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FILAMENTATION INSTABILITY OF LASER BEAMS IN NONLOCAL NONLINEAR MEDIA*

WEN SHUANG-CHUN(文双春)† and FAN DIAN-YUAN(范滇元)
National Laboratory on High Power Laser and Physics, Shanghai Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, Shanghai 201800, China

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The filamentation instability of laser beams propagating in nonlocal nonlinear media is investigated. It is shown that the filamentation instability can occur in weakly nonlocal self-focusing media for any degree of nonlocality, and in defocusing media for the input light intensity exceeding a threshold related to the degree of nonlocality. A linear stability analysis is used to predict the initial growth rate of the instability. It is found that the nonlocality tends to suppress filamentation instability in self-focusing media and to stimulate filamentation instability in self-defocusing media. Numerical simulations confirm the results of the linear stability analysis and disclose a recurrence phenomenon in nonlocal self-focusing media analogous to the Fermi–Pasta–Ulam problem.

Keywords: filamentation instability, nonlocal nonlinearity, nonlinear propagation
PACC: 4265J, 4265T, 4265S

I. INTRODUCTION

Filamentation instability (FI) occurs when an intense laser beam propagates in a third-order optical nonlinear medium, making the wave shape unstable to intensity modulations and breaking up the beam into a large number of filaments with a very high intensity which can eventually lead to local damage of the laser medium.[1] Thus FI is a key problem for the use of high-power laser beams. Theory on the filamentation process in local nonlinear media was well established by Bespalov and Talanov,[2] which basically considers the gain experienced by a small perturbation on a uniform plane wave. The theory is a linear one and can correctly predict the behaviour of the FI during the initial propagation. In this scope, the growth rate of filaments increases with the light intensity and becomes maximum for transverse wavenumber, usually called spatial frequency, proportional to the wave amplitude. These theoretical expectations received a clear experimental confirmation in the 1970s through nonlinear media served by, for example, CS_2 cells.[3] The so-called "angel-hair" damage often observed in high-power Nd:glass lasers used in the inertial confinement fusion (ICF), such as the SG-II laser facility in China and the national ignition facility (NIF) in USA,[4] also provided direct confirmation of the theory. It is worthwhile noting that FI also occurs in other physical systems, such as laser–plasma interactions,[5] light–atom interactions,[6] etc.

The purpose of this paper is to analyse FI in spatially nonlocal nonlinear media. A medium whose refractive index change Δn at a given point $\boldsymbol{r} = (x, y)$ is solely determined by the light intensity at $\boldsymbol{r}$ is called spatially local. More generally, a medium's refractive index change Δn at position $\boldsymbol{r}$ depends on the intensity at $\boldsymbol{r}$ as well as beyond $\boldsymbol{r}$; that is to say, a medium exhibits nonlocality to some extent. The influence of nonlocality clearly depends on the characteristic beam size relative to the characteristic size of the nonlocal response. The highly (weakly) nonlocal limit is realized when the characteristic size of the nonlocal response is much larger (smaller) than that of the beam. A number of materials which exhibit nonlocal nonlinear response have been reported. Examples of such materials include photorefractive materials,[7–13] liquid-crystalline materials,[14] and materials that exhibit laser-induced thermal nonlinearities,[15] etc. Theoretical and experimental investigations on light interactions with nonlocal nonlinear media have disclosed some new phenomena different from those with local ones. For instance, Snyder and Mitchell[16] have

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†E-mail: wenschun@sina.com.cn

shown that, in highly nonlocal nonlinear media, the beam evolution is described by the simple equation for a linear harmonic oscillator and thus "solitons were made simple".[17] In a photorefractive medium, Duree *et al.*[7] observed a new type of spatial soliton which is intensity independent. The influence of the nonlocality of the nonlinear response on the dynamics of beams propagation in plasmas[18] and in atomic vapours[19] have also been identified. To our knowledge, the phenomenon of FI in nonlocal nonlinear media has not been well studied. In this paper, we demonstrate, analytically and numerically, the existence of the phenomenon in both focusing and defocusing nonlocal media. Firstly, we derive, under the condition of weak nonlocality, a modified two-dimensional nonlinear Schrödinger equation (NLSE) which is valid for beam propagation in nonlocal materials reported to date; all of which have a correlation length of a few micrometres[17] and thus exhibit weak nonlocality. Secondly, we formulate the problem of filamentation instabilities in nonlocal self-focusing and defocusing media. Our theoretical formalism follows closely that of Bespalov and Talanov. Finally, we extend the linear stability analysis of the former section to the nonlinear regime by numerically solving the propagation equation.

