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DYNAMICS OF A TRAPPED ATOM INTERACTING WITH AN ELECTROMAGNETIC WAVE OF CIRCULAR POLARIZATION

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In this paper the dynamics of a trapped two-level atom interacting with a beam of electromagnetic wave of circular polarization is studied. In a frame rotating with the atomic transition frequency ω_0, the motion of the atom is governed by a nonlinear coupled Jaynes-Cummings (JC) Hamiltonian. By means of this nonlinear Hamiltonian, we study the atomic inversion, squeezing effect of the vibration mode and that of the atomic dipole, and the phase property as well as its fluctuation of the vibration mode. In the Lamb-Dicke limit, we get the results similar to those of the JC model; beyond the Lamb-Dicke limit, we find the evolution of these quantities becomes very complicated. The nonlinearity weakens the collapse-revival phenomenon of the atomic inversion and the squeezing effect; the evolution of the cosine phase operator becomes less predetermined with its fluctuation not enlarged.

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I. INTRODUCTION

The Jaynes-Cummings (JC) model[1], which describes a two-level system coupling to a boson mode, is one of the possible nonlinear models and has been studied extensively in the past two decades. The importance of this model in optics lies in the fact that it may be the simplest solvable model which describes the essential physics of radiation-matter interactions. Various phenomena, such as collapse and revival of the atomic inversion[2] as well as that of mean photon number of the field[3], squeezing effect of the field variables[4] and the dipole momentum[5], phase property of the field[6], super-Poissonian distribution[7], have been studied and observed.[8] Moreover, related models have been also considered, such as nonlinearity coupled model[9], and multiquantum model[10], three-level model[11] as well as two-level model with the fine structure.[12] Recently, the motion of a cold atom or a cold ion in a harmonic potential has drawn much attention.[13—18] In the Lamb-Dicke limit[19], the atom/ion motion is described by the JC dynamics[20], in which its quantized center-of-mass motion in the trap potential plays the role of the boson mode, and various phenomena have been studied.[21] Beyond the Lamb-Dicke limit, the atom/ion motion is governed by a nonlinearly coupled JC Hamiltonian, which is first pointed out by Vogel *et al.* and then studied by Onofrio *et al.* with Linblad approach.[22]

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The purpose of this paper is to present an overall and detailed study of the dynamics of a trapped atom interacting with an electromagnetic wave of circular polarization, and to get an insight into how the nonlinearity affecting the evolution of the atom. Compared with other work, we do not restrict our study in the Lamb-Dicke limit, a fact that allows us to study the effect of the nonlinearity; next, we use the circularly polarized electromagnetic wave and the effecitve Hamiltonian we get has a simpler form. We will show that the effect of the nonlinearity is very complicated. The nonlinearity gives itself an intangible veil.

This paper is arranged as follows: we first deduce the model Hamiltonian in Sec.Ⅱ and in the following sections, we study the atomic inversion, the squeezing effect, and the phase property as well as its fluctuation of the atom in turn and, especially, focus on the influence of the nonlinearity; Sec.Ⅵ is our conclusion and discussion.

Ⅱ. THE EFFECTIVE HAMILTONIAN

Consider a two-level atom with mass m, transition angular frequency ω_0 and dipole moment $\boldsymbol{D}$, where $\boldsymbol{D} = (D_x, D_y, D_z)$. The atom experiencing harmonic motion at frequency ν in z direction is irradiated by a circularly polarized electromagnetic wave with wave vector k, angular frequency ω_L. The electromagnetic wave propagates along the positive z direction and its electric field $\boldsymbol{E}$ is assumed to be of the form $\boldsymbol{E} = (E_x, E_y)$, where $E_x = A\cos(\omega_L t - kz)$, $E_y = A\sin(\omega_L t - kz)$ and A is the amplitude of $\boldsymbol{E}$.

