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SYMMETRIES AND DROMION SOLUTION OF A (2+1)-DIMENSIONAL NONLINEAR SOHRÖDINGER EQUATION*

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The Painlevé property, infinitely many symmetries and exact solutions of a (2+1)-dimensional nonlinear Schrödinger equation, which are obtained from the constraints of the Kadomtsev-Petviashvili equation, are studied in this paper. The Painlevé property is proved by the Weiss-Kruskal approach, the infinitely many symmetries are obtained by the formal series symmetry method and the dromion-like solution which is localized exponentially in all directions is obtained by a variable separation method.

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I. INTRODUCTION

Study of the integrable properties of nonlinear models is one of the important works in the nonlinear science. Recently, starting from the symmetry constrains of the Kadomtsev-Petviashvili (KP) equation, some new nonlinear models are obtained.[1,2] In this paper, we discuss some interesting properties of the following (2+1)-dimensional nonlinear Schrödinger equation(NLSE)[1]

$$\begin{aligned} &u_{xx} = uQ_x - iu_t \ , \\ &u^*_{xx} = u^*Q_x + iu^*_t \ , \\ &Q_y = uu^* \ . \end{aligned} \tag{1}$$

In section Ⅱ., we check the Painlevé property of the NLS equation (1). In section Ⅲ, infinitely many truncated symmetries with arbitrary functions of t for the NLSE are given by using formal series symmetry method.[3−5] These symmetries constitute an infinite-dimensional Lie algebra. We also discuss the existence of another set of symmetries with arbitrary functions of space y. Only one special symmetry of this type is given. In section Ⅳ, a variable separation procedure for the NLSE is used to its bilinear form.[6] The rich dromion structures of the equation are revealed. The dromions may have a free shape in one or more directions. Multi-dromion solutions of Eq.(1) can be driven by perpendicular line, non-perpendicular line and curved line ghost solitons.

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II. PAINLEVÉ PROPERTIES FOR THE (2+1)-DIMENSIONAL NONLINEAR SCHÖDINGER EQUATION

Though model (1) can be obtained from the symmetry constraints of the KP equation, we still like to check its integrability by means of the Painlevé analysis. By using the standard Weiss-kruskal approaches[7,8], the expansion about the singular manifold has the form

$$Q = \phi^{\alpha}\sum_{j=0}^{\infty} Q_j\phi^j, \quad u = \phi^{\beta}\sum_{j=0}^{\infty} u_j\phi^j, \quad u^* = \phi^{\gamma}\sum_{j=0}^{\infty} u_j^*\phi^j \ , \tag{2}$$

where $Q_j = Q_j(x,y,t)$, $u_j = u_j(x,y,t)$, $u_j^* = u_j^*(x,y,t)$, $\phi = \phi(x,y,t)$ are analytic functions of (x,y,t). Substituting Eqs.(2) into Eqs (1) leads to

$$\alpha = \beta = \gamma = -1 \quad Q_0 = -2\phi_x, \quad u_0u_0^* = 2\phi_x\phi_y \ , \tag{3}$$

and the recursion relations have the forms

$$\begin{aligned}
&j(j+1)(j-3)(j-4)\phi_x^3\phi_y u_j = (j^2-3j-2)\phi_x\phi_y\Big[-u_{j-2,xx} - (j-2)(2u_{j-1,x}\phi_x + u_{j-1}\phi_{xx}) \\
&+\sum_{l=0}^{j} u_{j-l-1}Q_{lx} + \sum_{l=1}^{j-1}(l-1)u_{j-l}Q_l\phi_x - i(u_{j-2,t} + (j-2)u_{j-1}\phi_t)\Big] + u_0^2 \\
&\Big[-u_{j-2,xx}^* - (j-2)(2u_{j-1,x}^*\phi_x + u_{j-1}^*\phi_{xx}) + \sum_{l=0}^{j} u_{j-l-1}Q_{lx} + \sum_{l=1}^{j-1}(l-1)u_{j-l}^*Q_l\phi_x \\
&+ i(u_{j-2,t}^* + (j-2)u_{j-1}^*\phi_t)\Big] + (j-3)ju_0\phi_x^2\left(-Q_{j-1,y} + \sum_{l=1}^{j-1} u_{j-l}u_l^*\right)
\end{aligned} \tag{4}$$

