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RELATIVISTIC MEAN-FIELD APPROACHES FOR LIGHT HYPERNUCLEI

SHEN YAO-SONG(沈姚崧) and REN ZHONG-ZHOU(任中洲)
Department of Physics, Nanjing University, Nanjing 210008, China

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We study the ground state properties of light hypernuclei theoretically using the relativistic mean-field theory with TM1 and NL-SH force parameters. It is found that these new parameters can satisfactorily reproduce the ground-state properties of the light hypernuclei even near the neutron drip-line. The parameter sets TM1 and NL-SH give a very similar description of hypernuclei. The properties of hypernuclei in baryon-hyperon systems are insensitive to the force parameters of effective nucleon-nucleon interactions. We also take into account the influence of pairing forces on the Λ hyperon and find that it leads to an even-odd effect in light hypernuclei.

PACC: 2180; 2110; 2160; 2100

1. INTRODUCTION

The study of hypernuclei has attracted much attention in theory and experiment since the first observation of the decay of a hypernucleus in nuclear emulsion.[1] Theoretically, in many studies, the relativistic mean-field (RMF) theory has proved to be very successful for describing many nuclear properties during the last decade.[2-6] The success of the RMF theory in the nuclear system inspired quite naturally attempts to describe the more general baryon-system with strangeness within a same formalism.[7-16]

Recently, Sharma *et al.*[3,4] have proposed a new set of RMF force parameters named NL-SH and Sugahara *et al.*[6] have proposed another RMF parameter set TM1 by introducing the nonlinear term for the ω-meson into the RMF theory. Both the new RMF parameter sets can satisfactorily reproduce ground-state properties of nuclei, such as the binding energy and the isotope shifts of charge distributions. The above RMF parameter sets have been used to investigate the properties of hypernuclei.[7,8] However, there is no detailed comparison between the two parameter sets.

Although no enough experimental information is available for hypernuclei, as far as light hypernuclei is concerned, some ground-state properties of hypernuclei have been measured using the gamma spectroscopy.[11,17,18]

In this paper, we will investigate the ground-state properties of light hypernuclei in elements He, Li, Be, B, and O with the new RMF parameter sets TM1 and NL-SH. Our purposes are: (i) to study the ground-state properties of light hypernuclei not only for nuclear cores near the stable line, but also for those far from the stable line in the RMF theory; (ii) to discuss the common points of and the difference between the two RMF parameter sets for hypernuclei.

This paper is organized in the following way. Section Ⅱ is a short description on the framework of the RMF theory for hypernuclei. In section Ⅲ, we give numerical

results and discussions. Section Ⅳ is the summary.

II. FRAMEWORK OF THE RELATIVISTIC MEAN-FIELD (RMF) THEORY WITH HYPERNUCLEI

Our starting point in describing the nuclear many-body problem with the strangeness degree of freedom is the effective Lagrangian density for interacting nucleons and the σ, ω, and ρ mesons

$$\mathscr{L}_{\text{total}} = \mathscr{L}_{\text{N}} + \mathscr{L}_{\Lambda}, \tag{1}$$

$$\begin{aligned}\mathscr{L}_{\text{N}} = {} & \bar{\Psi}_{\text{N}}\Big(\mathrm{i}\gamma^{\mu}\partial_{\mu} - M_{\text{N}} - g_{\sigma\text{N}}\sigma - g_{\omega\text{N}}\gamma^{\mu}\omega_{\mu} - g_{\rho\text{N}}\gamma^{\mu}\rho_{\mu}^{a}\tau^{a} - e\gamma^{\mu}A^{\mu}\frac{1}{2}(1-\tau^{3})\Big)\Psi_{\text{N}} \\ & + \frac{1}{2}\partial^{\mu}\sigma\partial_{\mu}\sigma - \frac{1}{2}m_{\sigma}^{2}\sigma^{2} - \frac{1}{3}g_{2}\sigma^{3} - \frac{1}{4}g_{3}\sigma^{4} + \frac{1}{4}c_{3}(\omega_{\mu}\omega^{\mu})^{2} \\ & - \frac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu} + \frac{1}{2}m_{\omega}^{2}\omega^{\mu}\omega^{\mu} - \frac{1}{4}R^{a\mu\nu}\cdot R_{\mu\nu}^{a} + \frac{1}{2}m_{\rho}^{2}\rho^{a\mu}\cdot\rho_{\mu}^{a} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu},\end{aligned} \tag{2}$$