The Hamiltonian for the cold trapped atom interacting with the electromagnetic wave is given by

$$H = H_0 + H_{\rm int} \,, \tag{1}$$

where H_0 describes the free motion of the electronic and the center-of-mass degrees of freedom of the atom

$$H_0 = \frac{1}{2}\hbar\omega_0\sigma_z + \frac{p^2}{2m} + \frac{1}{2}m\nu^2 z^2 \,, \tag{2}$$

in which σ_z is the Pauli inversion operator, ω_0 is the atomic transition frequency and ν the trap frequency. The first term in Eq.(2) describes the atom internal energy; the last two terms are related to the atom motion energy in the harmonic trap potential, z is the coordinate variable of position related to the trap. Using the standard method, the last two terms in H_0 can be expressed in a simpler form as follows:

$$H_v = \hbar\nu(a^\dagger a + 1/2) \,. \tag{3}$$

For simplicity we neglect the constant $1/2$ in Eq.(3) and call operators $N = a^\dagger a$, a and $a^\dagger$, respectively, the phonon number, annihilation and creation operators of the vibration state.

The second term in total Hamiltonian describing the interaction of the atom with the electromagnetic wave is given by

$$H_{\rm int} = -\boldsymbol{D}\cdot\boldsymbol{E} = -\frac{1}{2}(\boldsymbol{D}^+\boldsymbol{E}^- + \boldsymbol{D}^-\boldsymbol{E}^+) \,, \tag{4}$$

where the dipole approximation is used and we have set $D^\pm = (D_x \pm {\rm i}D_y)$, $E^\pm = (E_x \pm {\rm i}E_y)$.[23] Substituting $E_{x,y}$ into $E^\pm$ and noting that $D^\pm = \mu\sigma_\pm$ (where $\sigma_\pm$ are Pauli raising and lowering operators, μ denotes the magnitude of dipole momentum), we get the

interaction Hamiltonian as follows:

$$H_{\rm int} = -\frac{1}{2}\hbar\Omega_L[\sigma_+ {\rm e}^{-{\rm i}(\omega_L t - kz)} + {\rm H.c.}] \,, \tag{5}$$

where we have introduced $\Omega_L = \mu \cdot A/\hbar$ to describe the interaction intensity. Then the total Hamiltonian can be expressed as

$$H = \frac{1}{2}\hbar\omega_0\sigma_z + \hbar\nu a^\dagger a - \frac{1}{2}\hbar\Omega_L[\sigma_+ {\rm e}^{-{\rm i}(\omega_L t - kz)} + {\rm H.c.}] \,. \tag{6}$$

In a frame rotating with the atomic transition frequency ω_0, the interaction Hamiltonian becomes

$$\begin{aligned} \widetilde{H}_{\rm int} &= -\hbar\lambda[\sigma_+ {\rm e}^{-{\rm i}\Delta t}\exp({\rm i}\eta a^\dagger {\rm e}^{-{\rm i}\nu t})\exp({\rm i}\eta a {\rm e}^{{\rm i}\nu t}) + {\rm H.c.}] \\ &= -\hbar\lambda\left[\sum_{l,l'}\frac{({\rm i}\eta)^{l+l'}}{l!l'!}{\rm e}^{-{\rm i}\nu t(l-l'+\Delta/\nu)}a^{\dagger l}a^{l'}\sigma_+ + {\rm H.c.}\right] \,, \end{aligned} \tag{7}$$

where the detuning Δ is given by $\Delta = \omega_0 - \omega_L$, the Lamb-Dicke parameter η is defined as $\eta = \omega_L/c(\hbar/2m\nu)^{1/2}$ and we have set $\lambda = \frac{1}{2}\Omega_L\exp(-\eta^2/2)$. Noticing that we have not yet specified the detuning Δ, we choose it in such a way that one of the terms in the summation of Eq.(7) is time-independent whereas all the others are oscillating. Keeping the time-independent terms only and neglecting all the oscillating terms (the so-called JC model approximation[22]), we get the effective Hamiltonian as

$$\widetilde{H}_{\rm int} = -\hbar\lambda[f^{(n)}(a^\dagger, a)\sigma_+ a^k + {\rm H.c.}] \,, \tag{8}$$

where $f^{(n)}(a^\dagger, a)$ is a normal-ordered function of the phonon operators

$$f^{(n)}(a^\dagger, a) = \sum_l^\infty \frac{({\rm i}\eta)^{2l+k}}{l!(l+k)!}a^{\dagger l}a^l \,. \tag{9}$$

