speech signal contains direct current and sinusoidal signals of different frequencies, the sensitive error of the chaotic synchronization is composed of errors of the direct current signal and all kinds of sinusoidal signals with different frequencies. The speech signal on the synchronizing signal frustrates the synchronization at the receiver, as can be seen in Fig.8(c). So a speech signal is not recovered exactly at the receiver.

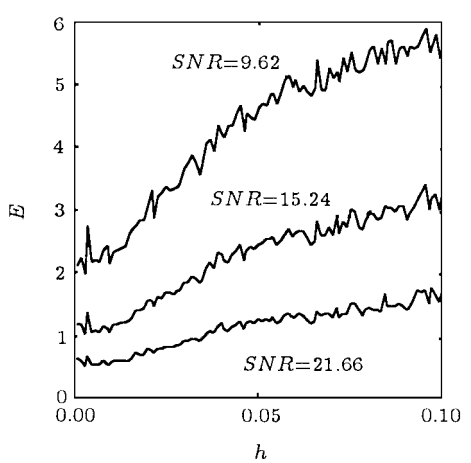


Fig.7. The curves of the synchronizing sensitive mean square error for adding the speech signal to the output signal $x_1(k)$ of the drive chaotic system.

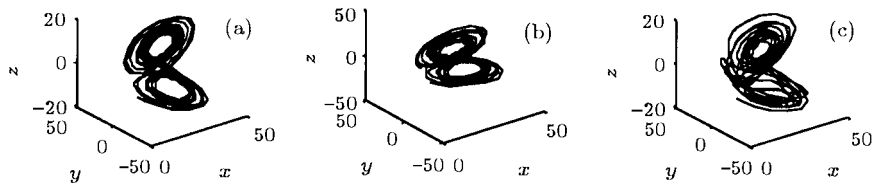


Fig.8. Three-dimensional phase plots of the sensitive error of chaotic synchronization for adding the speech signal to the output signal $x_1(k)$ of the drive chaotic system: (a) 3D phase plot of the drive chaotic system; (b) 3D phase plot where the speech signal is added to the output signal $x_1(k)$ of the drive chaotic system; (c) 3D phase plot of the response chaotic system.

IV. CONCLUSION

We have demonstrated that the chaotic mask is meaningless for the message signal on a chaotic synchronizing signal frustrated synchronization at the receiver. In chaos masking other signals on the synchronizing signal continually change the initial conditions of the response chaotic system. The response chaotic system evolves continually under new initial conditions. For a direct current signal, the response chaotic system iterates once and the trajectory will present a transition. Although the direct current voltage is not changed, at each time the delta of trajectory transition is different because the chaotic system is a nonlinear system. So the direct current signal on the synchronizing signal will result in the sensitive error of synchronization at the receiver. If we take the chaotic system of partial differential equations as a masking chaotic system, there is not only a nonlinear relation but also an integral relation between the output and the input signals of the chaotic system. Under the action of system integration, the singular components of the input signal are smoothed. Since the speech signal, which contains many singular components, is added to the synchronizing signal, the effect of synchronization is better than the direct current signal under the same modulation ratio (see Figs.4(c) and 8(c)). A nonlinear partial differential system is a super damped-frequency band filter. If the frequency of the sinusoidal signal on a synchronizing signal drops into the damped-frequency band, this sinusoidal signal is well inhibited. So the sinusoidal signal on the synchronizing signal is extracted exactly by chaos synchronization.

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