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TELEPORTATION OF A TWO-PARTICLE ENTANGLED STATE*

YE LIU(叶 柳)[a)b)], YAO CHUN-MEI(姚春梅)[a)c)], and GUO GUANG-CAN(郭光灿)[a)]
[a)] *Laboratory of Quantum Communication and Quantum Computation, University of Science and Technology of China, Hefei 230026, China*
[b)] *Department of Physics, Anhui University, Hefei 230039, China*
[c)] *Department of Physics, Changde Normal University, Changde 415000, China*

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A scheme for teleporting a two-particle entangled state via a three-particle entangled state is proposed. It is shown that the probability of successful teleportation is twice the modulus square of the smaller Schmidt coefficient of the entangled three-particle state.

Keywords: teleportation, entangled state, probability
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I. INTRODUCTION

Teleportation, proposed by Bennett *et al.*,[1] is a process that allows the transmission of an unknown state from a sender (Alice) to a receiver (Bob). For the teleportation of a quantum state, Alice and Bob must share an Einstein–Podolsky–Rosen (EPR) pair. Alice carries out a joint Bell-state measurement on her EPR particle and the unknown quantum system, and sends her measured result to Bob, who in turn can perform a unitary operation on his particle to obtain the original state. Based on the quantum teleportation protocol, quantum teleportation of a photon has been demonstrated experimentally by Bouwmeester *et al.*[2] and Boschi *et al.*[3] using parametric down-conversion. Davidovich *et al.*[4] have presented a scheme for the teleportation of an unknown atomic state between two high-Q cavities initially prepared in an entangled photon-number state. Cirac *et al.*[5] and other authors[6−12] have made other proposals for the quantum teleportation of an atomic state or field state. Gorbachev *et al.*[13] proposed a scheme of quantum teleportation of a two-particle entangled state, where a three-particle Greenberger–Horne–Zeilinger (GHZ) state is used as the communication channel. In the proposal of standard teleportation, if the entangled state used as the quantum channel is not maximally entangled, the faithful teleportation is not possible. However, in practice, maximally entangled states are always difficult to prepare. So teleporting a quantum state using partly entangled states is of practical importance. Recently Li *et al.*[14] have proposed a scheme to teleport the quantum state by using one pair of entangled (but not maximally) two-state particles. Feng *et al.*[15] have made another scheme for the realization of the quantum teleportation of an atomic state by using two partly entangled states. Lu *et al.*[16] have shown a scheme of teleporting a two-particle entangled state. In their proposal, two pairs of entangled particles are used as the quantum channel. Shi *et al.*[17] presented the scheme of probabilistic teleportation of a two-particle entangled state by using a three-particle entangled state. In this paper, we propose another scheme to realize the teleportation of a two-particle entangled state, where two kinds of quantum channels are considered.

II. TELEPORTATION OF A TWO-PARTICLE ENTANGLED STATE BY A THREE-PARTICLE GHZ STATE

Firstly we set up a three-particle GHZ state to be used as quantum channel between Alice and Bob, which is in the following state

$$|\psi\rangle_{123} = \frac{1}{\sqrt{2}}(|000\rangle_{123} + |111\rangle_{123}). \quad (1)$$

We suppose Alice has an entangled particle pair, which consists of particles 4 and 5. She wants to teleport the unknown state $|\psi\rangle_{45}$ of the particle pair to Bob. The state $|\psi\rangle_{45}$ may be expressed as

$$|\psi\rangle_{45} = \alpha|00\rangle_{45} + \beta|11\rangle_{45}, \quad (2)$$

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where $|\alpha|^2+|\beta|^2=1$. Particle 3 of the state $|\psi\rangle_{123}$ belongs to Alice, and the other two particles 1 and 2 belong to Bob.

