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LASER-CONTROLLED STATE EXCITATION AND QUANTUM COMPUTATION*

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Based on the dependence of the molecular excitation probabilities on the laser phases and amplitudes in the model of a three-mode laser interacting with a three-level molecule, we propose a scheme to implement quantum XOR logic gate and to construct quantum error prevention code.

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I. INTRODUCTION

The laser coherent control theory is generally used to analyze and control the probability of a chemical reaction, or the molecular photodissociation and ionization processes. Here we propose the use of laser to coherently control molecular states by selective state excitation. While the state is taken as a quantum bit (qubit) in quantum computation, the control is demonstrated to be capable of constructing a quantum Fredkin logic gate.[1]

Recently there has been growing interest in quantum computating.[2−8] To build a quantum computer one must find way to realize fundamental quantum logic gates. Quantum exclusive-OR(XOR) or controlled-NOT gate is one of the basic and important logic gates and plays a key role in quantum computation. It has been proved that any gate, such as Toffoli logic gate and Fredkin logic gate, can consist of all of one-bit quantum gates and the XOR gate. Thus the XOR gate and one-bit gates are universal.[9] XOR is a two-bit gate, in which there are two interactive qubits, a control bit and a target bit. If the control bit is in state $|0>$, the target bit is invariant. When the control bit is in state $|1>$, the target bit flips. Thus XOR can be expressed by a unitary transformation, the corresponding operator $U_{\rm XOR}$ is defined as follows

$$\begin{aligned} U_{\rm XOR}|00\rangle = |00\rangle, \qquad U_{\rm XOR}|01\rangle = |01\rangle\ , \\ U_{\rm XOR}|10\rangle = |11\rangle, \qquad U_{\rm XOR}|11\rangle = |10\rangle\ , \end{aligned} \tag{1}$$

and the truth table of XOR gate is

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control bit	Target bit	output of target bit
|0>	|0>	|0>
|0>	|1>	|1>
|1>	|0>	|1>
|1>	|1>	|0>

Alternatively, the function of the XOR gate can be represented by the following 4×4 matrix

$$U_{\mathrm{XOR}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \tag{2}$$

Though quantum computation has attracted much attention as a result of rapid progress made in designing efficient quantum algorithms, and demonstrating basic logic gates, it is still far from practical use. The biggest difficulty is the fragility of the quantum states required to process information. All the proposed implementations will suffer from the interaction with the environment, and even a weak coupling may result in decoherence.[10−12] Direct measurement of a qubit will also destroy its coherence. Moreover, other sources of errors (e.g., timing of laser pulse in the linear trap computer) will add to the problem. Fortunately, schemes for quantum error correction have recently been developed, and they allow certain independent erroes be corrected. Nine-[13], eight-[14], seven-[15], six-[16], and five[17]-qubit codes have been discovered with perfectly correct single qubit errors of any kind. All of these proposals hardly relate the schemes with real physical systems.

In this paper, based on the coherent interaction of laser with molecules we propose a model to realize XOR gate and construct four-bit error prevention code.

Ⅱ. THEORETICAL MODEL OF LASER-CONTROLLED STATE EXCITATION

Shown in Fig.1 is a three-level system interacting with three laser fields.[18]

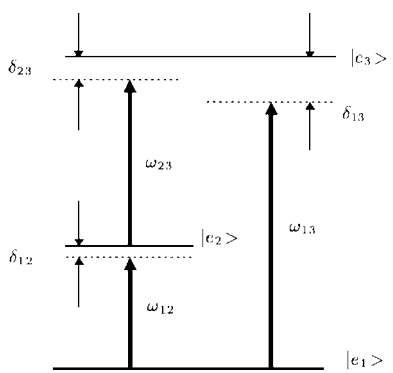

Fig.1. Interaction of a three-level system with three lasers; δ_{12}, δ_{23} and δ_{13} are detunings.

Let the light field acting on the system be

$$\boldsymbol{E} = \frac{1}{2}\boldsymbol{E}_{12}\mathrm{e}^{i(\omega_{12}t+\varphi_{12})} + \frac{1}{2}\boldsymbol{E}_{13}\mathrm{e}^{i(\omega_{13}t+\varphi_{13})} + \frac{1}{2}\boldsymbol{E}_{23}\mathrm{e}^{i(\omega_{23}t+\varphi_{23})} + c.c., \tag{3}$$

where E_{mn} are the laser amplitudes, ω_{mn} are frequencies, m, n=1, 2, 3 ($m \neq n$). Write the system wave function $|\Psi> = c_1|1> + c_2|2> + c_3|3>$, then we have the Schrödinger equation of motion

$$i\hbar\frac{\partial}{\partial t}|\Psi> = H|\Psi>, \tag{4}$$

where

$$H = H_0 + V = H_0 - \boldsymbol{\mu}\cdot\boldsymbol{E}\,. \tag{5}$$

In the interaction picture, with the rotating-wave approximation, the matrix equation is given by

$$i\begin{bmatrix}\dot{c}_1\\ \dot{c}_2\\ \dot{c}_3\end{bmatrix} = \begin{bmatrix}0 & V_{12} & V_{13}\\ V_{12}^* & 0 & V_{23}\\ V_{13}^* & V_{23}^* & 0\end{bmatrix}\begin{bmatrix}c_1\\ c_2\\ c_3\end{bmatrix}, \tag{6}$$

where $V_{mn} = \frac{-\mu E_{mn}}{2\hbar}\mathrm{e}^{i(\delta_{mn}t+\varphi_{mn})}$; $\delta_{mn} = \Omega_{mn} - \omega_{mn}$ are frequency detunings, and Ω_{mn} is the Bohr frequency between $|e_m>$ and $|e_n>$. In the following we specifically consider the resonant case, i.e. $\delta_{12} = \delta_{13} = \delta_{23} = 0$.