$$\mathscr{L}_{\Lambda} = \bar{\Psi}_{\Lambda}\Big(\mathrm{i}\gamma^{\mu}\partial_{\mu} - M_{\Lambda} - g_{\sigma\Lambda}\sigma - g_{\omega\Lambda}\gamma^{\mu}\omega_{\mu} + \frac{f_{\omega\Lambda}}{2M_{\Lambda}}\sigma^{\mu\nu}\partial_{\mu}\omega_{\nu}\Big)\Psi_{\Lambda}, \tag{3}$$

with

$$\Omega^{\mu\nu} = \partial^{\mu}\omega^{\nu} - \partial^{\nu}\omega^{\mu}, \tag{4}$$

$$R^{a\mu\nu} = \partial^{\mu}\rho^{a\nu} - \partial^{\nu}\rho^{a\mu} + g_{\rho}\,\epsilon^{abc}\,\rho^{b\mu}\,\rho^{c\nu}, \tag{5}$$

$$F^{\mu\nu} = \partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu}. \tag{6}$$

In Eqs. (1) and (2), $\mathscr{L}_{\text{N}}$ denotes the standard Lagrangian of the mean-field theory for nuclei. It involves nucleons Ψ_{N}, scaler mesons σ, vector mesons ω_{μ}, vector-isovector mesons ρ_{μ}^{a} and the photon A^{μ} for the Coulomb field. The isospin Pauli matrices are written as τ^{a}, τ^{3} being the third component of τ^{a}. $\mathscr{L}_{\Lambda}$ denotes the Lagrangian of the mean-field theory for Λ hyperon. The contributions of ρ-Λ and A-Λ interacting Lagrangian density are zero. The last term in Eq. (3) is an ω-Λ tensor coupling term.

The inputs in the calculation are the ML-SH and TM1 parameters which are listed in Table 1. The symbols $g_{\sigma\text{N}}$, $g_{\omega\text{N}}$ and $g_{\rho\text{N}}$ are the coupling constants of the σ, ω and ρ

Table 1. Parameter sets TM1 and NL-SH in the RMF theory.

	TM1	NL-SH
M/MeV	938.0	939.0
m_{σ}/MeV	511.198	526.059
m_{ω}/MeV	783.0	783.0
m_{ρ}/MeV	770.0	763.0
g_{σ}	10.0289	10.4436
g_{ω}	12.6139	12.9451
g_{ρ}	4.6322	4.3828
g_{2}/fm^{-1}	−7.2325	−6.9099
g_{3}	0.6183	−15.8337
c_{3}	71.3075	0.000

mesons with nucleons, respectively; g_2 and g_3 are the constants of nonlinear self-interactions of the σ meson, and c_3 is the constant of nonlinear self-interactions of the ω meson.

From the Lagrangian density in Eq.(1), we obtain the equations with static solutions[7-12] by standard techniques of a field variation.

$$\epsilon_{N\alpha}\phi_{N\alpha} = \gamma_0\Big(-i\gamma\cdot\nabla + M_N + g_{\sigma N}\sigma + g_{\omega N}\omega_0\gamma_0 + \frac{1}{2}g_{\rho N}R_{0,0}\tau_0 + e^2A_0\frac{1+\tau_0}{2}\gamma_0\Big)\phi_{N\alpha}, \tag{7}$$