The total state can be written as

$$\begin{aligned}|\psi\rangle =&|\psi\rangle_{123}\otimes|\psi\rangle_{45}\\ =&\frac{1}{2\sqrt{2}}[(|000\rangle_{345}+|111\rangle_{345})\\ &\cdot(\alpha|00\rangle_{12}+\beta|11\rangle_{12})\\ &+(|000\rangle_{345}-|111\rangle_{345})(\alpha|00\rangle_{12}-\beta|11\rangle_{12})\\ &+(|011\rangle_{345}+|100\rangle_{345})(\beta|00\rangle_{12}+\alpha|11\rangle_{12})\\ &+(|011\rangle_{345}-|100\rangle_{345})(\beta|00\rangle_{12}-\alpha|11\rangle_{12})]\\ =&\frac{1}{2\sqrt{2}}[|\Phi^+\rangle_{345}(\alpha|00\rangle_{12}+\beta|11\rangle_{12})\\ &+|\Phi^-\rangle_{345}(\alpha|00\rangle_{12}-\beta|11\rangle_{12})\\ &+|\Psi^+\rangle_{345}(\beta|00\rangle_{12}+\alpha|11\rangle_{12})\\ &+|\Psi^-\rangle_{345}(\beta|00\rangle_{12}-\alpha|11\rangle_{12})].\end{aligned}$$

Now, if Alice can perform a suitable measurement on her particles 3, 4 and 5, then Bob need only make standard rotations on his particles 1 and 2 to construct the unknown entangled state after he receives Alice's measurement result. The task is now to specify the kind of measurement that Alice should perform on her three qubits. First, Alice performs a controlled-NOT (CNOT) operation on particles 3 and 4, then a CNOT operation on particles 3 and 5. Afterwards she makes a Hadamard transformation, which transforms a quantum basis state into the superposition of two basis states, on particle 5 ($H|0\rangle=(|0\rangle+|1\rangle)/\sqrt{2}$, $H|1\rangle=(|0\rangle-|1\rangle)/\sqrt{2}$. Thus, the four measurement results become the following states

$$|\Phi^\pm\rangle_{345}\to(|00\rangle_{35}\pm|11\rangle_{35})|0\rangle_4\to(|0\rangle_3\pm|1\rangle_3)|0\rangle_4|0\rangle_5\to\begin{cases}|000\rangle_{345}\\ |100\rangle_{345},\end{cases}$$

$$|\Psi^\pm\rangle_{345}\to(|01\rangle_{35}\pm|10\rangle_{35})|1\rangle_4\to(|0\rangle_3\pm|1\rangle_3)|1\rangle_4|1\rangle_5\to\begin{cases}|011\rangle_{345}\\ |111\rangle_{345}.\end{cases}$$

Hence, the joint measurement can be achieved by detecting particles 3, 4 and 5 separately. With the outcome of the measurement on Alice's three particles transmitted to Bob, an appropriate rotation of particles 1 and 2 can be performed to reconstruct the initial entangled state; that is, teleportation is achieved.

III. TELEPORTATION OF A TWO-PARTICLE ENTANGLED STATE BY A THREE-PARTICLE ENTANGLED STATE

We consider that the pair of entangled particles which Alice wants to teleport is still in state (2), but the entangled particles used as the quantum channel are not maximally entangled, but prepared in the following state

$$|\psi\rangle_{123}=a|000\rangle_{123}+b|111\rangle_{123},\tag{3}$$

where $|a|^2+|b|^2=1$, $|a|>|b|$.

