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RADIATION OF DRESSED EXCITONS IN THE SEMICONDUCTOR MICROCAVITY*

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In this paper, we introduced the dressed exciton model of the semiconductor micro-cavity device. In the semiconductor micro cavity of vertical-cavity surface-emission device, the excitons first coupled with the cavity through an intra-electromagnetic field and formed the dressed excitons. Then these dressed excitons decayed into the vacuum cavity optical mode, as a multi-particle process. Through the quantum electrodynamics method, the dipole emission density and system energy decayed equation were obtained. And it was predicted that the excitons decay into a very narrow mode when the exciton-cavity coupling becomes strong enough.

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I. MODEL OF EXCITIONS IN MICRO-CAVITY

The dynamic response of atom in micro-cavity has been extensively studied both experimentally and theoretically, over the last two decades[1]. The concept of micro-cavity quantum-well (QW) semiconductor device was introduced several years ago. A classical single particle model was completed by Yamamoto *et al.*[2] In this paper, we will attack the problem of the dynamic process of the multi-exciton decay in the QW micro-cavity.

The novel behavior of micro-cavity effect in atom regime has led us to the consideration of the case of exciton in semiconductor micro-cavity. The Wanner exciton in the single quantum well can be considered as a two-level atom when the density of the excitons is so low that the direct interaction between two excitons can be neglected. These two-level atoms interact with each other through the electromagnetic field in the micro-cavity. In our model, the electromagnetic field is composed of two parts: one is the intra-action field and the other is an optical field. The photons of the intra-field cannot escape out of the cavity, because they are the exchange-particles of the cavity-atom system which has an ideal boundary. On the other hand, whether or not these photons of the optical field can be released from the cavity depends on the mirrors at the ends of the cavity. In this paper, we assume that the two-level atoms interact with each other via the intra-field at cavity frequency ω_L, which depends on the atom-cavity interaction, and form dressed excitons. Then the dressed excitons decay into the vacuum optical field.

The semiconductor micro-cavity device, for example, vertical-cavity surface-emitting structure in this paper, has only one mode in the spectrum regime concerned. And the two-level atom is in the excited state $|a\rangle$, or the ground state $|b\rangle$ with the transition frequency ω_0.

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The Hamiltonian H_{AL} of the atom + intrafield modes with eigenstates $|i,N\rangle$ $(i=1,2)$ satisfy the eigenvalue equation[3]

$$H_{AL}|i,N\rangle = \hbar[N_{\rm p}\omega_L - \frac{\Delta}{2} - (-1)^i\Omega]|i,N\rangle\,, \tag{1}$$

where

$$|1,N\rangle = \sin\theta|a,N\rangle + \cos\theta|b,N\rangle \quad \text{and} \quad |2,N\rangle = \cos\theta|a,N\rangle + \sin\theta|b,N\rangle$$

are the dressed atom states with $|l,N\rangle$ $(l=a,b)$ in which the atom is on $|l\rangle$ $(l=a,b)$ and there are $N_{\rm p}$ photons in the field mode. Here, θ is given by

$$\cos^2\theta = \frac{1}{2} - \frac{\Delta}{4\Omega}\,, \tag{2}$$

where $\Delta = \omega_L - \omega_0$ and $2\Omega = (\Delta^2 + 4\Omega_1^2)^{1/2}$, Ω being the detuned Rabi frequency and $\Omega_1 = g\sqrt{N_{\rm p}}$ the Rabi frequency at resonance; g is the atom-field coupling constant. In the case $\omega_L \approx \omega_0 \gg \Omega$, the dressed atom is reduced to another two-level atom and the Hamiltonian of the whole system is given by

$$H_0 = \hbar\omega\sum_{l=1}^{N} R_{lz} + H_{\rm optical}\,, \tag{3}$$

$$H_I = \boldsymbol{p}\cdot\boldsymbol{\varepsilon}(R_l^+ a + R_l^- a^+)\,, \tag{4}$$

where R_l^+, R_l^- and R_l are the raising, lowering and energy eigen operator. In quantum electrodynamics, the radiation of multi-atoms in $\boldsymbol{k}$ direction is derived as[4]

$$< I(\boldsymbol{k},t) >= I_0(\boldsymbol{k})\ \ \langle\psi_{\rm H}|U^-(T,0)S_{\vec{k}l}^+ S_{\vec{k}l}^- U(T,0)|\psi_{\rm H}\rangle\,, \tag{5}$$

in which

$$S_{\vec{k}l}^{\pm} = \sum_{l=1}^{N} R_l^{\pm}\exp[\pm{\rm i}(\boldsymbol{k}\cdot\boldsymbol{r}_l + \phi_l)]\,, \tag{6}$$