$$\epsilon_{\Lambda\alpha}\phi_{\Lambda\alpha} = \gamma_0\Big(-i\gamma\cdot\nabla + M_\Lambda + g_{\sigma\Lambda}\sigma + g_{\omega\Lambda}\omega_0\gamma_0 - \frac{f_{\omega\Lambda}}{2M_\Lambda}i\gamma\cdot\nabla\omega_0\Big)\phi_{\Lambda\alpha}, \tag{8}$$

$$(-\Delta + m_\sigma)\sigma = -g_{\sigma N}\rho_{sN} - g_{\sigma\Lambda}\rho_{s\Lambda} - g_2\sigma^2 - g_3\sigma^3, \tag{9}$$

$$(-\Delta + m_\omega)\omega_0 = g_{\omega N}\rho_{0N} + g_{\omega\Lambda}\rho_{0\Lambda} + c_3\omega_0^3 - \frac{f_{\omega\Lambda}}{2M_\Lambda}\rho_{T\Lambda}, \tag{10}$$

$$(-\Delta + m_\rho)R_{0,0} = \frac{1}{2}g_\rho\rho_{0,0}, \tag{11}$$

$$-\Delta A_0 = e^2\rho_{Pr,0}. \tag{12}$$

The densities ρ_{sN}, $\rho_{s\Lambda}$, ρ_{0N}, $\rho_{0\Lambda}$, $\rho_{T\Lambda}$, $\rho_{0,0}$ and $\rho_{Pr,0}$ are defined as

$$\rho_{sN} = \sum_{\alpha=1}^{\Omega} w_{N\alpha}\bar{\phi}_{N\alpha}\phi_{N\alpha}, \tag{13}$$

$$\rho_{s\Lambda} = \sum_{\alpha=1}^{\Omega} w_{\Lambda\alpha}\bar{\phi}_{\Lambda\alpha}\phi_{\Lambda\alpha}, \tag{14}$$

$$\rho_{0N} = \sum_{\alpha=1}^{\Omega} w_{N\alpha}\bar{\phi}_{N\alpha}\gamma_0\phi_{N\alpha}, \tag{15}$$

$$\rho_{0\Lambda} = \sum_{\alpha=1}^{\Omega} w_{\Lambda\alpha}\bar{\phi}_{\Lambda\alpha}\gamma_0\phi_{\Lambda\alpha}, \tag{16}$$

$$\rho_{0,0} = \sum_{\alpha=1}^{\Omega} w_{N\alpha}\bar{\phi}_{N\alpha}\gamma_0\tau_0\phi_{N\alpha}, \tag{17}$$

$$\rho_{Pr,0} = \sum_{\alpha=1}^{\Omega} w_{N\alpha}\bar{\phi}_{N\alpha}\frac{1+\tau_0}{2}\gamma_0\phi_{N\alpha}, \tag{18}$$

$$\rho_{T\Lambda} = \nabla\cdot\sum_{\alpha=1}^{\Omega} w_{\Lambda\alpha}\bar{\phi}_{\Lambda\alpha}i\boldsymbol{\alpha}\phi_{\Lambda\alpha}, \tag{19}$$

where α is the single particle states for nucleons or for Λ hyperon. As all the other derivations for the RMF theory can be found in Refs. [7-12], here we do not discuss it further.

The mean-field localizes the nucleus and thus breaks translational invariance which results in the center-of-mass of the whole nucleus oscillating in the mean-field, but the spurious excitation of the center-of-mass has to be eliminated. A simple and reliable treatment on the center-of-mass correction is[6,19]

$$E_{cm} = \frac{<P_{cm}^2>}{2Am}, \tag{20}$$

$$<P_{cm}^2> = \sum_\beta \omega\beta <\phi_\beta|\hat{p}^2|\phi_\beta>$$

$$- \sum_{\alpha\beta} \left(\omega_\alpha \omega_\beta + \sqrt{\omega_\alpha(1-\omega_\alpha)\omega_\beta(1-\omega_\beta)} \right) | < \phi_\alpha | \hat{p} | \phi_\beta > |^2,$$

where $P_{cm} = \sum_i \hat{p}_i$ is the total momentum operator.

