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OPTICAL REALIZATION OF WAVELET TRANSFORM WITH A SINGLE LENS

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Two optical set-ups to implement wavelet transform with a single lens have been proposed, in which the wavelet filter was placed in front of the imaging lens or on the frequency plane. The general formula of the complex field distribution of the output plane has been deduced. The analysing wavelet functions of the band-pass wavelet filters with double and circular slits have been discussed.

Keywords: Fourier transform, wavelet transform, wavelet filter
PACC: 4230, 4225

I. INTRODUCTION

Wavelet transform (WT) has been widely employed as a powerful analysis tool, such as in transient signal and image processing,[1−6] optical correlation,[7] and the presentation of fractal objects,[8−10] due to its performance for multiple resolution.[11] Actually, WT is a convolution of signal and wavelets; thus it can be realized with an optical set-up, both in the space domain by use of the shadow-casting system and in the spatial frequency domain by use of the optical wavelet filtering system.[7,10,12−15] Basically, a great deal of optical set-ups reported in the pioneering work are composed of at least two lenses.

In this paper, we discuss the analysing wavelet functions of the one-dimensional (1D) double-slit and two-dimensional (2D) rectangular aperture and circular slit pure-intensity band-pass wavelet filters, and introduce two methods to realize the optical wavelet transform (OWT) with a single lens, in which the wavelet filter is placed in front of the imaging lens or on the frequency plane.

II. THEORETICAL ANALYSIS

A. Double slit and circular slit band-pass wavelet filters

Several kinds of wavelet functions are often discussed in the WT, such as Morlet wavelets, Harr wavelets and Maxcan-hat wavelets. A kind of wavelet is often used in OWT, due to the Fourier transforms of these wavelets having the characteristic of pure-intensity and band-pass.

The 1D band-pass wavelet filter can be written as the form of double-slit:

$$P_s(y) = \mathrm{rect}(y/b_1) - \mathrm{rect}(y/b_2) \qquad (b_1 > b_2 > 0)$$

$$= \begin{cases} 1, & b_2 \le y \le b_1 \\ 0, & \text{other,} \end{cases} \tag{1}$$

and the Fourier transform (FT) of $P_s(y)$ (using the notation $\mathcal{F}$) is:

$$\mathcal{F}\{P_s(y)\} = b_1\mathrm{sinc}(b_1\eta) - b_2\mathrm{sinc}(b_2\eta), \tag{2}$$

A 2D rectangular aperture band-pass wavelet filter can be expressed by $P_s(x)\ P_s(y)$, and its Fourier transform is

$$\mathcal{F}\{P_s(x)P_s(y)\} = [b_1\mathrm{sinc}(b_1\xi) - b_2\mathrm{sinc}(b_2\xi)] \cdot [b_1\mathrm{sinc}(b_1\eta) - b_2\mathrm{sinc}(b_2\eta)]. \tag{3}$$

While a circular slit band-pass wavelet filter is expressed as:

$$P_{cs}(r) = \mathrm{circ}(r/b_1) - \mathrm{circ}(r/b_2) \qquad (b_1 > b_2 > 0)$$

$$= \begin{cases} 1, & b_2 \le r \le b_1 \\ 0, & \text{other.} \end{cases} \tag{4}$$

The Fourier–Bessell transform of $P_{cs}(r)$ (by the notation $\mathcal{B}$) is

$$\mathcal{B}\{P_{cs}(r)\} = b_1 J_1(b_1\rho)/\rho - b_2 J_2(b_2\rho)/\rho. \qquad (5)$$

Obviously, both $\mathcal{F}\{P_s(y)\}$ and $\mathcal{B}\{P_{cs}(r)\}$ are damped oscillating functions, and their mean values are equal to zero. Thus, the functions $\mathcal{F}\{P_s(y)\}$ and $\mathcal{B}\{P_{cs}(r)\}$ satisfy the admissibility condition of the analysing wavelet.

B. The wavelet filter placed in front of the lens

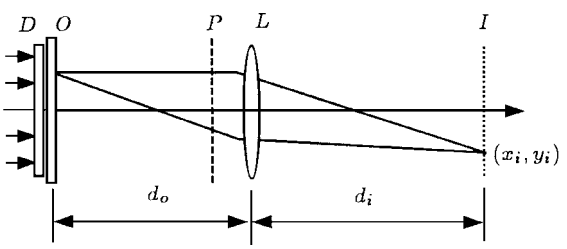


Fig.1. The geometry of OWT with the wavelet filter in front of the lens: D, diffuser; O, object; P, wavelet filter; L, imaging lens; I, image

