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USING RANDOM PROPORTIONAL PULSE FEEDBACK OF SYSTEM VARIABLES TO CONTROL CHAOS AND HYPERCHAOS*

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A method that allows one to control chaotic and hyperchaotic systems by a random proportional pulse feedback of system variables is proposed. This method is illustrated with the Rossler chaotic and the complex Lorenz–Harken hyperchaotic systems, and a better control result is obtained. The advantage of this method is that just one perturbed system variable is enough to obtain a stabilized periodic orbit.

Keywords: random proportional pulse feedback, control chaos, control hyperchaos
PACC: 0545, 4265

I. INTRODUCTION

Chaotic systems are characterized by their extreme sensitivity to perturbations on their initial conditions.[1−3] The inherent feature of chaotic systems, known as the "butterfly effect", is often troublesome and even unwanted in many cases of practical application. Quite recently, the control of chaotic and hyperchaotic systems has brought about increasing attention from basic as well as application viewpoints.[4−10] The systems with more than one positive Lyapunov exponent are called hyperchaotic systems. The control of hyperchaotic systems has become a new active research area.[7−9] From the control strategy viewpoint, the methods can be divided into two categories. One control strategy is to change the system parameters. The precondition of this method is that a typical chaotic attractor is embedded in an infinite number of unstable periodic orbits (UPOs) with different periods T_k. They can be stabilized by just one small and carefully chosen feedback perturbation applied to some system parameters as an external adjustment. Another control strategy is to change the variables of the system. For example, by the digital filter feedback method of the system variables, Luo *et al.*[5] successfully realized stable control over discrete chaotic and continuous nonlinear autonomous chaotic systems. By using the variable feedback method of compressed phase space, Luo[6] did so over various periodic orbits. Also, using the method of pulse feedback proportional to system variables, Guemez and Matias[7] studied to stabilize the periodic orbits of discrete chaotic and continuous nonlinear chaotic systems. The methods mentioned in Refs.[5], [6] and [7] belong to the system variable feedback control strategy. The common feature of these methods is that all of the system variables have to be controlled to obtain stable periodic orbits for multi-variable systems. In practical applications, it is very difficult to control all of the system variables at the same time. For high-dimensional chaotic and hyperchaotic systems, it will be valuable in practice that various periodic orbits can be obtained if perturbation feedback of only one or a few system variables are used.

II. CONTROL METHOD

In this paper, a new method for chaos and hyperchaos control is proposed, in which a random proportional pulse feedback of system variables is implemented, when the variables of the chaotic and hyperchaotic systems are at the positive maximum or at the negative minimum. The investigations indicate that one system variable is sufficient to control various stable periodic orbits for the three-variable chaotic system or the four-variable hyperchaotic system. Moreover, this method possesses, to some extent, a noise-resistance property, so it has great application potential.

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The algorithm we have proposed for the control of chaos and hyperchaos is described briefly as follows: if

$$((x_i^{(n)} > 0) \quad \text{and} \quad (x_i^{(n-1)} > 0) \quad \text{and} \quad (x_i^{(n-2)} > 0)$$
$$\text{and} \quad (x_i^{(n-1)} > x_i^{(n)}) \quad \text{and} \quad (x_i^{(n-1)} > x_i^{(n-2)}))$$
$$x_i'^{(n)} = x_i^{(n)}(1.0 + r), \tag{1}$$

or
if

$$((x_i^{(n)} < 0) \quad \text{and} \quad (x_i^{(n-1)} < 0) \quad \text{and} \quad (x_i^{(n-2)} < 0)$$
$$\text{and} \quad (x_i^{(n-1)} < x_i^{(n)}) \quad \text{and} \quad (x_i^{(n-1)} < x_i^{(n-2)}))$$
$$x_i^{'(n)} = x_i^{(n)}(1.0 + r), \tag{2}$$

we have from Eq.(2) that $x_i^{(n-1)}$ is the positive maximum or negative minimum of the variable x_i. Since $x_i^{(n-1)} \approx x_i^{(n)}$, when the integration step size is very small, we select $x_i^{(n)}$ as the positive maximum or negative minimum of variable x_i in the process of chaos control.

Ⅲ. CONTROL EXAMPLES AND CONTROL RESULTS

In this section, the validity of the control method stated above is verified by its application to the following two chaotic systems. The four-order Runge–Kutta algorithm is used for numerical calculation with double-precision data. In order to ensure the stability of the control result, 200,000 iteration steps are operated.

A. Rossler system

The equation of the Rossler chaotic system is given by

$$\begin{aligned} \mathrm{d}x/\mathrm{d}t &= -(y + z), \\ \mathrm{d}y/\mathrm{d}t &= x + 0.15y, \\ \mathrm{d}z/\mathrm{d}t &= 0.2 + z(x - 10.0). \end{aligned} \tag{3}$$

Equation (3) has a positive Lyapunov exponent $\lambda = 0.13$,[11] and therefore the system is chaotic. Let the integration step size Δt=0.004. If the random proportional pulse feedback of system variables is implemented only for variable z at its positive maximum or negative minimum, i.e., $z'^{(n)} = z^{(n)}(1.0 + r)$, a stabilized period -2 will be obtained when $r = -0.9$, as shown in Fig.1. If the same control is implemented for two variables z and y, i.e., $z'^{(n)} = z^{(n)}(1.0+r_1), y'^{(n)} = y^{(n)}(1.0 + r_2)$, a stabilized period -4 will be obtained when $r_1 = -0.18$ and $r_2 = -0.08$, as shown in Fig.2.