This Hamiltonian is multipurpose and we can used it to get some Hamiltonians of interest. Especially, if we choose $\Delta = \nu$, i.e., $k = 1$, the effective Hamiltonian becomes

$$\widetilde{H}_{\rm int} = -{\rm i}\eta\hbar\lambda[f_1^{(n)}(a^\dagger, a)\sigma_+ a - {\rm H.c.}] \,, \tag{10}$$

with $f_1^{(n)}(a^\dagger, a)$ being given by

$$f_1^{(n)}(a^\dagger, a) = \sum_{l=0}^\infty \frac{(-\eta^2)^l}{l!(l+1)!}a^{\dagger l}a^l \,. \tag{11}$$

This Hamiltonian represents a nonlinear coupled JC interaction, which is of central interest of this paper and will be studied in more details in the following. Comparing with the Hamiltonian given by Vogel[22], we can see the nonlinear function $f^{(n)}(a^\dagger, a)$ in this case misses a Hermitian Conjugation, a fact discriminates our study among those corresponding contributions.

Before doing further, we should pay some attention to the rapid oscillating terms that are discarded in the derivation of the effective Hamiltonian. This treatment implies that the

transitions between states $|+,n\rangle$ and $|-,n+k\rangle$ involving the exchange of $k=\Delta/\nu$ vibrational quanta are resonantly enhanced, whereas all the off-resonant coupling are rapidly oscillating with frequency ν as well as its integer times can be disregarded.[22] This approximation, named as the JC model approximation in the sense that using this approximation one can get a JC-type interaction, is the rotating wave approximation in nature. This approximation is effective when the trapped frequency is high and the Lamb-Dicke parameter η is small.

In the manifold $\{|n,+\rangle,|n+1,-\rangle\}$, the time evolution operator $U(t)=\exp(-\mathrm{i}\widetilde{H}_{\mathrm{int}}\,t/\hbar)$ can be expressed as the following 2×2 matrix

$$U_n(t)=\begin{pmatrix}\cos\Omega_n t & \sin\Omega_n t\\ -\sin\Omega_n t & \cos\Omega_n t\end{pmatrix}, \tag{12}$$

in which the vibration Rabi-frequency Ω_n is given by

$$\Omega_n=\eta\lambda\left[1+\sum_{l=1}^{n}\begin{pmatrix}n\\ l\end{pmatrix}\frac{(-\eta^2)^l}{(l+1)!}\right]\sqrt{1+n}\,, \tag{13}$$

where the summation in the bracket results from the nonlinear function $f^n(a^\dagger,a)$. Noticing that in the JC model the Rabi frequency is given by $\Omega_n'=\lambda\sqrt{1+n}$ (where λ is the coupling strength), we can see that the important modification consists in the replacement of the vibration Rabi frequency Ω_n, which is determined by the phonon number n and also depends on the Lamb-Dicke parameter η. In the Lamb-Dicke limit only the leading term works thus Ω_n reduces to a monotonic function of n, same as that of the JC Hamiltonian; increasing η, we find giant nonlinearity. Contrasting to those generalized JC models[3] (of which the Rabi frequency becomes $\Omega_n'=\lambda(1+n)$ for the intensity-dependent coupling JC model, $\Omega_n'=\lambda\sqrt{(n+1)(n+2)}$ for the two-photon JC model), Ω_n shows an oscillatory behavior and there even exist regions of n's values, where the coupling almost breaks down, similar to that of Ref.[22].

Using the time evolution operator in Eq.(12), we can get the time evolution of a given state of the atom. We only consider the case that the atom is fully inverted and its vibration state is initially prepared in a coherent state, then the time-evolved state of the atom becomes

$$|\psi(t)\rangle=\mathrm{e}^{-\bar{n}/2}\sum_n\frac{\alpha^n}{\sqrt{n!}}[\cos\Omega_n t|+,n\rangle-\sin\Omega_n t|-,n+1\rangle]\,, \tag{14}$$

where $\bar{n}=|\alpha|^2$ is the mean phonon number of the vibration mode, describing the overall state of the atom center-of-mass motion.