$$\begin{aligned}
&j(j+1)(j-3)(j-4)\phi_x^3\phi_y u_j^* = (j^2-3j-2)\phi_x\phi_y\Big[-u_{j-2,xx}^* - (j-2)(2u_{j-1,x}^*\phi_x + u_{j-1}^*\phi_{xx}) \\
&+\sum_{l=0}^{j} u_{j-l-1}^*Q_{lx} + \sum_{l=1}^{j-1}(l-1)u_{j-l}^*\phi_xQ_l + i(u_{j-2,t}^* + (j-2)u_{j-1}^*\phi_t)\Big] + u_0^{*2} \\
&\Big[-u_{j-2,xx} - (j-2)(2u_{j-1,x}\phi_x + u_{j-1}\phi_{xx}) + \sum_{l=0}^{j} u_{j-l-1}Q_{jx} + \sum_{l=1}^{j-1}(l-1)u_{j-l}Q_l\phi_x \\
&- i(u_{j-2,t} + (j-2)u_{j-1}\phi_t)\Big] + j(j-3)u_0^*\phi_x^2(-Q_{j-1,y} + \sum_{l=1}^{j-1} u_{j-l}u_l^*)\,,
\end{aligned} \tag{5}$$

$$\begin{aligned}
&(j+1)(j-1)(j-4)Q_j\phi_y\phi_x^2 = j(j-3)\phi_x^2\left(-Q_{j-1,y} + \sum_{l=1}^{j-1} u_{j-l}u_l^*\right) + u_0^* \\
&\Big[-u_{j-2,xx} - (j-2)(2u_{j-1,x}\phi_x + u_{j-1}\phi_{xx}) + \sum_{l=0}^{j} u_{j-l-1}Q_{lx} + \sum_{l=1}^{j-1}(l-1)u_{j-l}Q_l\phi_x \\
&- i(u_{j-2,t} + (j-2)u_{j-1}\phi_t)\Big] + u_0\Big[-u_{j-2,xx}^* - (j-2)(2u_{j-1,x}^*\phi_x + u_{j-1}^*\phi_{xx}) + \sum_{l=0}^{j} u_{j-l-1}^*Q_{lx} \\
&+ \sum_{l=1}^{j-1}(l-1)u_{j-l}^*Q_l\phi_x + i(u_{j-2,t}^* + (j-2)u_{j-1}^*\phi_t)\Big]\,.
\end{aligned} \tag{6}$$

From Eqs. (4)–(6), we see that "resonances" occur at $j = -1, 0, 1, 3, 4$. Resonances are those values of j at which it is possible to introduce an arbitrary function into the expansions (2). For each nontrivial resonance there occurs a compatibility condition which must be satisfied if the solution has a single-valued expansion. The resonance at $j = -1$ corresponds to the arbitrary function ϕ defining the singularity manifold for the (2+1)=dimensional NLS equation. After detailed calculation, we find the compatibility conditions at j=0,3,4 are satisfied identically for Eqs.(4) and (5), and the compatibility conditions at j=1,4 are also satisfied for Eq.(6). According to the Weiss-Kruskal approach, Eqs.(1) possess Painlevé properties. And the (2+1)-dimensional nonlinear Schrödinger equation system (1) is integrable. On the other hand, symmetry study plays an important role. For every known integrable model there are infinitely many symmetries. For many (2+1)-dimensional integrable models, they possess generalized W_∞ symmetry algebra.[3] While for other kinds of (2+1)-dimensional integrable models they possess only some types of Kac-Mcody-Virasoro symmetry algebra instead of generalized W_∞ algebra.[4] In the next section we would like to study the symmetry algebra of Eq.(1).