II. GOVERNING EQUATION FOR BEAM PROPAGATION IN WEAKLY NONLOCAL NONLINEAR MEDIA

In the paraxial approximation, the propagation of a two-dimensional beam in nonlocal nonlinear media, after suitable normalization, can be modelled with the following NLSE[20,21]

$$\mathrm{i}\partial_z u + \frac{1}{2}\nabla_\perp^2 u + \sigma u \int R(\boldsymbol{r}' - \boldsymbol{r}) I(\boldsymbol{r}', z)\mathrm{d}\boldsymbol{r}' = 0, \quad (1)$$

where $u(r, z)$ is the normalized envelope of optical field and $I = |u|^2$ is its intensity; $\boldsymbol{r}$ and z are the transverse and propagation coordinates, respectively; $\nabla_\perp^2 = \partial_r^2 + r^{-1}\partial_r = \partial_x^2 + \partial_y^2$ is the transverse Laplacian, ∂_j denotes the partial derivative with respect to j. $R(\boldsymbol{r})$ is the response function of the nonlocal medium, which is real, symmetric, i.e. $R(\boldsymbol{r})$=$R(-\boldsymbol{r})$, and localized whose width determines the degree of nonlocality. In addition, $\sigma = 1$ (-1) corresponds to a self-focusing (defocusing) nonlinearity.

The integrodifferential Eq.(1) can be transformed into an ordinary differential equation by using the Taylor expansion of $I(\boldsymbol{r}', z)$ around the point $\boldsymbol{r}' = \boldsymbol{r}$:

$$\begin{aligned} I(\boldsymbol{r}', z) =& I(\boldsymbol{r}) + \nabla_\perp I(\boldsymbol{r}) \cdot (\boldsymbol{r}' - \boldsymbol{r}) \\ &+ \frac{1}{2}[\nabla_\perp I(\boldsymbol{r})\nabla_\perp I(\boldsymbol{r})] : (\boldsymbol{r}' - \boldsymbol{r})(\boldsymbol{r}' - \boldsymbol{r}) + \cdots \end{aligned} \quad (2)$$

Truncating the Taylor expansion to a given order requires that the nonlocal influence of the nonlinearity is restricted to a limited region of a given linear dimension. When the nonlocality is weak, i.e. when the response function $R(\boldsymbol{r})$ is narrow compared to the extent of the beam, we can truncate the Taylor expansion after the second order.[20] By doing so and noting that due to the invariance of $R(\boldsymbol{r})$ under the exchange, $\boldsymbol{r} = -\boldsymbol{r}$, the odd-order terms of Eq.(2) do not contribute to the third term of Eq.(1), we obtain the following modified NLSE

$$\mathrm{i}\partial_z u + \frac{1}{2}\nabla_\perp^2 u + \sigma[|u|^2 + \gamma\nabla_\perp^2(|u|^2)]u = 0, \quad (3)$$

where we have defined the nonlocality parameter

$$\begin{aligned} \gamma =& \frac{1}{2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R(x, y)x^2 \mathrm{d}x\mathrm{d}y \\ =& \frac{1}{2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R(x, y)y^2 \mathrm{d}x\mathrm{d}y > 0, \end{aligned}$$

and assumed that the response function is normalized, $\int R(\boldsymbol{r})\mathrm{d}\boldsymbol{r}$=1. For weakly nonlocal media $\gamma \ll 1$ is a small parameter. Note that for $R(\boldsymbol{r}) = \delta(\boldsymbol{r})$, $\gamma = 0$ and Eq.(3) reduced to two-dimensional standard NLSE describing the beam propagation in local Kerr media. Thus the weak nonlocality appears as a perturbation to the local nonlinear refractive index change.

III. LINEAR STABILITY ANALYSIS AND THE INFLUENCE OF NONLOCALITY ON FI GAIN SPECTRA

We consider the propagation of an optical beam through a nonlocal nonlinear medium. As is always done in FI theory, we express the optical field as a perturbed plane wave which is defined as the most elementary solution of Eq.(3)

$$u(r, z) = u_0[1 + a(r, z)]\mathrm{e}^{\mathrm{i}\sigma I_0 z}, \quad (4)$$

where u_0 and $I_0 = u_0^2$ denote the initial amplitude and intensity of the plane wave, respectively; while a is a small perturbation. Substituting Eq.(4) into Eq.(3) and using the linearizing approximation $|a| \ll 1$, we obtain the evolution equation for perturbation a as

follows:

$$\partial_z a = \mathrm{i}\left(\frac{1}{2} + \sigma\gamma I_0\right)\nabla_\perp^2 a + \mathrm{i}\sigma\gamma I_0 \nabla_\perp^2 a^* + \mathrm{i}\sigma I_0 (a + a^*). \quad (5)$$