where $|\psi_{\rm H}\rangle_l = |\psi\rangle_1\otimes|\psi\rangle_2\otimes|\psi\rangle_3\cdots\otimes|\psi\rangle_l$. For the lth dressed exciton in the given system, we can write

$$|\psi\rangle_l = \exp\left({\rm i}\frac{\chi_l}{2}\right)\sin\left(\frac{\varphi}{2}\right)|\psi_N\rangle + \exp\left(-{\rm i}\frac{\chi_l}{2}\right)\cos\left(\frac{\varphi}{2}\right)|\psi_{N+1}\rangle\,, \tag{7}$$

where $|\psi_N\rangle$ and $|\psi_{N+1}\rangle$ are the dressed states for N and $N+1$ photons respectively, χ_l is the phase constant and the parameter φ denotes the probability density of the state. The radiation density, as an implicit function of t, can be given as

$$I(\boldsymbol{k},t) = I_0(\boldsymbol{k})\frac{N}{2}\left\{(1-\cos\varphi)(1-\sin 2\theta) + \frac{1}{2}N\sin^2\varphi\cos^2 2\theta\left[\Gamma(\boldsymbol{k},\boldsymbol{k}') - \frac{1}{N}\right]\right\}\,, \tag{8}$$

and the energy of the system is

$$W(t) = \left\langle\psi_{\rm H}(t)\right|\sum_{l=1}^{n} R_{lz}\left|\psi_{\rm H}(t)\right\rangle = -\frac{N}{2}\cos\varphi(t)\,, \tag{9}$$

where we have defined

$$\Gamma(\boldsymbol{k},\boldsymbol{k}') = |(\mathrm{e}^{\mathrm{i}(\boldsymbol{k}+\boldsymbol{k}')\cdot\boldsymbol{r}})_{av}|^2 \ ,$$

in which the average is to be taken over the positions $\boldsymbol{r}_l$ of all excitons. And other quantities are definded as

$$I(t) = \int I(\boldsymbol{k},t)\mathrm{d}\Omega_{\vec{k}} \ , \tag{10}$$

$$I_0 = \int I(\boldsymbol{k})\mathrm{d}\Omega_{\vec{k}} \ , \tag{11}$$

in which the integration is over all directions. We also define the shape factor μ by the relation

$$\left(\frac{1}{N}+\mu\right) I_0 = \int I(\boldsymbol{k})\Gamma(\boldsymbol{k},\boldsymbol{k}')\mathrm{d}\Omega_{\vec{k}} \ . \tag{12}$$

By integrating Eq.(8) over all directions of $\boldsymbol{k}$ and substituting φ by employing Eq.(9), we get

$$I(t) = \frac{\mu}{\tau_0}\left(\frac{N}{2}+W(t)\right)\left[\left(\frac{N}{2}-W(t)\right)\cos^2 2\theta + \frac{1-\sin 2\theta}{\mu}\right] \ , \tag{13}$$

where $\tau_0 = \hbar\omega/I_0$ is the single bare exciton lifetime, which is about 2 ps in a GaAs QW. Finally, the radiation-reaction dynamical law of evolution for the dressed exciton emitting system is given as

$$-\frac{\mathrm{d}W(t)}{\mathrm{d}t} = \frac{\mu}{\tau_0}\left(\frac{N}{2}+W(t)\right)\left[\left(\frac{N}{2}-W(t)\right)\cos^2 2\theta + \frac{1-\sin 2\theta}{\mu}\right] \ . \tag{14}$$

Until now, we have introduced our model and derived its evolution law. This law will reduce to the superradiation time-evolution equation in Ref.[4], when θ is approaching to $\pi/2$. In that case, the dressed exciton reduces to bare one, which will be discussed in detail later.

Ⅱ. SCHEMATIC OF SEMICONDUCTOR MICRO-CAVITY

Before discussing the solutions, we present the schematic structure of the ideal semiconductor micro-cavity. There are several structures of semi-conductor which can be the candidates for the micro-cavity, such as vertical cavity surface emitting laser (VCSEL), micro-plate and photonic band-gap material. In this paper, for example, we consider the VCSE structure, which is presented in Fig.1. The effective length of the cavity L is $\lambda/2$, λ being the wavelength of the emitting light. And r is the radius of the resource area. The QW lies on the antinode of the electromagnetic field in the cavity, and its width d is about 8 nm.