Figure 1 is the geometry of OWT with the wavelet filter aperture P placed in front of the imaging lens L, and with a diffused illumination. d_o and d_i are the object and the image distances respectively, and r represents the distance between the aperture P and the imaging lens L. For an arbitrary point object $\delta(x_o, y_o)$ on the object plane, the complex field distribution on the back surface of the aperture can be expressed as

$$\begin{aligned} U_a(x_a, y_a) =& [1/\mathrm{j}\lambda(d_o - r)]\exp\{\mathrm{j}k(d_o - r)\} \\ & \cdot \exp\{\mathrm{j}k[(x_a - x_o)^2 \\ & + (y_a - y_o)^2]/2(d_o - r)\} P(x_a, y_a), \end{aligned} \qquad (6)$$

where $P(x_a, y_a)$ is the transmittance function of the aperture. The field distribution on the back surface of the imaging lens is

$$\begin{aligned} U_l(x_l, y_l) =& (1/j\lambda r)\exp(\mathrm{j}kr) \iint U_a(x_a, y_a) \\ & \cdot \exp\{\mathrm{j}k[(x_l - x_a)^2 + (y_l - y_a)^2]/2r\} \\ & \cdot \exp\{\mathrm{j}k(x_l^2 + y_l^2)/2f\}\mathrm{d}x_a\mathrm{d}y_a, \end{aligned} \qquad (7)$$

where f is the focal length of the lens, and the field distribution on the image plane is

$$\begin{aligned} & U_i(x_i, y_i) \\ =& (1/\mathrm{j}\lambda d_i)\exp(\mathrm{j}kd_i) \iint U_l(x_l, y_l) \\ & \cdot \exp\{\mathrm{j}k[(x_i - x_l)^2 + (y_i - y_l)^2]/2d_i\}\mathrm{d}x_l\mathrm{d}y_l \\ =& [1/(\mathrm{j}\lambda)^3(d_o - r)rd_i]\exp\{\mathrm{j}k(d_o + d_i)\} \\ & \cdot \iiiint \exp\{\mathrm{j}k[(x_a - x_o)^2 \\ & + (y_a - y_o)^2]/2(d_o - r)\} P(x_a, y_a) \\ & \cdot \exp\{\mathrm{j}k[(x_l - x_a)^2 + (y_l - y_a)^2]/2r\} \\ & \cdot \exp\{\mathrm{j}k(x_l^2 + y_l^2)/2f\} \\ & \cdot \exp\{\mathrm{j}k[(x_i - x_l)^2 + (y_i - y_l)^2]/2d_i\}\mathrm{d}x_a\mathrm{d}y_a\mathrm{d}x_l\mathrm{d}y_l \\ =& A\Phi \iint P(x_a, y_a)\exp\{-\mathrm{j}2\pi[(x_i + Mx_o)x_a \\ & + (y_i + My_o)y_a]/\lambda d'\}\mathrm{d}x_a\mathrm{d}y_a \\ =& (\lambda d')^2 A\Phi \iint P(\lambda d'\tilde{x}_a, \lambda d'\tilde{y}_a) \\ & \cdot \exp\{-\mathrm{j}2\pi[(x_i + Mx_o)\tilde{x}_a + (y_i + My_o)\tilde{y}_a]\}\mathrm{d}\tilde{x}_a\mathrm{d}\tilde{y}_a \\ =& h(x_i + Mx_o, y_i + My_o), \end{aligned} \qquad (8)$$

where, $1/d_o + 1/d_i = 1/f$, $\tilde{x}_a = x_a/\lambda d'$, $\tilde{y}_a = y_a/\lambda d'$, A is a constant, Φ is a phase factor, M is the magnification of the system, and d' is an effective length. These are defined, respectively, as follows

$$\begin{aligned} A =& [1/(\mathrm{j}\lambda)^3(d_o - r)rd_i]\exp\{\mathrm{j}k(d_o + d_i)\} \\ & \cdot \iint \exp\{\mathrm{j}k(x^2 + y^2)/d'\}\mathrm{d}x\mathrm{d}y, \quad (r > 0) \end{aligned} \qquad (9)$$

$$\begin{aligned} \Phi =& \exp\{\mathrm{j}k(x_o^2 + y_o^2)/2(d_o - r)\} \\ & \cdot \exp\{\mathrm{j}k[(1 - d_o/f)(x_i^2 + y_i^2)/2d']\}, \end{aligned} \qquad (10)$$

$$M = [f(d_i + r) - d_i r]/f(d_o - r) = d_i/d_o, \qquad (11)$$

$$d' = r + d_i(f - r)/f. \qquad (12)$$

Equation (8) shows the response on the image plane of a point object $\delta(x_o, y_o)$. For the object field $O(x_o, y_o)$, its image field distribution is