We take the perturbation form as

$$a(r,z) = a_1 \mathrm{e}^{\mathrm{i}\vec{q}\cdot\vec{r}+gz} + a_2^* \mathrm{e}^{-\mathrm{i}\vec{q}\cdot\vec{r}+g^* z}, \quad (6)$$

where $\boldsymbol{q} = (q_x, q_y)$ is the transverse wave vector (its magnitude is usually called spatial frequency in FI theory), and g is the instability growth rate. Inserting Eq.(6) into Eq.(5) we obtains the instability determinant, $M = \{m_{lp}\}(l,p = 1,2)$, of the perturbed plane wave solution with

$$m_{11} = g + \mathrm{i}\left(\frac{1}{2} + \sigma\gamma I_0\right)q^2 - \mathrm{i}\sigma I_0,$$
$$m_{12} = \mathrm{i}\sigma\gamma I_0 q^2 - \mathrm{i}\sigma I_0,$$
$$m_{21} = -\mathrm{i}\sigma\gamma I_0 q^2 + \mathrm{i}\sigma I_0,$$
$$m_{22} = g - \mathrm{i}\left(\frac{1}{2} + \sigma\gamma I_0\right)q^2 + \mathrm{i}\sigma I_0.$$

The expression for the instability growth rate g produced by the determinant is

$$g = q\sqrt{\sigma I_0 - \left(\frac{1}{4} + \sigma\gamma I_0\right)q^2}. \quad (7)$$

For instability to occur, g must be strictly real and greater than zero. Thus we obtain the cut-off frequency for instability growing from Eq.(7)

$$q_\mathrm{c} = 2\sqrt{\frac{\sigma I_0}{1 + 4\sigma\gamma I_0}}. \quad (8)$$

Several interesting conclusions can be drawn from Eqs.(7) and (8). Firstly, we consider the case of nonlocal self-focusing media, i.e. $\sigma = 1$. In this case, the cut-off frequency q_c is the upper limit frequency for perturbation growth and it is always larger than zero for all values of optical intensity I_0 and non-negative nonlocality parameter γ. Thus FI can always occur, as in local media, in nonlocal focusing ($\sigma = 1$) media. In the case when the cw solutions are unstable, it is interesting to find a particular mode that provides for a maximum instability growth rate. Such a perturbation mode is most risky in the process of beam filamentation. To this end, we set $\sigma = 1$ and evaluate the frequency of the mode (usually called the fastest growing frequency) and the corresponding maximum growth rate from Eq.(7), which are

$$q_m = \sqrt{\frac{2I_0}{1 + 4\gamma I_0}}, \quad (9)$$

$$g_m = I_0/\sqrt{1 + 4\gamma I_0}. \quad (10)$$

For local nonlinearity ($\gamma = 0$), Eqs.(7)–(10) reduce to the expressions for FI in self-focusing Kerr media.[2,22–24]. Obviously, nonlocality exerts fundamental influences on the properties of FI gain spectra, as illustrated in Eqs.(7)–(10) as well as in Fig.1 which shows the variation of the perturbation growth rate with the spatial frequency for different values of nonlocality parameters. We clearly see from Fig.1 that, as the nonlocality parameter increases, the maximum growth rate, the fastest growth and the cut-off frequencies decrease. Thus the nonlocality tends to suppress FI. This tendency is also illustrated in Fig.2 which shows how the cut-off frequency for perturbation growth tends to zero, which means the FI bandwidth shrinks, as the nonlocality parameter increases. Thus we can expect that there should be no FI in a highly nonlocal self-focusing media.

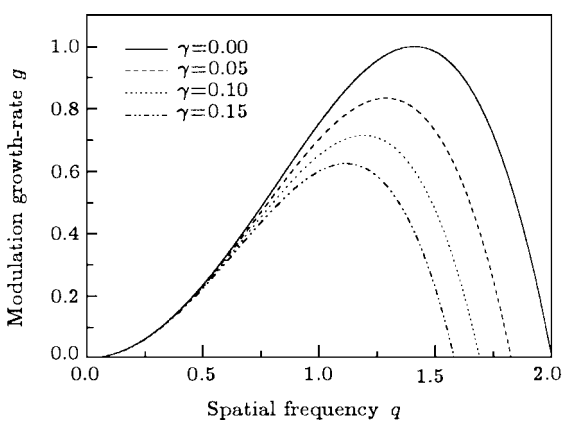


Fig.1. Gain spectra of the filamentation instability in self-focusing media for different nonlocalities. We have used $I_0 = 1.0$.

