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NEW KINDS OF DIRAC ENERGY LEVELS AND THEIR CROSSING REGIONS*

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In the space-time of a non-Kerr–Newman black hole, the Dirac energy levels and their crossing regions are investigated. Near the event horizon of the black hole there are crossing Dirac energy levels, which lead to the occurrence of non-thermal radiation.

Keywords: black hole, energy level, radiation
PACC: 9760L, 0420

I. INTRODUCTION

In recent years, Manko and co-workers[1−3] have proven that, under the condition of the Robinson–Carter theorem, a special kind of black hole may exist in the universe. The typical event horizon of the black hole is not a smooth hyper-surface but contains one or two singular points. Accordingly, in the static condition it is different from the Schwarzschild black hole, while in the stationary condition it differs from the Kerr and Kerr–Newman black holes. According to the exact solution of the Einstein–Maxwell equation referring to a charged spinning mass, Dirac energy levels of an asymptotically flat non-Kerr–Newman black hole are calculated by computer. The property of non-thermal radiation and the influences that singular points have on Dirac energy levels in the black hole are presented.

II. CALCULATIONS AND DIAGRAMS

The space-time line element of a non-Kerr–Newman black hole expressed in the prolate spheroid coordinates is

$$\begin{aligned} \mathrm{d}s^2 =& k^2 f^{-1}\left[\mathrm{e}^{2\gamma}(x^2-y^2)\left(\frac{\mathrm{d}x^2}{x^2-1}+\frac{\mathrm{d}y^2}{1-y^2}\right)\right. \\ &\left.+(x^2-1)(1-y^2)\mathrm{d}\varphi^2\right]-f(\mathrm{d}t-\omega\mathrm{d}\varphi)^2, \quad (1) \end{aligned}$$

where k is a real constant, and f, γ and ω are metric functions. According to Ref.[1], the electric and magnetic potentials A_μ=(A_0,0,0,A_3) and metric function f, γ and ω are respectively

$$A_0+\mathrm{i}A_3=-\frac{2\beta B}{A}, \quad (2)$$

$$\begin{aligned} A =& [(1-\beta^2)x+(1+\beta^2)](x^2-y^2)^4-a^2(x^2-1)^3 \\ &\cdot[(1-\beta^2)(x^3+3x)+(1+\beta^2)(3x^2+1)] \\ &-2\mathrm{i}\alpha y(x^2-1)\{(x^4+6x^2y^2+y^4) \\ &\cdot[(1-\beta^2)(x^2+1)+2(1+\beta^2)x] \\ &-4x(x^2+y^2)[(1+\beta^2) \\ &\cdot(x^2+1)+2(1-\beta^2)x]\}, \quad (3) \end{aligned}$$

$$\begin{aligned} B =& (x^2-y^2)^4-\alpha^2(x^2-1)^3(3x^2+1)+4\mathrm{i}\alpha xy \\ &\cdot(x^2-1)(x^4+2x^2-4x^2y^2-y^4+2y^2), \quad (4) \end{aligned}$$

$$f=(1-\beta^2)^2(x^2-1)\frac{C}{D},$$

$$\mathrm{e}^{2\gamma}=\frac{x^2-1}{x^2-y^2}\frac{C}{(1-\alpha^2)^2(x^2-y^2)^8},$$

$$\omega=-\frac{4k\alpha(1-y^2)E}{(1-\alpha^2)(1-\beta^2)^2C},$$

$$\begin{aligned} C =& [(x^2-y^2)^4-\alpha^2(x^2-1)^4]^2 \\ &-4\alpha^2(1-y^2)(x^2-1)^3(x^4+6x^2y^2+y^4)^2, \end{aligned}$$

$$\begin{aligned} D =& \{[(1-\beta^2)x+1+\beta^2](x^2-y^2)^4 \\ &-\alpha^2(x^2-1)^3[(1-\beta^2)(x^3+3x) \\ &+(1+\beta^2)(3x^2+1)]\}^2 \\ &+4\alpha^2y^2(x^2-1)^2\{(x^4+6x^2y^2+y^4) \\ &\cdot[(1-\beta^2)(x^2+1)+2(1+\beta^2)x] \\ &-4x(x^2+y^2)[(1+\beta^2)(x^2+1)+2(1-\beta^2)x]\}^2, \end{aligned}$$