Ⅲ. SYMMETRIES AND ALGEBRA OF THE (2+1)-DIMENSIONAL NLS EQUATION

A. Symmetries with arbitrary functions of t

A symmetry

$$\sigma = \begin{pmatrix} p \\ q \\ r \end{pmatrix} \tag{7}$$

of the (2+1)-dimensional NLS is defined as, if σ satisfies the linearlized equations (1),

$$\begin{aligned} p_{xx} &= pQ_x + ur_x - ip_t \ , \\ q_{xx} &= qQ_x + u^* r_x + iq_t \ , \\ r_y &= pu^* + uq \ . \end{aligned} \tag{8}$$

This means that Eqs.(1) are form-invariant under the transformation

$$(u, u^*, Q) \longrightarrow (u, u^*, Q) + \epsilon(p, q, r)^T \ ,$$

where ϵ is an infinitesimal. Similarly to the KP equation[3], we look for the symmetries of Eqs.(1), which have the form

$$\begin{aligned} p_n(f) &= \sum_{k=0}^{\infty} f^{(n-1-k)} P_n[k] \ , \\ q_n(f) &= \sum_{k=0}^{\infty} f^{(n-1-k)} Q_n[k] \ , \\ r_n(f) &= \sum_{k=0}^{\infty} f^{(n-k)} R_n[k] \ , \end{aligned} \tag{9}$$

with f being an arbitrary function of t, $f^{(k)} = \left(\frac{\partial^k}{\partial t^k}\right) f$ and $P_n[k]$, $Q_n[k]$ and $R_n[k]$ are functions of x, y, u, u^*, Q and their derivatives, but they are not time-dependent explicitly. The explicit time dependencies of $p_n(f)$, $q_n(f)$ and $r_n(f)$ have been exported out in $f(t)$ and its derivatives.

Substituting Eq.(9) into Eq.(8) and considering the non-trivial condition $\sigma \neq 0$ yields the only possibility:

$$i\sum_{k=0}^{\infty} f^{(n-k)} P_n[k] + \sum_{k=0}^{\infty} f^{(n-1-k)}(iP_{nt}[k] + P_{nxx}[k] - P_n[k]Q_x) - \sum_{k=0}^{\infty} f^{(n-k)} uR_{nx}[k] = 0 \,, \tag{10}$$

$$i\sum_{k=0}^{\infty} f^{(n-k)} Q_n[k] + \sum_{k=0}^{\infty} f^{(n-1-k)}(iQ_{nt}[k] - Q_{nxx}[k] + Q_n[k]Q_x) + \sum_{k=0}^{\infty} f^{(n-k)} u^* R_{nx}[k] = 0 \,, \tag{11}$$

$$\sum_{k=0}^{\infty} f^{(n-k)} R_{ny}[k] - \sum_{k=0}^{\infty} f^{(n-1-k)} u^* P_n[k] - \sum_{k=0}^{\infty} f^{(n-1-k)} uQ_n[k] = 0 \,. \tag{12}$$

Since $f = f(t)$ is an arbitrary function of t, Eqs.(10)–(12) should be true at any order of differentiation of $f(t)$. That means the following equations should be satisfied.

$$R_{ny}[0] = 0; \quad i.e. \quad R_n[0] = g_n(x) \,, \tag{13}$$

$$P_n[0] = -iuR_{nx}[0] \,, \tag{14}$$

$$Q_n[0] = iu^* R_{nx}[0] \,, \tag{15}$$

$$P_n[k] = -P_{nt}[k-1] + iP_{nxx}[k-1] - iP_n[k-1]Q_x - iuR_{nx}[k] \,, \tag{16}$$

$$Q_n[k] = -Q_{nt}[k-1] - iQ_{nxx}[k-1] + iQ_n[k-1]Q_x + iu^* R_{nx}[k] \,, \tag{17}$$

$$R_{ny}[k] = u^* P_n[k-1] + uQ_n[k-1] \,, \tag{18}$$

where $D_y D_y^{-1} = D_y^{-1} D_y = 1$. Equations (16)–(18) can be solved recursively. The result reads

$$\begin{pmatrix} P[k] \\ Q[k] \\ R[k] \end{pmatrix} = \begin{pmatrix} -D_t + iD_x^2 - iQ_x - iu(D_y^{-1}u_x^* + D_y^{-1}u^* D_x) & -iuD_y^{-1}(u_x + uD_x) & 0 \\ iu^* D_y^{-1}(u_x^* + u^* D_x) & -D_t - iD_x^2 + iQ_x + iu^* D_y^{-1}(u_x + uD_x) & 0 \\ D_y^{-1}u^* & D_y^{-1}u & 0 \end{pmatrix}$$

$$\times \begin{pmatrix} P_n[k-1] \\ Q_n[k-1] \\ 0 \end{pmatrix} \equiv L \begin{pmatrix} P_n[k-1] \\ Q_n[k-1] \\ 0 \end{pmatrix}$$

$$= L^2 \begin{pmatrix} P_n[k-2] \\ Q_n[k-2] \\ 0 \end{pmatrix} = \cdots = L^k \begin{pmatrix} P_n[0] \\ Q_n[0] \\ 0 \end{pmatrix} \equiv L^k \begin{pmatrix} -iug_x(x) \\ iu^* g_x(x) \\ 0 \end{pmatrix}, \quad k = 1, 2, \ldots \tag{19}$$