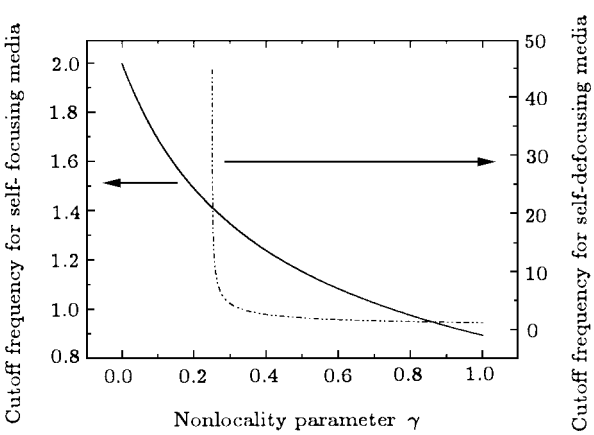


Fig.2. Variations of the upper limit frequency for FI in self-focusing media and the lower limit frequency for FI in self-defocusing media with the nonlocality parameter for $I_0 = 1.0$.

Let us now see what happens in self-defocusing media by setting ($\sigma = -1$). Recall that, in the local case, FI never occurs in self-defocusing media. There are changes in the nonlocal case. As seen from Eq.(7), whether or not the perturbations grow depends sensitively on optical intensity I_0 as well as the nonlocality parameter γ. There is a critical value $I_0^{\rm cr}$ of the intensity,

$$I_0^{\rm cr} = 1/(4\gamma), \qquad (11)$$

for $I_0 > I_0^{\rm cr}$. FI may appear, since for sufficiently large frequency $q > q_{\rm c}$, $g > 0$. Fig.3 shows the dependence of the growth rate g on the spatial frequency q and the nonlocality parameter γ. We see that, for $q > q_{\rm c}$, the growth rate increases rapidly with the spatial frequency; and with the increase of the nonlocality parameter the cut-off frequency shifts to zero. The cut-off frequency shift is also clearly shown in Fig.2. Note that the cut-off frequency in the current case is the lower limit frequency for perturbation growth, thus in sharp contrast to the self-focusing case, the nonlocality tends to promote FI in self-defocusing media.

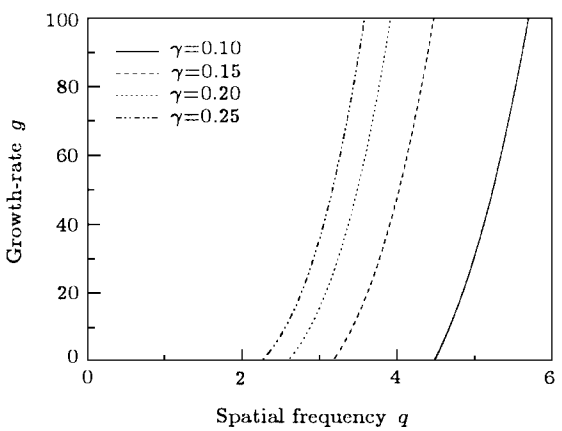


Fig.3. Variations of the growth rate for FI in self-defocusing media with the spatial frequency for different nonlocality parameters. We have used $I_0 = 5.0$.

IV. NUMERICAL SIMULATIONS

To verify the linear stability analysis detailed above and to predict the evolution of FI beyond the applicability of the linear analysis, we present here a numerical analysis. We solve Eq.(3) numerically, utilizing the two-dimensional split-step Fourier method.[25] The initial field distribution at the entrance to the nonlocal nonlinear medium is a sinusoidally modulated plane wave,

$$u = \sqrt{I_0}[1 + \varepsilon_1 \sin(q_x x)][1 + \varepsilon_2 \sin(q_y y)], \qquad (12)$$

with ε_1 and ε_2 being the amplitudes of the perturbation. In general, the perturbation should be in the form of a product $\sin(q_x x)\sin(q_y y)$. We have chosen the form of Eq.(12) because it results in the smallest spatial dimensions of a unit cell when periodic boundary conditions are imposed. In all cases we set $q_x = q_y$, and their values are chosen such that $q = q_m$ in self-focusing media, and $q > q_{\rm c}$ in self-defocusing media.