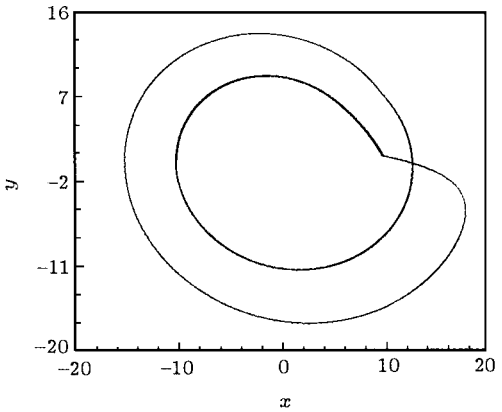


Fig.1. The stabilized orbit of period -2.

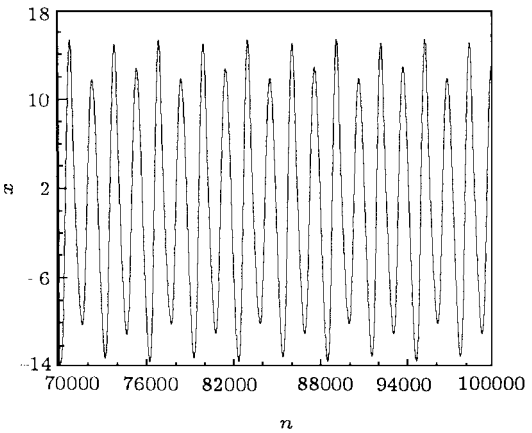


Fig.2. The stabilized orbit of period -4.

B. Complex Lorenz–Harken hyperchaotic system

The complex Lorenz–Harken (CLH) hyperchaotic system is given by

$$\begin{aligned} \mathrm{d}X/\mathrm{d}t &= -kX + kY, \\ \mathrm{d}Y/\mathrm{d}t &= -aY + (m - Z)X, \\ \mathrm{d}Z/\mathrm{d}t &= -bZ + 0.5(X^*Y + XY^*), \end{aligned} \tag{4}$$

where X and Y are complex variables, and m and a are complex parameters: $m = m_1 + \mathrm{i}m_2, a = 1 - \mathrm{i}e$. By transformation:

$$X = x_1\exp(\mathrm{i}\phi), \quad Y = (x_2 + \mathrm{i}x_3)\exp(\mathrm{i}\phi), \quad Z = x_4,$$

the CLH system can be changed into the nonlinear autonomous dynamic equation, as given by

$$\begin{aligned} \mathrm{d}x_1/\mathrm{d}t &= -kx_1 + kx_2, \\ \mathrm{d}x_2/\mathrm{d}t &= m_1x_1 - x_2 - ex_3 + kx_3^2/x_1 - x_1x_4, \\ \mathrm{d}x_3/\mathrm{d}t &= m_2x_1 + ex_2 - x_3 - kx_2x_3/x_1, \\ \mathrm{d}x_4/\mathrm{d}t &= -bx_4 + x_1x_2. \end{aligned} \tag{5}$$

When k=0.6, m_1=91.0, m_2 = −1.5 and e=2.5, the above system has two positive Lyapunov exponents: λ_1=0.029 and λ_2=0.002. Therefore, Eq.(5) is a hyperchaotic system.[12] Let the integration step size Δt=0.0005, and if we implement the control method proposed above to variable x_4, the period -1 is obtained when $r = -0.15$, as shown in Fig.3. Applying the same control method to the variables x_3 and x_4 at the same time, the stabilized orbit of period -3 is obtained when $r_1 = -0.015$ and $r_2 = -0.015$, as shown in Fig.4.

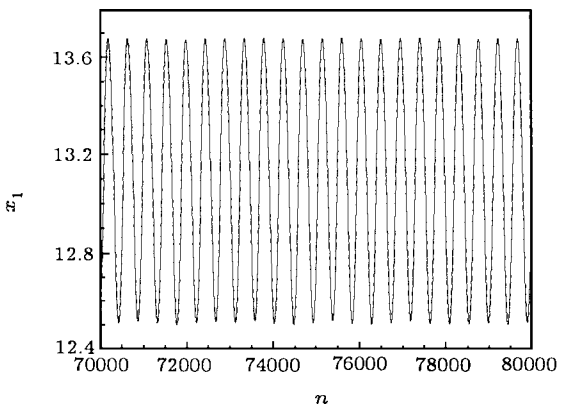


Fig.3. The stabilized orbit of period -1.