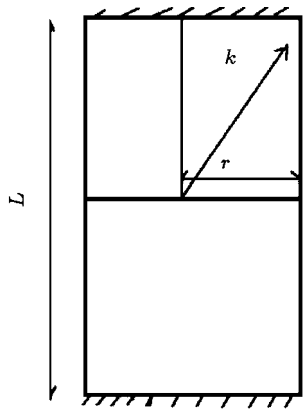

Fig.1. Schematic structure of VCSE structure.

We define two parameters: $H = \frac{2\pi}{\lambda} \cdot d$ and $A = \frac{2\pi}{\lambda} \cdot r$. The shape factor in Eq.(12) is[4]

$$\mu = \frac{6(N-1)}{NA^2H^2} \int_{-1}^{1} \frac{\mathrm{d}x(1+x^2)}{(1-x)^2(1-x^2)} \times \sin^2\left[\frac{1}{2}H(1-x)\right] \times J_1^2\left[A(1-x^2)^{1/2}\right] , \qquad (15)$$

In the QW, the number of the excitons is defined as[5]

$$N = \frac{8a^2}{a_0^2} \doteq 800\,A ,$$

where a_0 is the free exciton radius of about 10 nm.

Ⅲ. RESULTS AND DISCUSSION

The energy evolution of the system is derived from Eq.(14),

$$W(t') = -\frac{1}{2}\left[\frac{\exp[(t'-t_0)(N\mu\cos^2 2\theta + a)] - \left(1 + \dfrac{2a}{N\mu\cos^2 2\theta}\right)}{\exp[(t'-t_0)(N\mu\cos^2 2\theta + a)] + 1}\right] , \qquad (16)$$

where $t' = \dfrac{t}{\tau_0}$ and $a = (1 - \sin 2\theta)$, t_0 is a constant which is defined by the initial condition. Then, substituting $W(t)$ into Eq.(13), $I(t)$ can be expressed as an explicit function of time t.

The result can be discussed in two regimes, according to different coupling strength. The exciton-cavity coupling constant is defined as

$$g = -\varepsilon \cdot d\cos\gamma\sqrt{\frac{\hbar w_L}{2\epsilon_0 V_{\text{optical}}}} . \qquad (17)$$

The confficient γ represents the angle between $\boldsymbol{\varepsilon}_L$ and $\boldsymbol{d}_{ab}$. In this case, the coupling effect factor θ in Eq.(2) can be written as

$$\cos\theta = \sqrt{\frac{1}{2} - \frac{1}{2}\frac{1}{\sqrt{1 + \dfrac{\hbar w_L}{2\epsilon_0 \Delta^2} \cdot \dfrac{N_{\text{photon}}}{V_{\text{optical}}} \cos^2\gamma}}}\,, \tag{18}$$

where $\dfrac{N_{\text{photon}}}{V_{\text{optical}}}$ is the photon density in the cavity and θ depends on $\dfrac{\hbar w_L}{2\epsilon_0 \Delta^2} \cdot \dfrac{N_{\text{photon}}}{V_{\text{optical}}} \cos^2\gamma = \text{CF}\cos^2\gamma/A^2$, CF being the coupling factor.

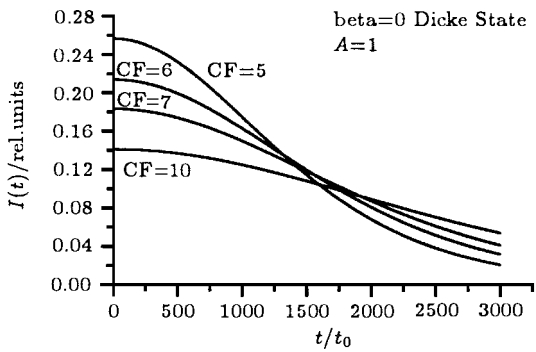

Fig.2. The lifetimes of dressed exciton of different coupling factor. The unit of CF is 1×10^5.

A. The strong coupling regime

When $\cos^2\gamma = 1$ and $\text{CF}/A^2 \gg 1$, $\theta \to \pi/4$, the system enters the strong coupling regime. This means that the high density of photon in the cavity and the low detuning between the cavity mode and excitons eigen mode, Δ, enhance the coupling factor CF. The initial sate is set as $W(t)|_{t=0} = 0$, which is equivalent to the Dicke state.[6] The t_0 is derived as

$$t_0 = \frac{-1}{N\mu\cos^2(2\theta) + a} \ln\left(1 + \frac{2a}{N\mu\cos^2(2\theta) + a}\right)\,, \tag{19}$$

In this regime, the photons emitted by the dressed exciton are squeezed into a very narrow spectrum, when the coupling is strong enough. In Fig.2 we show the numeral results of the light emitting process with different coupling strength. With increasing coupling strength, the lifetime of the dressed exciton is prolonged. When the coupling factor CF is up to 5×10^5, the lifetime $t(\sim 1000\tau_0)$ is about 2 ns, and the FWHM is reduced to 1 GHz. Besides the decreasing tendency of lifetime, upon increasing A, is shown. In other words, with the same coupling strength, the transverse confinement of the cavity will enhance the decay with very narrow fluorescence spectrum.