Figure 4 shows the result obtained for self-focusing media. It shows the evolution of the normalized intensity profile on a transverse dimension, i.e. $I(x,0,z)/I_0$, with the propagation distance. The input intensity and the nonlocality parameter are set to 4.0 and 0.02, respectively. Thus, according to Eq.(9), the fastest growing frequency $q_m = 2.5$. As can be seen in Fig.4, the initial perturbation is amplified until most of the energy is transferred from the pump plane wave into the sidebands, indicating that the perturbation rapidly grows during the initial propagation, breaking up the plane wave into a sequence of filaments. At that point, the reverse process takes place, and energy flows back into the pump until the initial state is recovered, and so on. Thus, FI exists as well as an analogue of the Fermi–Pasta–Ulam recurrence phenomenon[26] in nonlocal nonlinear self-focusing media.

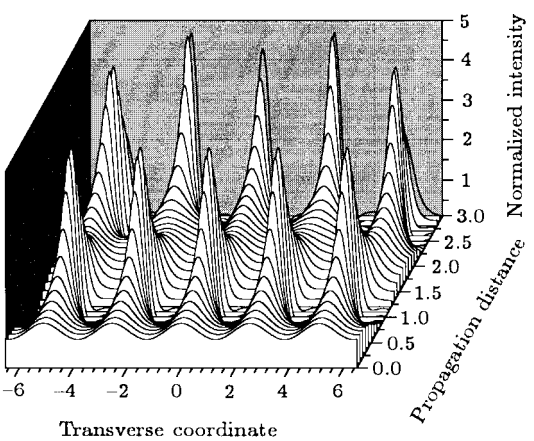


Fig.4. Evolution in nonlocal self-focusing media of the normalized intensity profile $I(x,0,z)/I_0$ for $I_0 = 4.0$, $\gamma = 0.02$, $\varepsilon_1 = \varepsilon_2 = 0.1$, $q = 2.5$.

The linear stability analysis obtained above predicted that the perturbation, if the conditions for FI occurring are satisfied, grows much more rapidly in a self-defocusing nonlocal media than in a self-focusing one. To this end, we set the initial amplitude of the perturbation to be a relatively very small value and propagate the modulated plane wave to a very short distance in self-focusing media. In Fig.5 we show the evolution of the normalized intensity profile with the

propagation distance. It clearly shows that the modulated plane wave rapidly breaks up into numerous filaments with very high intensity. Unlike those in the self-focusing case, the filaments exhibit somewhat chaotic characteristics in the self-defocusing media. In addition, we have observed the long-term behaviour of the filamentation process. We find that as the propagation distance further increases, the beam eventually breaks up; no phenomenon of recurrence exists in the nonlocal self-defocusing media.

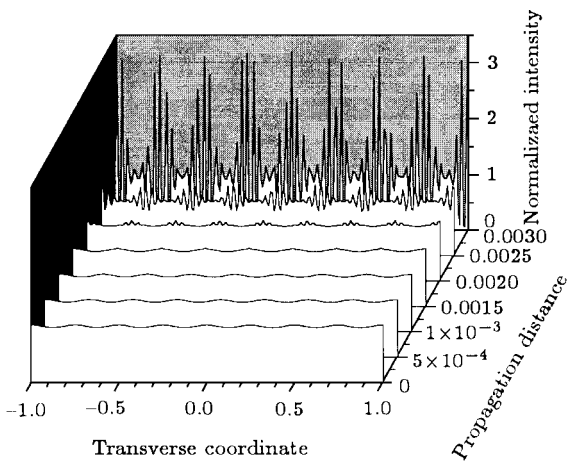


Fig.5. Evolution in nonlocal self-defocusing media of the normalized intensity profile $I(x,0,z)/I_0$ for $I_0 = 4.0$, $\gamma = 0.1$, $\varepsilon_1 = \varepsilon_2 = 0.01$, $q = 8\pi$.

V. CONCLUSIONS

We have analysed the stability of the modulated plane wave in nonlocal nonlinear media. We have shown that, as in local self-focusing Kerr media, FI always occurs in nonlocal self-focusing media; yet the nonlocality reduces the maximum growth-rate, the fastest frequency and the cut-off frequency of the perturbations, exhibiting a tendency to suppress the instability. More interesting is that FI may occur also in nonlocal self-defocusing media if the input light intensity is larger than a threshold related to the degree of nonlocality. In contrast to the former case, nonlocality tends to stimulate FI in self-defocusing media.

On the other hand, we have numerically studied the FI process. Numerical simulation confirms the analytical results and shows that a laser beam propagating in a nonlocal self-focusing medium will experience a periodic filamentation process such as the Fermi–Pasta–Ulam recurrence; while in a nonlocal self-defocusing medium it will immediately break up into numerous filaments and exhibit a chaotic behaviour.

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