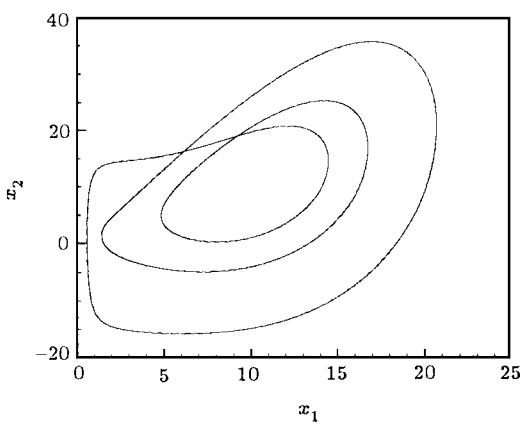


Fig.4. The stabilized orbit of period -3.

IV.CONTROL MECHANISM

So far, the maximum Lyapunov exponent (MLE) is mainly used to characterize chaos and hyperchaos control. This is mainly a common mechanism for chaos and hyperchaos control for most current methods. The chaos control is realized when the MLE in a controlled chaotic system is converted to the negative one. In this paper, we also use this method to serve as a measure of the chaos control process. Using the algorithm of calculation, the LE of a chaotic system was presented in Ref.[11]. The MLE of the Rossler system is converted from 0.1286 to −0.0201 when it is controlled to period -2, as shown in Fig.5.

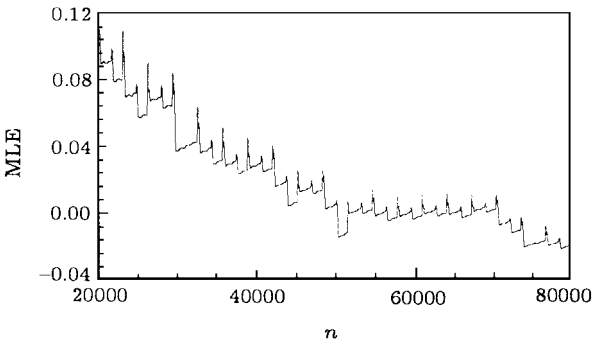


Fig.5. The time evolution of the MLE of the Rossler system when it is controlled to period -2.

V. THE INFLUENCE OF NOISE ON THE CONTROL RESULTS

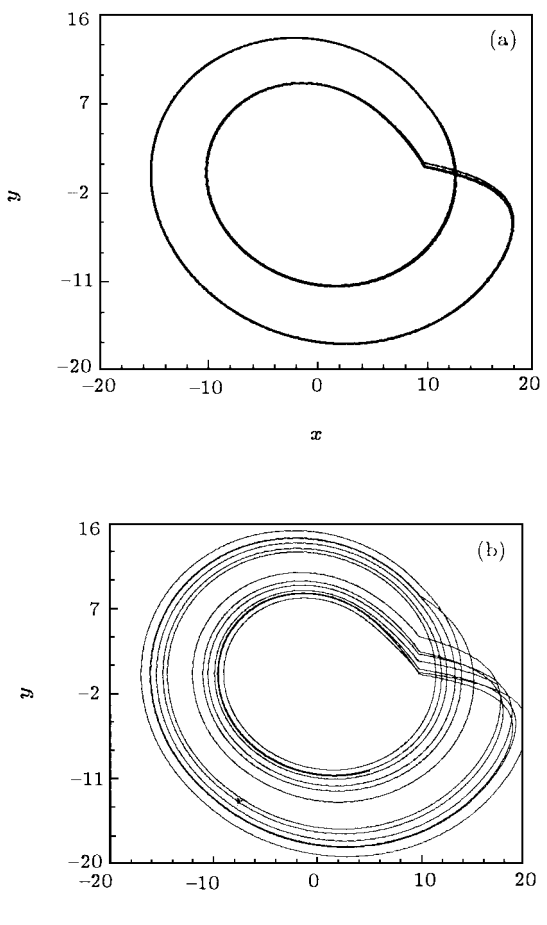


Fig.6. The influence of noise on the orbit of period -2 of the Rossler system.

To decide whether a chaos control method is applicable or not, the noise-resistance property is one of

the key features. The influence of the noise expressed by Eq.(6) on the control results of the Rossler system is studied. We have

$$x_i = x_i + \Gamma\sigma(0,1) \qquad (i = 1, 2, ..., n), \qquad (6)$$

where Γ is the noise strength and $\sigma(0,1)$ is the Gaussian noise with a width of 1 and mean value of zero. The numerical simulation shows that the orbit of period -2 is stable when Γ=0.001 and unstable when Γ=0.002, as shown in Figs.6(a) and 6(b). The orbit of period -4 is stable only when Γ <0.0002. This demonstrates that the control method in this paper has robustness to some extent, and that the stability of the low periodic orbits is quite good.

VI.CONCLUSIONS

Stable control over the continuous autonomous chaotic and hyperchaotic systems is realized by the random proportional pulse feedback method implemented at the positive maximum or negative minimum of the system variables. The advantage of this method is that various stabilized periodic orbits will be obtained as long as only one variable of the system is controlled, whereas all the variables of the system would have to be controlled to obtain the stabilized periodic orbit if other variable feedback control methods, such as the PPSV method, the phase space compress method etc., were used. Furthermore, to some extent, this method possesses a noise-resistance property and thus it has a wide application prospect.

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