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$$
\begin{aligned}
E =&(1-\beta^4)x\{(x^2-y^2)^5[3x^4+x^2+x^2y^2\\
&+3y^2-\alpha^2(3x^4+x^2y^2+x^2+3y^2)]\\
&-\alpha^2(x^2-1)^3[5x^8+10x^6+10x^6y^2+x^4\\
&+76x^4y^2+x^4y^4+10x^2y^2+10x^2y^4\\
&+5y^4-a^2(5x^8+10x^6y^2+10x^6+x^4y^4\\
&+76x^4y^2+x^4+10x^2y^4+10x^2y^2+5y^4)]\}\\
&-a^2(x^2-1)^3+(1+\beta^4)\{(x^2-y^2)^5[3x^4+6x^2y^2\\
&-y^4-a^2(6x^4+3x^2y^2-3x^2+y^2+1)]\\
&\cdot[8x^8+5x^6-x^6y^2+45x^4y^2-145x^4y^4\\
&+15x^2y^4-51x^2y^6-y^6-3y^8\\
&-a^2(15x^8+45x^6y^2-5x^6+5x^4y^4+55x^4y^2\\
&+10x^4+10x^2y^4-5x^2y^2-5x^2+y^4+y^2+1)]\}.
\end{aligned}
\tag{5}
$$

From Eqs.(1) and (5) the non-zero components of the contrary metric tensor are respectively

$$
\begin{aligned}
g^{00} =&\frac{16\alpha^2E^2(1-y^2)(x^2-1)(1-a^2)^{-2}-D^2}{DC(x^2-1)(1-\beta^2)^2},\\
g^{03} =&-\frac{4\alpha E}{kD(1-a^2)},\\
g^{11} =&\frac{(1-a^2)^2(1-\beta^2)^2(x^2-1)(x^2-y^2)^8}{k^2D},\\
g^{22} =&\frac{(1-a^2)^2(1-\beta^2)^2(1-y^2)(x^2-y^2)^8}{k^2D},\\
g^{33} =&\frac{C(1-\beta^2)^2}{k^2(1-y^2)D}.
\end{aligned}
\tag{6}
$$

Formulae (1)–(5) fully determine the stationary metric of a charged spinning black hole. Using the transformation of coordinates[1−3]

$$
x=(r-M)/k,\ y=\cos\theta, \tag{7}
$$

in the limit $r\to\infty$, we have

$$
\begin{aligned}
M =&\frac{k(1-3a^2)(1+\beta^2)}{(1-\alpha^2)(1-\beta^2)},\\
J =&\frac{2k^2\alpha(5\alpha^2-3)(1+\beta^2)}{(1-\alpha^2)^2(1-\beta^2)},\\
Q =&\frac{2k\beta(3\alpha^2-1)}{(1-\alpha^2)(1-\beta^2)},\\
\mu =&\frac{4k^2\alpha\beta(5\alpha^2-3)}{(1-\alpha^2)^2(1-\beta^2)},
\end{aligned}
\tag{8}
$$

where M is the total mass, J is the angular momentum, Q is the electronic charge and μ is the magnetic dipole moment. They characterize the charged rotating source. In these formulae, α and β are real constants ($0\le\alpha<1, 0\le\beta<1$). Note that the solution expressed by Eqs.(2)–(5) possesses the event horizon given by the hyper-surface x=1; however, the horizon contains two singular points (the poles $y=\pm1$). Therefore, it differs from the Kerr–Newman black hole.[4,5]

In the curved space-time expressed in Eqs.(1) and (5), the motion equation of a particle with mass m_0 and electric charge e is[6,7]

$$
g^{\mu\nu}\left(\frac{\partial s}{\partial x^\mu}-eA_\mu\right)\left(\frac{\partial s}{\partial x^\nu}-eA_\nu\right)+m_0^2=0. \tag{9}
$$