Ⅲ. ATOMIC INVERSION

The atomic inversion is the occupation probability difference for the atom lying in its two electronic states

$$<\sigma_z>=\mathrm{e}^{-\bar{n}}\sum_n\frac{\bar{n}^n}{n!}\cos 2\Omega_n t\,, \tag{15}$$

which is formally the same as that of the JC model, except for different Rabi frequency Ω_n.

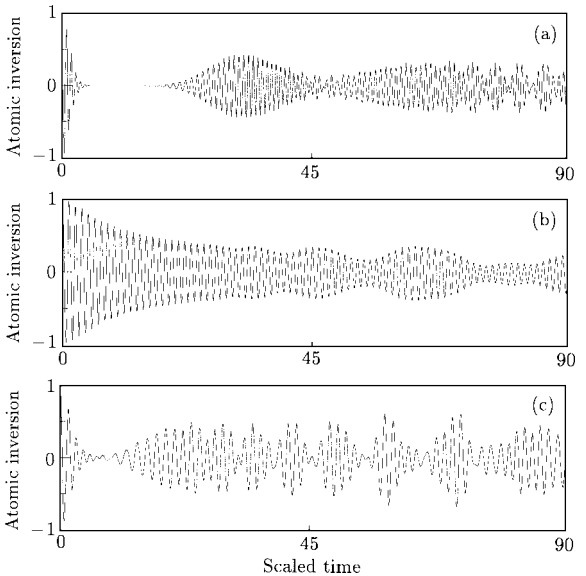

Fig.1. The atomic inversion vs scaled time $\eta\lambda t$ of the trapped atom for (a) $\bar{n}$ = 15, η=0.1; (b) $\bar{n}$=15, η=0.2; (c) $\bar{n}$=5, η=0.2.

Figure 1 shows the atomic inversion for different mean phonon number $\bar{n}$ for several η. From this figure, we see that the nonlinearity weakens the collapse-revival phenomenon of the atomic inversion. In the Lamb-Dicke limit, where the nonlinearity is weak, the atomic inversion is similar to that of the JC model, the collapse-revival phenomenon is obvious. With increasing η, the collapse-revival period becomes short rapidly with less perfect cycles lying, thus when $\eta \geq 0.5$ no obvious cycle exists. We also notice that in the case of large η, the atomic inversion sometimes get its extreme values, say ± 1, which means the atom being definitely in excited (or ground) state at the moment. This phenomenon does not appear in that of JC model. On the other hand, for the same mean photon number $\bar{n}$, the inversions are obviously not the same for different η.

IV. SQUEEZING EFFECT

We now examine the squeezing effect of the atom and pay special attention to the influence of the nonlinearity on squeezing of the vibration mode and that of the atomic dipole.

A. Squeezing of the vibration mode

We study the squeezing of the vibration operator defined as

$$X_1 = \frac{1}{2}(a + a^\dagger), \quad X_2 = \frac{1}{2\mathrm{i}}(a - a^\dagger) \,. \tag{16}$$

It is easy to show their variances satisfying the uncertainty relation

$$< (\Delta X_1)^2 >< (\Delta X_2)^2 > \geq \frac{1}{16} \,. \tag{17}$$

Squeezing occurs whenever $< (\Delta X_i)^2 >< 1/4 (i = 1, 2)$. For the variances of these two operatoprs in a coherent state V_c is $1/4$, it is convenient to adopt the following Q-parameter:

$$Q = \frac{V - V_c}{V_c} \tag{18}$$

to mark their squeezing degree. Squeezing occurs whenever $Q< 0$, and we study the squeezing of X_1 alone.

Using the state given by Eq.(14), we get the mean value of these operators

$$< X_1^2 > = \frac{\alpha^2 + \alpha^{*2}}{4} \mathrm{e}^{-\bar{n}} \sum_n \frac{\bar{n}^n}{n!} [\cos(\Omega_{n+2} - \Omega_n)t] + \frac{2\bar{n} + 1}{4} , \tag{19}$$

$$< X_1 > = \frac{\alpha + \alpha^*}{2} \mathrm{e}^{-\bar{n}} \sum_n \frac{\bar{n}^n}{n!} \cos(\Omega_{n+1} - \Omega_n)t . \tag{20}$$

Also, we find that these quantities are formally the same as those in JC model, expected for different Ω_n.