For the type of linear differential equations

$$\frac{\mathrm{d}C(t)}{\mathrm{d}t} = A(t)C(t)\,, \tag{7}$$

an approximate solution method was studied systematically by Pechukas and Light.[19−21] Suppose the molecule is initially in the state $|e_1>$, we obtain the probabilities of state $|e_2>$ and $|e_3>$ at time t[18]

$$\begin{aligned} P_2 &= |c_2(t)|^2 = \frac{\mu^2E_{12}^2t^2}{4\hbar^2} + \frac{\mu^4E_{13}^2E_{23}^2t^4}{64\hbar^4} - \frac{\mu^3E_{12}E_{23}E_{13}t^3}{8\hbar^3}\sin(\varphi_{12}+\varphi_{23}-\varphi_{13})\,,\\ P_3 &= |c_3(t)|^2 = \frac{\mu^2E_{13}^2t^2}{4\hbar^2} + \frac{\mu^4E_{12}^2E_{23}^2t^4}{64\hbar^4} + \frac{\mu^3E_{12}E_{23}E_{13}t^3}{8\hbar^3}\sin(\varphi_{12}+\varphi_{23}-\varphi_{13})\,. \end{aligned} \tag{8}$$

Ⅲ. USING LASER-CONTROLLED STATE EXCITATION TO IMPLEMENT XOR GATE

From Eq.(8) it can be seen that P_2 and P_3 depend on time t. When t is getting large, the populations change till nearly saturated finally. Thus the probabilities cannot be controlled with continuous laser. On the other hand, if we use pulsed lasers the probabilities can be controlled by changing the laser parameters. If we alter the intensities, the pulse widths τ and the laser phases, so that under condition 1, i.e.,

$$\varphi_{12}+\varphi_{23}-\varphi_{13} = 2k\pi+\pi/2, k = 0, \pm1, \pm2\cdots, \quad 4\hbar E_{12} = \mu\tau E_{13}E_{23}\,,$$

we obtain

$$P_2 = 0, \quad P_3 = \left(\frac{\mu E_{13}\tau}{2\hbar} + \frac{\mu^2E_{12}E_{23}\tau^2}{8\hbar^2}\right)^2. \tag{9}$$

or under the condition 2, i.e.,

$$\varphi_{12}+\varphi_{23}-\varphi_{13}=2k\pi+3\pi/2, k=0,\pm1,\pm2\cdots, \quad 4\hbar E_{13}=\mu\tau E_{12}E_{23}\ ,$$

we get

$$P_2=\left(\frac{\mu E_{12}\tau}{2\hbar}+\frac{\mu^2 E_{13}E_{23}\tau^2}{8\hbar^2}\right)^2, \quad P_3=0\ . \tag{10}$$

This means that we can selectively either populate the state $|e_3>$ or just excite $|e_2>$ by changing the experimentally controllable laser parameter.

The idea to implement quantum XOR gate by using the laser-controlled state excitation is that we regard the laser parameter control as a control bit, and take the combination ($|e_3\rangle$, $|e_2\rangle$) as the target bit of an XOR gate. We designate the condition 1 as control bit $|1>$ and the corresponding combination in Eq.(9) as target bit $|1>$. Similarly we assign the condition 2 as control bit $|0>$ and Eq.(10) as target bit $|0>$ (see Table 1). Target bit will be flipped ($|1>\leftrightarrow|0>$) when laser amplitudes and phases are controlled from condition 1 to condition 2.

Table 1. Relation between laser coherent excitation and XOR gate

Qubit of XOR gate	$\|1>$	$\|0>$
Laser control condition as control bit	$\varphi_{12}+\varphi_{23}-\varphi_{13}=(2k+1/2)\pi$ $4\hbar E_{12}=\mu\tau E_{13}E_{23}$	$\varphi_{12}+\varphi_{23}-\varphi_{13}=(2k+3/2)\pi$ $4\hbar E_{13}=\mu\tau E_{12}E_{23}$
($\|e_3\rangle$, $\|e_2\rangle$) probability as target bit	(P_3, 0)	(0, P_2)

IV. FOUR-BIT ERROR PREVENTION CODE BY USING LASER-CONTROLLED STATE EXCITATION

As in section Ⅲ, we define ($|e_3\rangle$, $|e_2\rangle$) as qubit $|1>$ when Eq.(8) satisfies condition 1 and ($|e_3\rangle$, $|e_2\rangle$) as qubit $|1>$ under the condition 2. In general case the laser fields do not meet the two conditions, so that both $P_2\neq0$ and $P_3\neq0$, which represents a superposition state ($\alpha|0\rangle\pm\beta|1\rangle$). Similarly, more qubits could be produced by extend the above arguments to different molecules or different states of a single molecule.