Putting Eq.(6) into Eq.(9) can obtain

$$
\begin{aligned}
&\frac{16\alpha^2E^2(1-y^2)(1-\alpha^2)^{-2}(x^2-1)-D^2}{DC(x^2-1)(1-\beta^2)^2}\left(\frac{\partial s}{\partial t}-eA_0\right)^2\\
&+\frac{(1-\alpha^2)^2(1-\beta^2)^2(x^2-1)(x^2-y^2)^8}{k^2D}\left(\frac{\partial s}{\partial x}\right)^2\\
&+\frac{(1-\alpha^2)^2(1-\beta^2)^2(1-y^2)(x^2-y^2)^8}{k^2D}\left(\frac{\partial s}{\partial y}\right)^2\\
&+\frac{C(1-\beta^2)^2}{k^2(1-y^2)D}\left(\frac{\partial s}{\partial\varphi}-eA_3\right)^2\\
&-\frac{8\alpha E}{kD(1-\alpha^2)}\left(\frac{\partial s}{\partial t}-eA_0\right)\left(\frac{\partial s}{\partial\varphi}-eA_3\right)+m_0^2=0,
\end{aligned}
\tag{10}
$$

where s is Hamilton principal function. We then use the transformation of tortoise coordinates[8−10]

$$
x_*=x+\frac{1}{2\kappa_0}\ln(x-1),\quad y_*=y-y_0. \tag{11}
$$

Meanwhile we let

$$
\frac{\partial s}{\partial t}=-\varepsilon,\quad \frac{\partial s}{\partial y_*}=n,\quad \frac{\partial s}{\partial\varphi}=m, \tag{12}
$$

where ε is the energy of the particle, κ_0 is a real constant, and n and m are the components of general momentum of the particle. Therefore, putting Eqs.(11) and (12) into Eq.(10) and considering that s and $\dfrac{\partial s}{\partial x}$ are real numbers, we get[11]

$$
\begin{aligned}
&\frac{D^2-16\alpha^2E^2(1-y^2)(1-\alpha^2)^{-2}(x^2-1)}{DC(x^2-1)(1-\beta^2)^2}(\varepsilon-A_0)^2\\
&-\frac{C(1-\beta^2)^2}{k^2(1-y^2)D}(m-eA_3)^2\\
&-\frac{(1-\alpha^2)^2(1-\beta^2)^2(x^2-1)(x^2-y^2)^8}{k^2D}n^2\\
&-\frac{8\alpha E(\varepsilon+aA_0)(m-eA_3)}{kD(1-\alpha^2)}-m_0^2\ge0.
\end{aligned}
\tag{13}
$$

Taking the equals sign and solving it, we get

$$\varepsilon_{\pm}/m_0 = \frac{e}{m_0}A_0 + \frac{4\alpha CE(m-eA_3)(x^2-1)(1-\beta^2)^2 \pm C(1-\beta^2)^2 H^{\frac{1}{2}}}{m_0 k[D^2 - 16\alpha^2E^2(1-y^2)(x^2-1)(1-\alpha^2)^{-2}]}. \tag{14}$$

Considering the sign of inequality we can get

$$\varepsilon > \varepsilon_+ \quad \text{or} \quad \varepsilon < \varepsilon_-, \tag{15}$$

while

$$\begin{aligned} H =& 16\alpha^2E^2(m-eA_3)^2(x^2-1)^2(1-\alpha^2)^{-2} + D^{-1}C^{-1}(x^2-1)(1-\beta^2)^{-2} \\ & \cdot [D^2 - 16\alpha^2E^2(x^2-1)(1-y^2)(1-\alpha^2)^{-2}][Dn^2(1-y^2)(1-\beta^2)^2(1-\alpha^2)^2(x^2-y^2)^8 \\ & + D^{-1}C(1-\beta^2)^2(m-eA_3)^2(1-y^2)^{-1} + (m_0kD)^2]. \end{aligned} \tag{16}$$