By means of numerical study, we get the Variation of Q-parameter with time. We find that squeezing appears only when η is very small. For the nonlinearity acting as a dissipation factor, it is easy to understand that squeezing does not appear for larger η.

B. Squeezing of the atomic dipole

The dispersive and absorptive components of the atomic dipole can be written as

$$\sigma_1 = \frac{1}{2}(\sigma_+ + \sigma_-), \quad \sigma_2 = \frac{1}{2\mathrm{i}}(\sigma_+ - \sigma_-) , \tag{21}$$

respectively. The variance of $\sigma_i (i = 1, 2)$ satisfies

$$< (\Delta\sigma_i)^2 >< \frac{1}{4} | < \sigma_z > | . \tag{22}$$

We adopt the following S_i parameter

$$\begin{aligned} S_1 &= \frac{1 - 4(\mathrm{Re} < \sigma_- >)^2}{| < \sigma_z > |} , \\ S_2 &= \frac{1 - 4(\mathrm{Im} < \sigma_- >)^2}{| < \sigma_z > |} \end{aligned} \tag{23}$$

to mark the squeezing in dispersive and absorptive components of the atomic dipole. Squeezing occure when $S_i<1$.

By means of the time-evolved state in Eq.(14), $< \sigma_- >$ works out to be

$$< \sigma_- >= -\mathrm{e}^{-\bar{n}} \sum_n \frac{\bar{n}^n}{n!} \frac{\alpha}{\sqrt{n+1}} \sin \Omega_n t \cos \Omega_{n+1} t , \tag{24}$$

and $< \sigma_z >$ is given by Eq.(15). Our numerical calculation shows that squeezing occure in a short time at the very beginning for larger η. In Fig.2 we depict S_1-parameter for α is real with η=0.1 and 0.2, respectively. Because the nonlinearity plays a dispersive role, which

demolishes squeezing of the vibration mode, it also decreases the squeezing of the atomic dipole. However, due to the stronger squeezing effect of the atomic dipole, squeezing occurs in a modest manner in the case of large η.

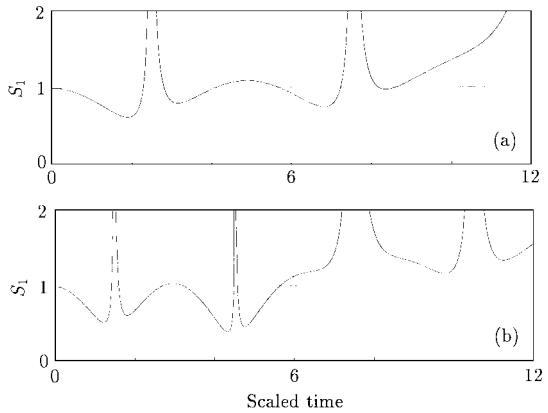

Fig.2. The evolution of the S_1-parameter vs scaled time $\eta\lambda t$ for an initially coherent state $\alpha>$ when α is real. Other conditions are (a) $\bar{n}$=10 and η=0.1; (b) $\bar{n}$=10 and η=0.2.

V. PHASE EVOLUTION AND ITS FLUCTUATION

In the interaction picture, the P-S cosine phase operator is defined as[6]

$$\cos\phi = \frac{1}{2}[(aa^{\dagger})^{-1/2}a + \text{H.c.}] \,. \tag{25}$$

After a short calculation, by the aid of Eqs.(14) and (25), we get the mean value of this operator as follows:

$$< \cos\phi >= \frac{\alpha+\alpha^*}{2}\mathrm{e}^{-\bar{n}} \sum_n \frac{\bar{n}^n}{n!}\frac{\cos(\Omega_{n+1}-\Omega_n)t}{\sqrt{n+1}} \,. \tag{26}$$