Now the question is: What type of function $g_n(x)$ will result in truncated series symmetries? Fortunately, if $g_n(x)$ is taken to be

$$g_n(x) = \frac{x^n}{2^{n-1} n!}, \tag{20}$$

Eqs.(19) are nontrivial truncated symmetries. According to Eqs.(9) with Eq.(20), after some detailed calculation, we can get five truncated symmetries.

$$K_0(f) = 2 \begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix}, \tag{21}$$

$$K_1(f) = \begin{pmatrix} -iuf \\ iu^* f \\ x\dot{f} \end{pmatrix}, \tag{22}$$

$$K_2(f) = \frac{1}{4} \begin{pmatrix} 4u_x f - 2ixu\dot{f} \\ 4u_x^* f + 2ixu^* \dot{f} \\ 4Q_x f + x^2 \ddot{f} \end{pmatrix}, \tag{23}$$

$$K_3(f) = \frac{1}{24} \begin{pmatrix} 24u_t f + 6(u + 2xu_x)\dot{f} - 3ix^2 \ddot{f} u \\ 24u_t^* f + 6(u^* + 2xu^*)\dot{f} + 3ix^2 u^* \ddot{f} \\ 24Q_t f + 12(Q + xQ_x)\dot{f} + x^3 \ddot{f} \end{pmatrix}, \tag{24}$$

$$K_4(f) = \frac{1}{2^3 4!} \begin{pmatrix} f(192iu_{xt} + 96u_x Q_x - 48uQ_{xx} - 48iuQ_t + 96u\partial_y^{-1}(u^* u_x - uu_x^*)_x) \\ f(-192iu_{xt}^* + 96u_x^* Q_x - 48u^* Q_{xx} + 48iu^* Q_t - 96u^* \partial_y^{-1}(u^* u_x - uu_x^*)_x) \\ f(192i\partial_y^{-1}(u^* u_{xt} - uu_{xt}^*) + 48Q_x^2 - 96\partial_y^{-1}(Q_y Q_{xx})) \end{pmatrix}$$

$$+ \begin{pmatrix} \dot{f}(96xu_t + 96iu_x - 48iuQ) + \ddot{f}24(x^2 u_x + xu) - f^{(3)} 4iux^3 \\ \dot{f}(96xu_t^* - 96iu_x^* + 48iu^* Q) + 24(x^2 u_x^* + xu^*)\ddot{f} + f^{(3)} 4ix^3 u^* \\ \dot{f}(96xQ_t + 96i\partial_y^{-1}(u_x u^* - uu_x^*)) + 24(x^2 Q_x + 2xQ)\ddot{f} + x^4 f^{(4)} \end{pmatrix}. \tag{25}$$

Take f=1 in Eqs.(21)–(25), five time-independent symmetries of the (2+1)-dimensional NLS can be obtained. If we take $f = t$ in $K_4(f)$, we get

$$K_4(t) = tK_4(1) + M. \tag{26}$$

One can readily verify that

$$M = \frac{1}{2^3 4!}\begin{pmatrix} 96xu_t + 96iu_x - 48iuQ \\ 96xu_t^* - 96iu_x^* + 48iu^*Q \\ 96xQ_t + 96i\partial_y^{-1}(u_x u^* - uu^*) \end{pmatrix} \tag{27}$$

is a master symmetry of Eqs.(1). Now using the standard master symmetry approach[10,11], we get a set of the time-independent commuting symmetries.

$$\frac{1}{2}(1-n)K_n = \frac{1}{2}(1-n)K_n(1) = [M, K_{n-1}] = [K_4(t), K_{n-1}] \quad n = 1, 2, \ldots, \quad K_{-1} = 0 \tag{28}$$

$$[K_m, K_n] = 0\,, \tag{29}$$

where the Lie product $[A, B]$ is defined as

$$[A, B] \equiv \frac{\partial}{\partial \epsilon}[A(F + \epsilon B) - B(F + \epsilon A)]\,. \tag{30}$$