B. The medium coupling regime

The dipole direction effect in microcavity, which was first observed by Jhe[8], has been discussed.[7] In our model, the effect is derived from the atom-cavity interact-constant g, which is presented in Eq.(17). When the dipole is vertical to the cavity, g equals to zero. This means the coupling of exciton and cavity approaches zero, and the dressed excitons

degenerate to bare. In Fig.3, the line labeled beta= 0 is the radiation rate of the bare excitons and the lifetime is about $0.0028\tau_0$, where we define direction=$\cos^2\gamma$. With the decrease of γ, from $\pi/2$ to 0, the coupling increases to CF, which is set to be 3.5×10^3. Figure 4 shows that the lifetime is prolonged as much as $10\tau_0$, when the dipole is parallel to the cavity. In other words, when the excitons are polarized exactly normal to the cavity, they are enhanced to decay; but the parallel ones can survive much longer.

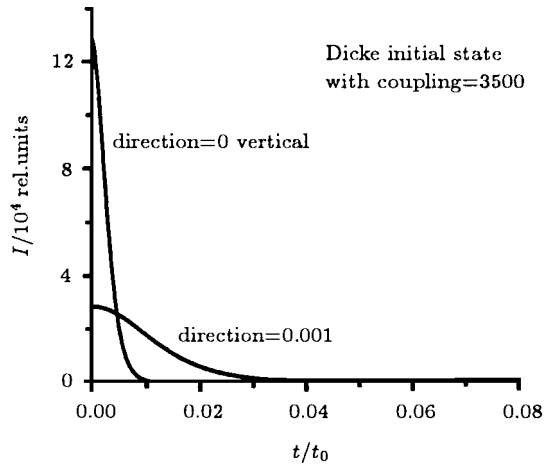

Fig.3. The radiation rate when the dipole is close to vertical to the cavity. The radiation starts from Dicke state, with CF=3.5×10^3. The lifetime of the dipole, which is vertical to the cavity, can be shortened to $0.0028\tau_0$ with direction=$\cos^2\gamma$.

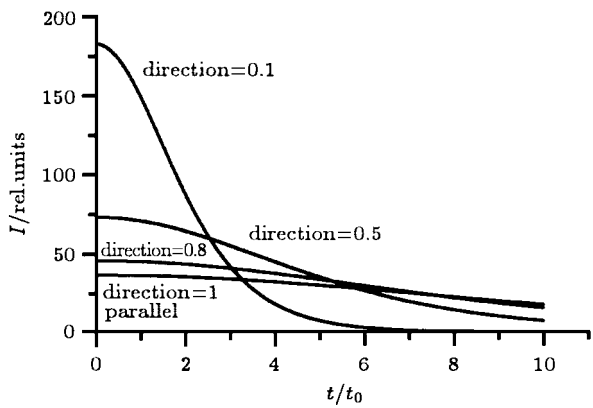

Fig.4. With medium CF= 3.5×10^3, the lifetime of the dipole which is parallel to the cavity is prolonged as much as $10\tau_0$. direction=$\cos^2\gamma$.

IV. CONCLUSION

In this paper, we first introduce the dressed exciton model of the semiconductor microcavity device. The excitons in QW of a VCSE structure couple to the cavity through the intra-field, in which the photon is emitted and absorbed by the exciton and the cavity, and form the dressed excitons. Then the dressed excitons emit photons into the vacuum cavity

optical mode, as a multi-particle process. Through the QED method, the dipole emission density and the system energy decay equation are obtained. We discussed the solution for two cases:

In the strong coupling regime, with Dicke initial state, the dressed excitons' lifetime is prolonged, with increasing coupling factor, as much as $1000\tau_0$. This predicts that the excitons directly radiate into a very narrow fluorescence spectrum mode.

And in the medium regime, the dipole direction effect of the micro-cavity decay rate is derived from our model. When the dipole is parallel to the cavity, the decay rate is hindered and the lifetime can be prolonged as much as $10\tau_0$. On the other hand, when the dipole is vertical, the decay is promoted and the lifetime becomes less than $0.001\tau_0$. This result can give a clear explanation of the Jhe's experiment of anisotropy of the cavity vacuum.

In conclusion, from our multi-particle model, several results are derived as the above. All the results show that this model gives a new method to understand the radiation process in micro-cavity.

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