First Alice performs a measurement of particles 3 and 4 on the Bell basis, then the state of particles 1, 2 and 5 will collapse into one of the following states

$$\langle\Phi^\pm|_{34}\psi\rangle_{123}\otimes|\psi\rangle_{45}=\frac{\alpha a}{\sqrt{2}}|000\rangle_{125}\pm\frac{\beta b}{\sqrt{2}}|111\rangle_{125},\tag{7}$$

$$\langle\Psi^\pm|_{34}\psi\rangle_{123}\otimes|\psi\rangle_{45}=\frac{\beta a}{\sqrt{2}}|001\rangle_{125}\pm\frac{\alpha b}{\sqrt{2}}|110\rangle_{125},\tag{8}$$

$$|\Phi^\pm\rangle_{34}=\frac{1}{\sqrt{2}}(|00\rangle_{34}\pm|11\rangle_{34}),$$

where

$$|\Psi^\pm\rangle_{34}=\frac{1}{\sqrt{2}}(|01\rangle_{34}\pm|10\rangle_{34}).$$

Equations (7) and (8) also can be written as

$$\begin{aligned}&\langle\Phi^+|_{34}\psi\rangle_{123}\otimes|\psi\rangle_{45}\\ =&\frac{1}{2\sqrt{2}}[(a|0\rangle_5+b|1\rangle_5)(\alpha|00\rangle_{12}+\beta|11\rangle_{12})\\ &+(a|0\rangle_5-b|1\rangle_5)(\alpha|00\rangle_{12}-\beta|11\rangle_{12})],\end{aligned}$$

$$\begin{aligned}&\langle\Phi^-|_{34}\psi\rangle_{123}\otimes|\psi\rangle_{45}\\ =&\frac{1}{2\sqrt{2}}[(a|0\rangle_5-b|1\rangle_5)(\alpha|00\rangle_{12}+\beta|11\rangle_{12})\\ &+(a|0\rangle_5+b|1\rangle_5)(\alpha|00\rangle_{12}-\beta|11\rangle_{12})],\end{aligned}$$

$$\begin{aligned}&\langle\Psi^+|_{34}\psi\rangle_{123}\otimes|\psi\rangle_{45}\\ =&\frac{1}{2\sqrt{2}}[(a|1\rangle_5+b|0\rangle_5)(\beta|00\rangle_{12}+\alpha|11\rangle_{12})\\ &+(a|1\rangle_5-b|0\rangle_5)(\beta|00\rangle_{12}-\alpha|11\rangle_{12})],\end{aligned}$$

$$\begin{aligned}&\langle\Psi^-|_{34}\psi\rangle_{123}\otimes|\psi\rangle_{45}\\ =&\frac{1}{2\sqrt{2}}[(a|1\rangle_5-b|0\rangle_5)(\beta|00\rangle_{12}+\alpha|11\rangle_{12})\\ &+(a|1\rangle_5+b|0\rangle_5)(\beta|00\rangle_{12}-\alpha|11\rangle_{12})].\end{aligned}$$

Now the measurement on Alice's side takes place in two steps. Firstly Alice performs a measurement on the Bell basis on her particles 3 and 4, and therefore

obtains any one of the four Bell states randomly and with equal probability. Then Alice performs an optimal positive operator value measurement (POVM), which provides the most general physically realizable measurement in quantum mechanics. The generalized measurement can distinguish conclusively between the two nonorthogonal states $a|0\rangle_5 + b|1\rangle_5(a|1\rangle_5 + b|0\rangle_5)$ and $a|0\rangle_5 - b|1\rangle_5(a|1\rangle_5 - b|0\rangle_5)$. Having the measurement result on particles 3, 4 and 5, Bob can transform the state of his two particles into the initial entangled state. Because there is always a possibility of an inconclusive result in the state discrimination procedure, the probability of successful teleportation is $2|b|^2$ (b is the smaller of the Schmidt coefficients).

IV. CONCLUSION

In summary, the scheme to teleport a two-particle entangled state by different quantum channels has been presented. One quantum channel is that Alice and Bob share the GHZ particles 1, 2 and 3. By performing the measurement of particles 3, 4 and 5, and a suitable rotation on particles 1 and 2, an unknown entangled state of particles 4 and 5 is teleported from Alice to Bob. Making use of an entangled three-particle state, the probabilistic teleportation of a two-particle entangled state can be realized. The probability of success is equal to the twice modulus square of the smaller coefficient of the entangled three-particle state.

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