Now replacing $K_4(t)$ in Eq.(28) by $K_4(f)$, we get the generaliztion of $K_n(f)$ for all time-independent $K_n(1)$:

$$\frac{1}{2}(1-n)K_{n+1}(f) = [K_4(f_1), K_n(1)] \quad f = \dot{f}_1 \quad K_{-1}(f) = 0 \quad n = 1, 2, \ldots \tag{31}$$

Similarly to the KP case in Eq.(31)[9], we have defined the Lie algebraic meaning of D_y^{-1} by

$$D_y^{-1}c = yc + h(x, t)\,, \tag{32}$$

where c is a constant and h is an arbitrary function of x and t which should be determined directly from the symmetry definition equations (8).

After finishing the detailed calculations, we can prove that the generalized symmetries $K_n(f)$ constitute a closed infinite-dimensional Lie algebra.

$$[K_n(f_1), K_m(f_2)] = -\frac{1}{2}K_{n+m-3}((m-1)\dot{f}_1 f_2 - (n-1)\dot{f}_2 f_1), \quad K_{n+m-3} = 0 \ \ (n+m-3 < 0)\,. \tag{33}$$

B. Existence of another set of symmetries with arbitrary functions y

In addition to the $K_n(f)$ symmetries, there may be other types of symmetries for the (2+1)-dimensional NLS. For instance, we can look for the symmetries of Eqs.(1) which have the form

$$\begin{aligned} p_n(h) &= \sum_{k=0}^{\infty} h^{(n-1-k)} P_n[k]\,, \\ q_n(h) &= \sum_{k=0}^{\infty} h^{(n-1-k)} Q_n[k]\,, \\ r_n(h) &= \sum_{k=0}^{\infty} h^{(n-k)} R_n[k]\,, \end{aligned} \tag{34}$$

where h is an arbitrary function of y, $h^{(k)} = \left(\frac{\partial^k}{\partial y^k}\right) h(y)$, and $P_n[k]$, $Q_n[k]$ and $R_n[k]$ are y-independent explicitly. The explicit y dependencies of $P_n[k]$, $Q_n[k]$ and $R_n[k]$ have been separated out in $h^{(k)}$. Substituting Eq.(39) into Eq.(8) yields

$$R[0] = 0, \quad R[1] = 0 \,, \tag{35}$$

$$P_{nxx}[0] = Q_x P_n[0] - iP_{nt}[0] \,, \tag{36}$$

$$Q_{nxx}[0] = Q_x Q_n[0] + iQ_{nt}[0] \,, \tag{37}$$

$$R_n[2] = u^* P_n[0] + uQ_n[0] \,, \tag{38}$$

$$P_{nxx}[k-1] = Q_x P_n[k-1] + uR_{nx}[k] - iP_{nt}[k-1] \,, \tag{39}$$

$$Q_{nxx}[k-1] = Q_x Q_n[k-1] + u^* R_{nx}[k] + iQ_{nt}[k-1], \quad k = 2, 3, \ldots \tag{40}$$

$$R_n[k] = -R_{ny}[k-1] + u^* P_n[k-2] + uQ_n[k-2]. \quad k = 3, 4, \ldots \tag{41}$$

To find all the possible solutions of Eqs.(35)–(41) is quite difficult. Here we give a special example. One can easily verify that

$$P_n[0] = u; \quad Q_n[0] = u^*; \quad P_n[1] = 2u_y; \quad Q_n[1] = 2u_y^*; \quad R_n[2] = 2Q_y; \tag{42}$$

$$P_n[k] = Q_n[k] = R_n[k+1] = 0, \quad k > 1 \tag{43}$$

i.e.

$$Y_0 = \begin{pmatrix} 2h(y)u_y + h(y)u \\ 2h(y)u_y^* + h(y)u^* \\ 2h(y)Q_y \end{pmatrix} \tag{44}$$

is a non-trivial solution of Eqs.(35)–(41). The commutation relations of the algebra constituted by $K_n(f)$ and $Y(0)$ are given by Eqs.(45) and (46).

$$[Y_0(h), K_n(f)] = 0, \quad n = 0, 1, 2, \ldots \tag{45}$$

$$[Y_0(h_1), Y_0(h_2)] = 2Y_0(h_1\dot{h}_2 - h_2\dot{h}_1) \,. \tag{46}$$

UP to now, we have not yet found any other solutions of Eqs.(35)–(41). Whether there exist any more nontrivial solutions of Eqs.(35)–(41) is worthy of further study.