III. NUMERICAL RESULTS

For the Λ hyperons, we now have three additional parameters, $g_{\sigma\Lambda}$, $g_{\omega\Lambda}$ and $f_{\omega\Lambda}$. The couplings of the σ and ω to the Λ hyperon, which come from $SU(3)$ model, are $\sigma_{\sigma\Lambda} = g_{\sigma\Lambda}/g_{\sigma N} = 0.621$ and $\alpha_{\omega\Lambda} = g_{\omega\Lambda}/g_{\omega N} = 0.667$.[7] There exist some suggestions, based on the quark model, that predict the tensor-to-vector coupling ratio $f_{\omega\Lambda}/g_{\omega\Lambda} = -1$.[20] In the following, we use the above three parameters together with the relativistic mean-field parameter sets TM1 and NL-SH, to describe the properties of ground states in light hypernuclei.

For open shell hypernuclei, pairing correlations have been included on using the BCS formalism. The contribution of pairing forces is zero for magic numbers 2 and 8. For protons and neutrons, $1p_{3/2}$ can be treated as a subshell and the pairing force is very small for nucleon number 6.[23] It is well known that the angular momentum and parity of nuclei with an odd nucleon only depend on the occupation of the last nucleon or hole while one valence nucleon or one hole is just outside the closed shell. So we do not take into account the effect of pairing forces with numbers 3, 5 and 7 for the above reason and for the conservation of angular momentum. This means that for proton and neutron the pairing correlations have to be taken into account only when the particle number is 4. In calculations, we use constant-force approaches with $G_p = 16.5/(11 + Z)$ and $G_n = 13.5/(11 + N)$.[22]

For Λ particles, the Λ-Λ interactions are not so clear as the baryon-baryon interactions. The pairing correlations for Λ particles in multi-Λ baryonic systems are neglected in previous works.[7-12] In our calculations, we will take into account the influence of pairing correlations for Λ particles with the similar treatment to that in baryons with $G_L = 10.0/(11 + L)$, L is the number of Λ particles. Because the spin-orbit splitting of 1p level is small for Λ particle, the pairing correlations have to be taken into account in the case that the numbers of Λ particle are 4 and 6.

The orbitals with positive single particle energies are cut off and , as a result, the active space is well defined since all orbitals are bound in our calculation.

For odd-hypernuclei, the subspace of occupied states is not invariant under time-reversal, and this gives rise to additional spatial components of the vector fileds. Warrier *et al*.[23] investigated the effect of vector current and pointed out that it is very small for odd-A nuclei. They concluded that it may be neglected if only binding energies and radii are concerned. In our calculations, we do not take into account the effect of vector current for the above reason.

In Fig. 1 we plot the binding energies of the single-particle states of hypernuclei as a function of the mass number $A^{-1/3}$ with $A = 9 - 209$, calculated using the parameter sets TM1 and NL-SH. The experimental data from (π^+, K^+) production[11,18] denoted

by full circles are also given for comparison. The figure shows a good agreement of theoretical results with experimental data for both sets TM1 and NL-SH. The difference between TM1 and NL-SH is very small and this indicates that theoretical results is not sensitive to the input of force parameters.

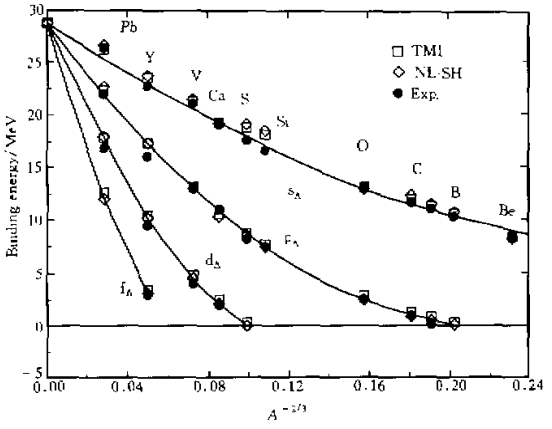

Fig.1. The s_Λ, p_Λ, d_Λ, and f_Λ single-particle energies plotted versus the atomic mass of the nuclei, by using the new RMF parameter sets TM1 and NL-SH in comparison to recent experimental data.[17,18]