$$\begin{aligned} U_i(x_i, y_i) =& \iint U_o(x_o, y_o)h(x_i + Mx_o, \\ & y_i + My_o)\mathrm{d}x_o\mathrm{d}y_o \\ =& (1/M)U_o(-x_i/M, -y_i/M) * h(x_i, y_i), \end{aligned} \qquad (13a)$$

in which, the notation $*$ represents convolution, $(1/M)U_o(-x_i/M, -y_i/M)$ represents the field distribution obtained from geometrical optics, and $h(x_i, y_i)$ is the impulse response function

$$\begin{aligned} h(x_i,y_i) =&(1/M)h(x_i,y_i)\\ =&A'\iint P(\lambda d'\tilde{x}_a,\lambda d'\tilde{y}_a)\\ &\cdot\exp\{-\mathrm{j}2\pi(x_i\tilde{x}_a+y_i\tilde{y}_a)\}\mathrm{d}\tilde{x}_a\mathrm{d}\tilde{y}_a\\ =&A'\mathcal{F}\{P(\lambda d'\tilde{x}_a,\lambda d'\tilde{y}_a)\}\big|_{\bar{x}=x_a/\lambda d',\bar{y}=y_a/\lambda d'}, \end{aligned} \tag{14}$$

where $A'=(1/M)(\lambda d')^2A\Phi$. Equations (13) and (14) indicate that the impulse response will be a wavelet function, and the field distribution on the image plane will be a convolution of the geometrical optical field and the wavelet, if the FT of the aperture equals an analysing wavelet.

Equations (10)–(14) still hold but with a corrected constant A, when the aperture is placed on the surface of the imaging lens (r=0).[16]

For example, if the 1D double-slit band-pass wavelet filter $P_s(y)$ is employed and placed in front of the imaging lens, then the field distribution on the image plane will be

$$\begin{aligned} U_i(x_i,y_i) =&A'U_o(-x_i/M,-y_i/M)*[b_1\mathrm{sinc}(b_1\eta)\\ &-b_2\mathrm{sinc}(b_2\eta)]\big|_{\eta=y/\lambda d'}; \end{aligned} \tag{13b}$$

while the aperture is placed right on the surface of the lens (r=0), and $d_o=d_i$. Then, $d'=d_i=2f, M=1$, and

$$\begin{aligned} U_i(x_i,y_i) =&A'(U_o(-x_i,-y_i)*[b_1\mathrm{sinc}(b_1\eta)\\ &-b_2\mathrm{sinc}(b_2\eta)]\big|_{\eta=y/2\lambda f}. \end{aligned} \tag{13c}$$

C. The wavelet filter placed on the frequency plane

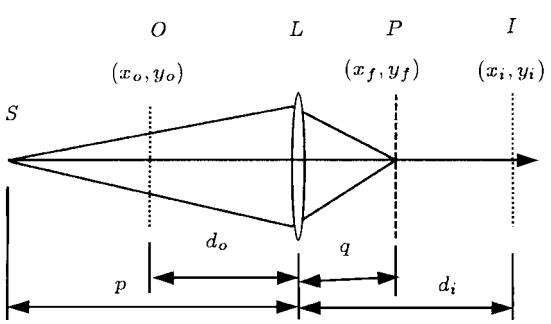


Fig.2. The geometry of OWT with the wavelet filter on the frequency plane: S, point illuminating source; O, object; L, imaging lens; P, wavelet filter; I, image

Figure 2 shows the geometry of OWT with the wavelet filter placed on the frequency plane, and with a point source illumination. The image plane of the point source is the frequency plane of the 2D transparency object, and the complex field distribution on the frequency plane $U_f(x_f,y_f)$ can be obtained by using Eq.(8). With the following substitutions of the parameters—$p\to d_o$, $d_o\to r$, $q\to d_i$, $d''\to d'$, $(0,0)\to(x_o,y_o)$, $T(x_o,y_o)\to P(x_a,y_a)$, $U_f(x_f,y_f)\to U_i(x_i,y_i)$—the field distribution is given by

$$\begin{aligned} U_f(x_f,y_f) =&B\exp\{\mathrm{j}k(1-p/f)(x_f^2+y_f^2)/2d''\}\\ &\cdot\iint T(x_o,y_o)\exp\{-\mathrm{j}2\pi(x_fx_o\\ &+y_fy_o)/\lambda d''\}\mathrm{d}x_o\mathrm{d}y_o, \end{aligned} \tag{15}$$

where $T(x_o,y_o)$ is the transmittance of the transparent object, and

$$d''=d_o+q(f-d_o)/f, \tag{16}$$

$$\begin{aligned} B =&[1/(\mathrm{j}\lambda)^3(p-d_o)d_oq]\exp\{\mathrm{j}k(p+q)\}\\ &\cdot\iint\exp\{\mathrm{j}k(x^2+y^2)/d''\}\mathrm{d}x\mathrm{d}y. \end{aligned} \tag{17}$$