IV. SPECIAL SOLUTION OF EQUATIONS (1) FROM VARIABLE SEPARATION APPROACH

In (2+1)-dimensional case, the soliton solutions have much more abundant structures.[6] In this section, we study the soliton solutions of Eqs.(1) by means of the variable separator approach.

From the Painlevé analysis of section Ⅱ, if we take the truncated expansion, i.e., the first terms of Eq.(2),

$$u = u_0\phi^{-1}, \quad u^* = u_0^*\phi^{-1}, \quad Q = Q_0\phi^{-1} \,, \tag{47}$$

Eqs.(1) can be written as

$$(D_x^2 + iD_t)u_0\phi = 0 \,, \tag{48}$$

$$(D_x^2 - iD_t)u_0^*\phi = 0 \,, \tag{49}$$

$$D_xD_y\phi\phi = -|u_0|^2 \,, \tag{50}$$

where D is the usual bilinear operator defined by

$$D_t^m D_t^n D_t^p \phi\phi \equiv (\partial_t - \partial_{t'})^m(\partial_x - \partial_{x'})^n(\partial_y - \partial_{y'})^p \phi(x,y,t)\phi(x',y',t')|_{(x=x',y=y',t=t')} \,. \tag{51}$$

In order to get some solutions of (2+1)-dimensional NLS different from exponential, we look for the solutions (48)–(50) by means of variable separation procedure. For this, u_0, u_0^*, ϕ can be assumed to possess the following variable separation form according to Ref.[6].

$$\phi = f_1(x,t) + g_1(y,t) + cf_1(x,t)g_1(y,t) \,, \tag{52}$$

$$u_0 = p(x,t)q(y,t)\exp(i(r(x,t) + s(y,t))) \,, \tag{53}$$

$$u_0^* = p(x,t)q(y,t)\exp(-i(r(x,t) + s(y,t))) \,, \tag{54}$$

where $f_1 = f_1(x,t)$, $p = p(x,t)$, $r = r(x,t)$ and $g_1 = g_1(x,t)$, $q = q(y,t)$, $s = s(y,t)$ are y and x independent respectively, and c is an arbitrary constant. Substituting Eqs.(52)–(54) into Eqs.(48)–(50), we find that $p = p(x,t)$ and $q = q(y,t)$ should be related to f and g by

$$p = 2^{1/4} f_{1x}^{1/2}, \quad q = 2^{1/4} g_{1y}^{1/2} \,. \tag{55}$$

After substituting Eqs.(52)–(54) in Eq.(48) with the consideration of Eq.(55) and vanishing of both real and imaginary parts respectively, we obtain

$$f_{1xx}^2 - 2f_{1x}f_{1xxx} + 4f_{1x}^2 r_x^2 + 4f_{1x}^2 r_t + 4f_{1x}^2 s_t = 0 \,, \tag{56}$$

$$\left[-2(1 - cg_1)g_{1y}^2 f_{1x}^2 + (f_1 + g_1 + cf_1g_1)f_{1x}g_{1y}^2 \frac{\partial}{\partial x}\right](f_{1t} + 2f_{1x}r_x)$$
$$+ \left[-2(1 + cf_1)f_{1x}^2 g_{1y}^2 + (f_1 + g_1 + cf_1g_1)f_{1x}^2 g_{1y}\frac{\partial}{\partial y}\right] g_{1t} = 0 \,. \tag{57}$$