In Tables 2 and 3 we list the Λ binding energies of single-Λ and double-Λ hypernuclei, and these quantities are defined as follows:

$$B_\Lambda({}^{A}_{\Lambda}Z) = B({}^{A}_{\Lambda}Z) - B({}^{A-1}Z), \tag{21}$$

$$B_{\Lambda\Lambda}({}^{A}_{\Lambda\Lambda}Z) = B_\Lambda({}^{A}_{\Lambda\Lambda}Z) - B_\Lambda({}^{A-1}_{\Lambda}Z), \tag{22}$$

$$\Delta B_{\Lambda\Lambda}({}^{A}_{\Lambda\Lambda}Z) = B_\Lambda({}^{A}_{\Lambda\Lambda}Z) - B_\Lambda({}^{A-1}_{\Lambda}Z), \tag{23}$$

In Table 2, the experimental data of Λ binding energies of single-Λ light hypernuclei in He, Li, Be, and B are also listed for comparison. We can see from Table 1 that the theoretical results agree well with the experimental data, and the results with parameter set NL-SH are slightly smaller than those with parameter set TM1. We also predict some neutron-rich hypernuclei and find the Λ binding energies increase with the increase of neutrons. This means that these unstable nuclei will get stable when they are bound with one or two Λ hyperons. In Table 3, we list two quantities $B_{\Lambda\Lambda}$ and $\Delta B_{\Lambda\Lambda}$ of double-Λ light hypernuclei. Up to now, the experimental binding energies in $B_{\Lambda\Lambda}$ and $\Delta B_{\Lambda\Lambda}$ are limited to three double-Λ hypernuclei, ${}^{6}_{\Lambda\Lambda}$He, ${}^{10}_{\Lambda\Lambda}$Be and ${}^{13}_{\Lambda\Lambda}$B.[9,17] $B^{\exp}_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\mathrm{He}) = 10.9 \pm 0.6$, $B^{\exp}_{\Lambda\Lambda}({}^{10}_{\Lambda\Lambda}\mathrm{Be}) = 17.7 \pm 0.4$, $B^{\exp}_{\Lambda\Lambda}({}^{13}_{\Lambda\Lambda}\mathrm{B}) = 27.6 \pm 0.7$ and $\Delta B^{\exp}_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\mathrm{He}) = 4.7 \pm 0.6$, $\Delta B^{\exp}_{\Lambda\Lambda}({}^{10}_{\Lambda\Lambda}\mathrm{Be}) = 4.3 \pm 0.4$, $\Delta B^{\exp}_{\Lambda\Lambda}({}^{13}_{\Lambda\Lambda}\mathrm{B}) = 4.9 \pm 0.7$. We can see from Table 3 that the theoretical results of $B_{\Lambda\Lambda}$ agree with the experimental data but the results of $\Delta B_{\Lambda\Lambda}$ are smaller than the experimental data. These results are also obtained in the works of Schaffner *et al*.[9] and are insensitive to the coupling constants.[9] We think that some density-dependent hyperon-hyperon correlations , which are beyond the present RMF

Table 2. The B_Λ(in MeV) of the single-Λ hypernuclei ${}_\Lambda$He, ${}_\Lambda$Li, ${}_\Lambda$Be, ${}_\Lambda$B are defined by Eq. (21). The theoretical values are compared with the experimental data taken from Ref. [11].