Suppose we encode a qubit using additional n qubits. The encoded logical states $|0_L\rangle$ and $|1_L\rangle$ are expressed as entangled states in the n-particle Hilbert space, i.e.

$$|0_L\rangle=\sum_{i=0}^{2^n-1}c_i|i\rangle, \quad |1_L\rangle=\sum_{i=0}^{2^n-1}d_i|i\rangle\ . \tag{11}$$

where $|i\rangle=|i_{n-1},\cdots,i_0\rangle$ form the basis of the Hilbert space, and i_j is the binary representation (0 or 1) of an integer i. We use four qubits for encoding in the following way[22],

$$|0\rangle\rightarrow|0_L\rangle=\frac{1}{2}(|00\rangle+|11\rangle)(|00\rangle+|11\rangle)\ , \tag{12a}$$

$$|1\rangle\rightarrow|1_L\rangle=\frac{1}{2}(|00\rangle-|11\rangle)(|00\rangle-|11\rangle)\ . \tag{12b}$$

The transformation changes system state $\alpha|0\rangle+\beta|1\rangle$ into a new state $\alpha|0_L\rangle+\beta|1_L\rangle$. To keep the system state within the subspace defined by Eq.(12), we use the single-particle

measuring method[22] and obtain the possible state of the system as

$$d_1(|00\rangle + |11\rangle)(|00\rangle + |11\rangle) + d_2(|00\rangle - |11\rangle)(|00\rangle - |11\rangle) + d_3(|00\rangle + |11\rangle)(|00\rangle - |11\rangle) + d_4(|00\rangle - |11\rangle)(|00\rangle + |11\rangle). \tag{13}$$

Then we disentangle the two subsystems in each term of Eq.(12) and make two separate Bell measurements on the subsystems. For example, when a unitary transformation that exchanges the two states ($|10> \leftrightarrow |11>$) and leaves the other states unchanged acts on the Bell states, it removes the entanglement, i.e.,

$$(|00> \pm |11>)/\sqrt{2} \rightarrow (|00> \pm |10>)/\sqrt{2} = (|0> \pm |1>) \otimes |0> /\sqrt{2}\,. \tag{14}$$

This operation actually corresponds to an XOR gate. The operation acts on the first two qubits, then on qubits 3 and 4. If two measurements give the same results, then $d_3 = d_4 = 0$, i.e., the state belongs to Eq.(12). The recovery of the state can be made by another XOR operation.

To realize the encoding (Eq.(12)), we propose a circuit shown in Fig.2. The output state will always be $\alpha|0_L\rangle + \beta|1_L\rangle$, when the input is $|Q\rangle = \alpha|0\rangle + \beta|1\rangle$. This circuit is a combination of several quantum logic gates, where R describes a rotation $|0> \rightarrow (|0> + |1>)/\sqrt{2}$ or $|1> \rightarrow (|0> - |1>)/\sqrt{2}$. A block dot together with a σ_x actually is an XOR gate. At an open circle the target bit flips if the control bit is $|0>$. The two-bit gate denoted by a line and two open squares is "control-phase gate", it functions as a unitary operator

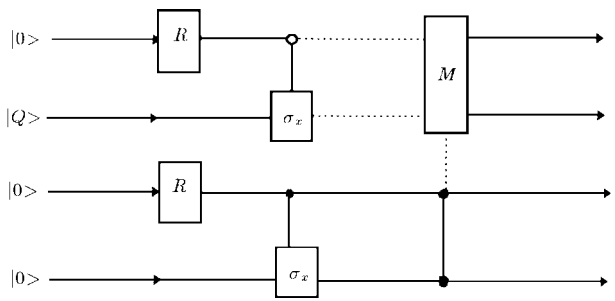

Fig.2. Circuit for the encoding of four qubits

$$U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{15}$$

acting on the basis of ($|00>$, $|01>$, $|10>$, $|11>$). This gate prevents the generation of the sate $-|1111>$, since $-|1111>$ is not a state in subspace defined by Eq.(12). M is a detector, which acts on the input states, i.e. $|-10> \xrightarrow{M} |11>$, $|01> \xrightarrow{M} |00>$. It implements a control-phase operation if and only if two bits are different. When the input two bits are the same, the input state does not change.

V. CONCLUSION

We have proposed a new way to realize the XOR gate and to construct the error prevention code. The model is based on the interaction of a three-level molecule with three laser fields. Although the entanglement feature of the model needs further verification, the scheme really has some attractive aspects. Firstly, the qubit is related only to the zero or not of the population. If there is no molecule populated in a level, the level is virtual, thus there exists no spontaneous emission. This can avoid the decoherence due to spontaneous emission. Secondly, the qubits are experimentally controllable by changing the laser phases and amplitudes.

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