According to the fact that f_1 and r are only functions of x and t and g_1 and s are only functions of y and t, we have the following four equations

$$s(t) = -c_2(t) \,, \tag{58}$$

$$f_{1t} + 2f_{1x}r_x + c_1(t)(1 + cf_1) = 0 \,, \tag{59}$$

$$g_{1t} - c_1(t)(1 + cg_1) = 0 \,, \tag{60}$$

$$f_{1xx}^2 - 2f_{1x}f_{1xxx} + 4f_{1x}^2 r_x + 4f_{1x}^2 r_t + 4f_{1x}^2 c_2(t) = 0 \,, \tag{61}$$

where $c_2(t)$ and $c_1(t)$ are two arbitrary functions of t. From Eqs.(58) and (59), we have ($c \neq 0$)

$$r(x,t) = -\int^x \frac{f_t}{2f_x}\mathrm{d}x + c_3(t) \,, \tag{62}$$

$$s(y,t) = -\int^t c_2(t)\mathrm{d}x + s(y)\,, \tag{63}$$

$$g_1 = B(g(y) + \int c_1(t)B^{-1}\mathrm{d}t)\,, \tag{64}$$

with

$$f = f(x,t) = \left(f_1(x,t) + \frac{1}{c}\right)B, \quad B = \exp\int^t cc_1(t')\mathrm{d}t'\,, \tag{65}$$

and $c_3(t)$ being integration functions of time t, $g(y)$ being integration functions of y. Substituting Eq.(62) into Eq.(61) leads to one (1+1)-dimensional equation in a trilinear form about field $f(x,t)$

$$f_{xx}f_t^2 + f_{tt}f_x^2 - 2f_tf_{xt}f_x + f_{xx}^3 + f_x^2f_{xxxx} - 2f_xf_{xx}f_{xxx} = 0\,. \tag{66}$$

The integrability of Eq.(66) is guaranteed by the integrability of the (2+1)-dimensional NLS equation.

Now the important conclusion is that when we get any solutions of $f(x,t)$ from Eq.(66), we can get a corresponding solution of the (2+1)-dimensional NLS equation while the variable x and y are completely separated to f and g.

To give out detail solutions of Eq.(66) is still a very difficult task. Here we only write down a special solution of Eq.(66), (and then solution of the (2+1)-dimensional NLS equation). The simplest solution of Eq.(66) possess exponential forms.

$$f = a_0 + a\exp(kx + \omega t)\,, \tag{67}$$

with arbitrary constants a_0, a, k, ω.

Substituting Eqs.(52) and (53) with Eqs.(55), (62)–(65) and (67) into Eq.(47) we get the so-called dromion solutions

$$u = (H_1H_2)^{-1/2}(2akg_y)^{1/2}\mathrm{sech}\frac{1}{2}\left(kx + \omega t + \ln\frac{H_2}{H_1}\right)\exp i\left(\frac{\omega}{2k}x + c_4(t) + s(y)\right)\,, \tag{68}$$

with

$$H_1 = a_0B^{-1} + a_0c\left(g(y) + \int c_1(t)B^{-1}\mathrm{d}t\right) - \frac{1}{c}\,, \tag{69}$$

$$H_2 = a\left(B^{-1} + c(g(y) + \int c_1B^{-1}\mathrm{d}t)\right)\,. \tag{70}$$

If we take $c_1 = 0$, $c_0 = \dfrac{1}{c}$ then

$$(H_1H_2)^{-1/2}(2akg_y)^{1/2} = \left[\frac{2kg_y}{g(1+cg)}\right]^{1/2} \equiv G(y)\,, \tag{71}$$

$$u = G(y)\mathrm{sech}\frac{1}{2}\left(kx + \omega t + \ln\frac{a(1+cg(y))}{g(y)}\right)\exp i\left(-\frac{\omega}{2k}x + c_4(t) + s(y)\right)\,. \tag{72}$$

In Eq.(71) because $g(y)$ is an arbitrary function of y, we can take $G(y)$ as an arbitrary function of y. To understand the meaning of solutions (72), we discuss some special cases.

A. Single dromion driven by one line soliton (parallel to the x axis) and one curve soliton

If $G(y)$ is fixed as a single line soliton which is parallel to the x-axis, and we combine the line soliton $G(y)$ and curve soliton $\mathrm{sech}^2\frac{1}{2}\left(kx+\omega t+\ln\frac{a(1+cg(y))}{g(y)}\right)$ together properly (i.e. multiply them together simply in the case of Eq.(72)), (become ghosts) and only a single peak localized in all directions, which is called a dromion, survives. The dromion is located at the interaction of the line and curve solitons. Because $G(y)$ is an arbitrary function of y, the single dromion still possesses rich structures. Here are three concrete simple examples,