Similarly, the phase fluctuation works out to be

$$\begin{aligned} &< \cos^2\phi >= \frac{1}{2} + \frac{\alpha^2+\alpha^{*2}}{4}\mathrm{e}^{-\bar{n}} \sum_n \frac{\bar{n}^n}{n!}\frac{\cos(\Omega_{n+2}-\Omega_n)t}{\sqrt{(n+1)(n+2)}} \,, \\ &< (\Delta\cos\phi)^2 >=< \cos^2\phi > - < \cos\phi >^2 \,. \end{aligned} \tag{27}$$

Our numerical calculation shows that in large time scale, the phase of the vibration mode oscillates around zero; in small time scale, it behaves as a damped oscillator. For larger η, the oscillating frequency becomes small and the damping rate gets larger. Focusing on the phase fluctuation, we also find that in large time scale it oscillates around the value of about 0.5; in small time scale, it increases from zero to 0.5 then oscillates. The phase fluctuation increases in a smooth manner and its oscillating frequency is small in the larger η case, as shown in Figs.3 and 4.

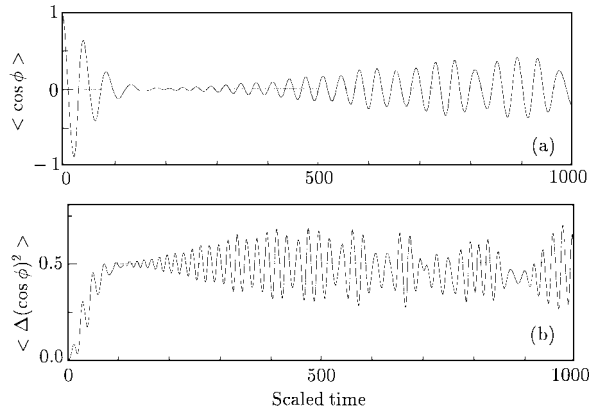

Fig.3. The evolution of (a) $< \cos\phi >$; (b) $< \Delta(\cos\phi)^2 >$ vs scaled time $\eta\lambda t$ for $\bar{n}$=10 and η=0.01.

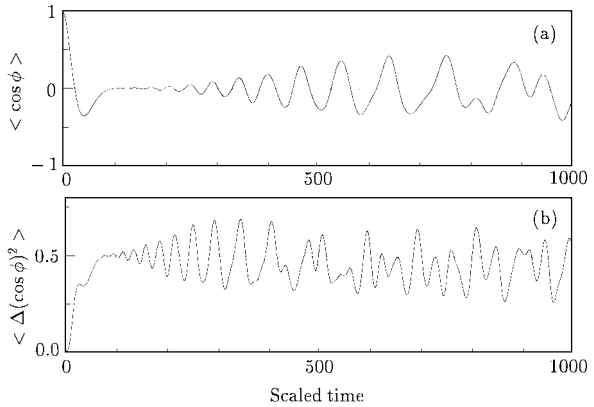

Fig.4. The evolution of (a) $< \cos\phi >$; (b) $< \Delta(\cos\phi)^2 >$ vs scaled time for $\bar{n}$=10 and η=0.2.

VI. CONCLUSION AND DISCUSSION

We have studied several quantum effects of a trapped atom under the action of the electromagnetic wave of circular polarization by means of a nonlinear coupled JC Hamiltonian and have shown the influence of nonlinearity on the atomic behavior. The nonlinearity, which works when the atom motion is beyond the Lamb-Dicke limit, plays a dominant role in the case of large η and its influence is weak for small η. Generally speaking, the nonlinearity behaves as a dissipation factor which demolishes the collapse-revival phenomenon of the atomic inversion as well as that of the phase, and the squeezing effect are also weakened. Further investigation will focus on the influence of the time-dependent terms in the interaction Hamiltonian as well as that of the dissipation that comes from the mismatch of the amplitude of the two components of the electromagnetic wave and the mismatch of the frequency of the electromagnetic wave to that of the trap potential. And we are unlikely to accept that the influence of the time-dependent terms is obvious in some special cases, for instance, in the case that the frequency of the trapping potential is small and η is large.

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