From Eqs.(2)–(4) we can obtain that in Eqs.(14)–(16) A_0 and A_3 are respectively

$$A_0 = -\frac{2\beta A_1B_1 - 16\alpha^2\beta xy^2A_2(x^2-1)^2(x^2+2x-4x^2y^2-y^2+2y^2)}{A_1^2 + 4\alpha^2A_2^2y^2(x^2-1)^2}$$

$$A_3 = -\frac{4\alpha\beta B_1A_2y(x^2-1) + 8A_1\alpha\beta xy(x^2-1)(x^4+2x^2-4x^2y^2-y^4+2y^2)}{A_1^2 + 4\alpha^2y^2(x^2-1)^2A_2^2}, \tag{17}$$

where

$$\begin{aligned} A_1 =& [(1-\beta^2)x + (1+\beta^2)](x^2-y^2)^4 \\ & - \alpha^2(x^2-1)^3[(1-\beta^2)(x^3+3x) \\ & + (1+\beta^2)(3x^2+1)], \\ A_2 =& (x^4+6x^2y^2+y^4)[(1-\beta^2)(x^2+1) + 2(1+\beta^2)x] \\ & - 4x(x^2+y^2)[(1+\beta^2)(x^2+1) + 2(1-\beta^2)x], \\ B_1 =& (x^2-y^2)^4 - \alpha^2(x^1-1)^3(3x^2+1). \end{aligned} \tag{18}$$

Formulae (14) and (15) are the exact expressions of Dirac energy levels in the curved space-time described by Eqs.(1)–(5). The relevant functions are expressed in Eq.(5) and Eqs.(16)–(18).

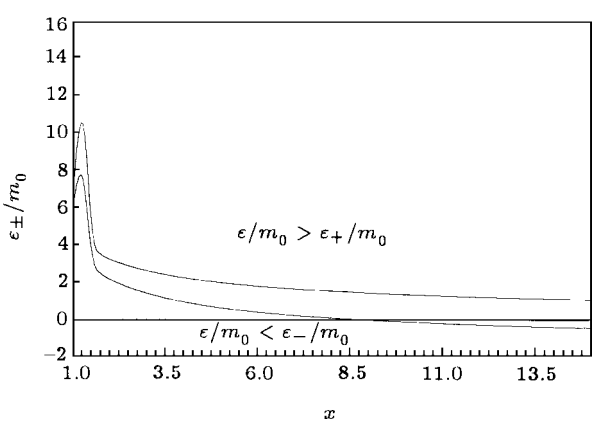


Fig.1. α=0.25, β=0.5×10^{-6}, y=0.99.

When α=0, from Eq.(8) we can obtain J=0 and μ=0. Under these conditions, the non-Kerr–Newman space-time turns into non-Reissner–Nordstrom space-time. When β=0, from Eq.(8) we can obtain Q=0 and μ=0. Under these conditions, the non-Kerr–Newman space-time turns into non-Kerr space-time. When $\alpha = \beta$=0, from Eq.(8) we can obtain Q=0, J=0 and μ=0. Under these conditions, the space-time turns into Schwarzschild space-time.[12,13] Under the Schwarzschild condition the event horizon is a smooth hyper-surface. Noting that in the limit $x \to 1$ we can obtain

$$\lim_{x\to 1} H = 0, \tag{19}$$

$$\lim_{x\to 1}(\varepsilon_{\pm}/m_0) = \frac{e}{m_0}\beta, \tag{20}$$

$$\lim_{x\to\infty}(\varepsilon_{\pm}/m_0) = \pm 1. \tag{21}$$

From above, the Dirac energy levels described in Eqs.(14) and (15) form the crossing region near the event horizon and at a distance are the same as the case of flat space-time.

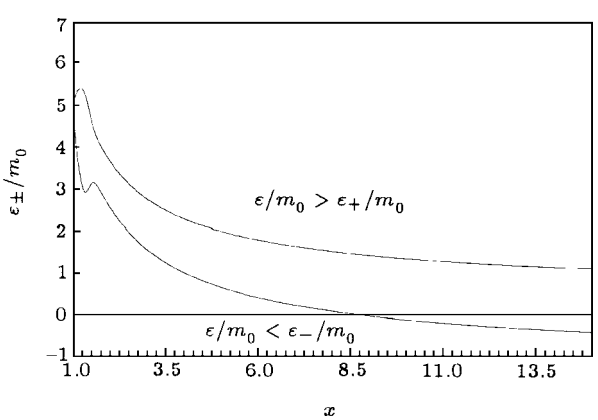


Fig.2. α=0.25, β=0.5×10^{-6}, y=0.45.