$$u_1=\mathrm{sech}^n(y-y_0)\mathrm{sech}\frac{1}{2}(kx+\omega t+\ln(a(g^{-1}+c)))\exp\left(i\left(-\frac{\omega}{2k}x+c_4(t)+s(y)\right)\right)$$
$$=h_1(y)\mathrm{sech}\frac{1}{2}(kx+\omega t+\ln(a(g^{-1}+c)))\exp\left(i\left(-\frac{\omega}{2k}x+c_4(t)+s(y)\right)\right)\ , \qquad (73)$$

$$u_2=\mathrm{sech}^n(\cosh(y-y_0-1))\mathrm{sech}\frac{1}{2}(kx+\omega t+\ln(a(g^{-1}+c)))\exp\left(i\left(-\frac{\omega}{2k}x+c_4(t)+s(y)\right)\right)$$
$$=h_2(y)\mathrm{sech}\frac{1}{2}(kx+\omega t+\ln(a(g^{-1}+c)))\exp\left(i\left(-\frac{\omega}{2k}x+c_4(t)+s(y)\right)\right)\ , \qquad (74)$$

$$u_3=((y-y_0)^{2n}+1)^{-1}\mathrm{sech}^2\frac{1}{2}(kx+\omega t+\ln(a(g^{-1}+c)))\exp\left(i\left(-\frac{\omega}{2k}x+c_4(t)+s(y)\right)\right)$$
$$=h_3(y)\mathrm{sech}^2\frac{1}{2}(kx+\omega t+\ln(a(g^{-1}+c)))\exp\left(i\left(-\frac{\omega}{2k}x+c_4(t)+s(y)\right)\right)\ , \qquad (75)$$

the first type of dromion solution, u_1, decays exponentially in all direction. The second type of dromion solution, u_2, decays much more quickly than the first in the y direction. While the third type of dromion solution, u_3, decays much slower than the first in the y direction.

B. Muti-dromion 'bounded' states

If $G(y)$ is selected to be N parallel line solitons (parallel to the x-axis), then we obtain an N-dromion 'bound' state

$$u_{NB}=\sum_{j=1}^{N}f_j(y)\mathrm{sech}^2\frac{1}{2}(kx+\omega t+\ln(a(g^{-1}+c)))\exp\left(i\left(-\frac{\omega}{2k}x+c_4(t)+s(y)\right)\right)\ , \qquad (76)$$

driven by N line ghost solitons and curved line ghost soliton. In Eq.(76), $f_j(y)$ can be selected quite freely, say h_1, h_2, h_3 in Eqs.(73)–(75). Because all N parallel line solitons are static in y direction, the N-dromion can only move with the same speed in the x direction as the curved line ghost soliton moves and they cannot pass through each other. In other words, the behaviour of these dromions looks like a bound state.

V. SUMMARY AND DISCUSSION

(2+1)-dimensional NLS equation can be obtained from the symmetry constraint of the KP equation.[1] We have discussed the Painlevé property, symmetries and solution for the (2+1)-dimensional NLS equations. Using Weiss-Kruskal approach, we checked the Painlevé property of (2+1)-dimensional NLS equations and proved that Eqs.(1) is integrable. Using a

formal series symmetry method given by Lou, a set of generalized symmetries with arbitrary functions of t is given. These symmetries constitute an infinite dimensional Lie algebra. The generalized symmetry of the y-translation contains and arbitrary function of space (y) instead of time (t), which means there may exists another set of generalized symmetries with an arbitrary function of y. Unfortunately we do not yet get any other non-trivial solution except Y_0. By taking the variable separation approach, the solutions of the NLS equation (1) can be obtained from a (1+1)-dimensional trilinear equation. Many types of dromion solutions for the (2+1)-dimensional NLS equation are just the special case in which the solutions of (1+1)-dimensional trilinear equation is fixed as the exponential. The dromions can be driven not only by perpendicular line ghost solitons but also by some nonperpendicular line and curved line ghost solitons. The dromions can also possess some quite free shapes. For instance, they may decay extremely faster or much slower than the exponentially decayed in the y direction because an arbitrary function is included in the general dromion solution. Here we only give out a special type of solution for the trilinear equation (65). To look for more solutions of the model, further studies are worth while to do.

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