As the band-pass filter P is placed on the frequency plane, the field distribution on the back plane of the filter P is

$$U'_f((x_f,y_f)=U_f(x_f,y_f)P(x_f,y_f). \tag{18}$$

Then the field distribution on the image plane of the transparency object is

$$\begin{aligned} &U_i(x_i,y_i)\\ =&[1/\mathrm{j}\lambda(d_o-q)]\iint U'_f(x_f,y_f)\\ &\cdot\exp\{\mathrm{j}k[(x_i-x_f)^2+(y_i-y_f)^2]/2(d_i-q)\}\mathrm{d}x_f\mathrm{d}y_f\\ =&B'\iint P(x_f,y_f)\Big[\iint T(x_o,y_o)\\ &\cdot\exp\{-\mathrm{j}2\pi(x_fx_o+y_fy_o)/\lambda d''\}\mathrm{d}x_o\mathrm{d}y_o\Big]\\ &\cdot\exp\{-\mathrm{j}2\pi(x_ix_f+y_iy_f)/M\lambda d''\}\mathrm{d}x_f\mathrm{d}y_f\\ =&B'(\lambda d'')^2\iint P(-M\lambda d''\tilde{x}_f,-M\lambda d''\tilde{y}_f)\\ &\cdot\Big[\iint T(\tilde{x}_o/M,\tilde{y}_o/M)\\ &\cdot\exp\{-\mathrm{j}2\pi(\tilde{x}_f\tilde{x}_o+\tilde{y}_f\tilde{y}_o)\}\mathrm{d}\tilde{x}_o\mathrm{d}\tilde{y}_o\Big]\\ &\cdot\exp\{-\mathrm{j}2\pi(x_i\tilde{x}_f+y_i\tilde{y}_f)/M\lambda d''\}\mathrm{d}\tilde{x}_f\mathrm{d}\tilde{y}_f\\ =&B'(\lambda d'')^2\mathcal{F}\{P(-M\lambda d''\tilde{x}_f,\\ &-M\lambda d''\tilde{y}_f)\mathcal{F}\{T(\tilde{x}_o/M,\tilde{y}_o/M)\}\}\\ =&B'(\lambda d'')^2T(-\tilde{x}_i/M,-\tilde{y}_i/M)*\mathcal{F}\{P(-M\lambda d''\tilde{x}_f,\\ &-M\lambda d''\tilde{y}_f)\}\big|_{\bar{x}=x_f/M\lambda d'',\bar{y}=y_f/M\lambda d''}, \end{aligned} \tag{19a}$$

in which the convolution theorem of FT is used, $\tilde{x}_o=Mx_o$, $\tilde{y}_o=My_o$, $1/p+1/q=1/f$, $1/d_o+1/d_i=1/f$,

and $M = d_o/f - 1 = d_o/d_i$, $B' = [B/j\lambda(d_o - q)]\exp\{jk(x_i^2 + y_i^2)/2(d_i - q)\}$. It is obvious that the form of Eq.(19) is similar to that of Eq.(13a).

For example, with the 1D double-slit wavelet filter $P_s(y)$ placed on the frequency plane, the field distribution on the image plane is expressed as

$$U_i(x_i, y_i) = (B'\lambda d'')T(-x_i/M, -y_i/M) * [b_1\mathrm{sinc}(b_1\eta) - b_2\mathrm{sinc}(b_2\eta)]\big|_{\eta=y/M\lambda d''}. \quad (19b)$$

Assuming that the illumination source is a plane wave (p=0, $q = f$), and $d_o = d_i$, then we have $d'' = f$, $M = 1$, and

$$U_i(x_i, y_i) = (B'\lambda d'')T(-x_i, -y_i) * [b_1\mathrm{sinc}(b_1\eta) - b_2\mathrm{sinc}(b_2\eta)]\big|_{\eta=y/\lambda f}. \quad (19c)$$

III. CONCLUSIONS

In summary, the double-slit and the circular slit wavelet filters are types of ideal band-pass optical filters; the FT of the former is the difference of two sinc functions and that of the latter is the difference of two Bessell functions, both satisfying the admissibility condition of the analysing wavelet.

WT could be realized by the optical coherent imaging system with a single lens, in which the analysing wavelet function is implemented by the FT of the optical band-pass wavelet filter placed in front of the imaging lens or on the spatial frequency plane. Accordingly, the complex field distribution on the output plane can be deduced for the two cases.

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