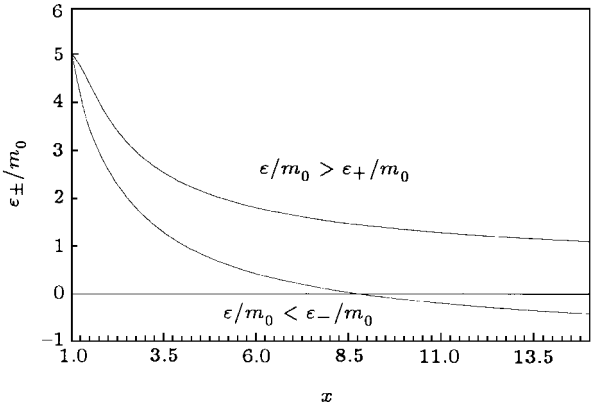


Fig.3. α=0.25, β=0.5×10^{-6}, y=0.

In order to describe the Dirac energy levels of non-Kerr–Newman space-time directly, letting m=0.5, m_0 k=100, n=0.65, $e/m_0 \sim 10^7$, and taking α=0.25, β=0.5×10^{-6}, with different values of y one can get various Dirac energy levels, as shown in Figs.1–3. From Figs.1–3 one can learn how the energy levels vary with the value of y. Near the singular points, the energy levels are influenced greatly and there are some phenomena of crossing energy levels. At the equator and other points (such as y=0.45) there are also phenomena of crossing energy levels. These three figures show that the maximum of crossing energy levels varies with the value of y. However, in the crossing regions of the above figures there are some new situations. The crossing of energy levels leads to the occurrence of non-thermal radiation. In the negative state, the particles with energy $\varepsilon_- > m_0$ or $\varepsilon_-/m_0 > 1$ can go through the forbidden region by the quantum tunnelling effect and become outgoing radiation, which is a kind of non-thermal radiation process.[14–18] Especially in Fig.1, the maximum is not at x=1 but x >1, which is a new situation of non-Kerr–Newman black hole space-time. From Figs.1–3, their asymptotic behaviour, $\lim_{x\to\infty} \varepsilon_\pm/m_0 = \pm 1$, is the same. In these figures, β=0.5×10^{-6}, in fact, $0 \le \beta < 1$. When $\beta \to 1$, the maximum of crossing energy levels will increase greatly, which shows that the bigger $\beta(0 \le \beta < 1)$ is, the bigger the energy of non-thermal radiation. This is a significant, interesting and meaningful conclusion.[19–21] What we want to explain is that when y=–0.45, the figure is similar to Fig.2; when y=–0.99, the figure is similar to Fig.1. When α=0 and β=0, the curved space-time turns into the Schwarzschild case where the distribution of Dirac energy levels, as in Fig.4, is in accordance with well-known conclusions.

III. CONCLUSION

Based on Figs.1–3, the Dirac energy levels and their crossing regions are discussed. The particles in the crossing region can become outgoing radiation by the quantum tunnelling effect, so the above non-Kerr–Newman black hole has non-thermal radiation. Along with the outgoing radiation, the non-Kerr–Newman black hole will eventually degenerate into a Schwarzschild black hole. As the above calculation shows, Dirac energy level figures and their crossing regions can enrich the studies of non-thermal radiation. What needs to be explained is that the principle of cosmic censorship and the law of thermodynamics may prevent the appearance of singular points. According to the solution of the Einstein–Maxwell equation given by Manko and other researchers, new kinds of Dirac energy levels, their crossing regions and the associated non-thermal radiation may be meaningful in physics.[22–25]

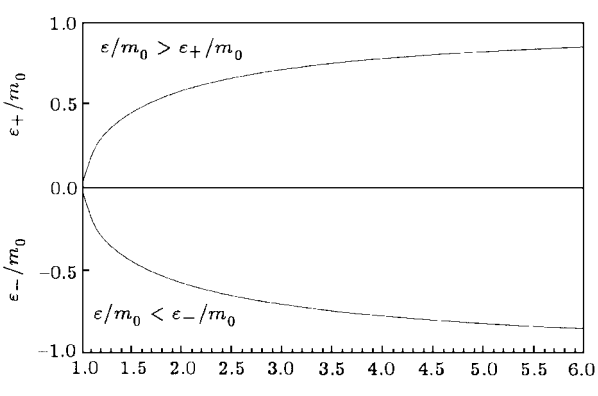


Fig.4. α=0, β=0.

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