	Exp.	TM1	NL-SH		Exp.	TM1	NL-SH
${}^{4}_{\Lambda}$He	2.42	1.35	1.06	${}^{7}_{\Lambda}$Li	5.64	5.57	5.26
${}^{5}_{\Lambda}$He	3.17	2.47	2.10	${}^{8}_{\Lambda}$Li	6.81	6.61	6.40
${}^{6}_{\Lambda}$He	4.42	4.17	3.90	${}^{9}_{\Lambda}$Li	8.59	8.00	7.76
${}^{7}_{\Lambda}$He		5.13	5.00	${}^{10}_{\Lambda}$Li		8.76	8.56
${}^{8}_{\Lambda}$He		6.70	6.46	${}^{11}_{\Lambda}$Li		9.48	9.30
${}^{9}_{\Lambda}$He		7.80	7.63	${}^{12}_{\Lambda}$Li		9.98	9.70
${}^{10}_{\Lambda}$He		8.14	8.08				
${}^{11}_{\Lambda}$He		8.83	8.55				
${}^{7}_{\Lambda}$Be	5.09	3.53	3.16	${}^{9}_{\Lambda}$B	7.89	7.93	7.74
${}^{8}_{\Lambda}$Be	6.91	6.63	6.33	${}^{10}_{\Lambda}$B	8.82	8.91	8.82
${}^{9}_{\Lambda}$Be	6.80	7.63	7.54	${}^{11}_{\Lambda}$B	10.24	10.02	10.05
${}^{10}_{\Lambda}$Be	9.30	9.03	8.88	${}^{12}_{\Lambda}$B	11.45	10.94	11.17
${}^{11}_{\Lambda}$Be		9.81	9.94	${}^{13}_{\Lambda}$B		11.32	11.30
${}^{12}_{\Lambda}$Be		10.35	10.34	${}^{14}_{\Lambda}$B		11.61	11.50
${}^{13}_{\Lambda}$Be		10.75	10.55				

Table 3. The $B_{\Lambda\Lambda}$ and $\Delta B_{\Lambda\Lambda}$(in MeV) of the double-Λ hypernuclei ${}_{\Lambda\Lambda}$He, ${}_{\Lambda\Lambda}$Li, ${}_{\Lambda\Lambda}$Be, ${}_{\Lambda\Lambda}$B, which are defined by Eqs. (22) and (23), respectively.

	TM1		NL-SH			TM1		NL-SH	
	$B_{\Lambda\Lambda}$	$\Delta B_{\Lambda\Lambda}$	$B_{\Lambda\Lambda}$	$\Delta B_{\Lambda\Lambda}$		$B_{\Lambda\Lambda}$	$\Delta B_{\Lambda\Lambda}$	$B_{\Lambda\Lambda}$	$\Delta B_{\Lambda\Lambda}$
${}^{5}_{\Lambda\Lambda}$He	3.51	0.71	2.89	0.77	${}^{8}_{\Lambda\Lambda}$Li	11.64	0.50	10.99	0.47
${}^{6}_{\Lambda\Lambda}$He	5.76	1.26	5.00	1.00	${}^{9}_{\Lambda\Lambda}$Li	13.71	0.44	13.22	0.42
${}^{7}_{\Lambda\Lambda}$He	8.95	0.61	8.22	0.62	${}^{10}_{\Lambda\Lambda}$Li	16.28	0.28	15.86	0.34
${}^{8}_{\Lambda\Lambda}$He	11.14	0.88	10.70	0.70	${}^{11}_{\Lambda\Lambda}$Li	17.89	0.37	17.70	0.58
${}^{9}_{\Lambda\Lambda}$He	13.83	0.43	13.33	0.41	${}^{12}_{\Lambda\Lambda}$Li	19.22	0.26	18.74	0.24
${}^{10}_{\Lambda\Lambda}$He	15.94	0.34	15.60	0.34	${}^{13}_{\Lambda\Lambda}$Li	20.15	0.19	19.61	0.21
${}^{11}_{\Lambda\Lambda}$He	16.97	0.35	16.48	0.32					
${}^{12}_{\Lambda\Lambda}$He	17.98	0.32	17.38	0.28					
${}^{8}_{\Lambda\Lambda}$Be	10.95	0.59	10.33	0.57	${}^{10}_{\Lambda\Lambda}$B	16.20	0.34	15.83	0.35
${}^{9}_{\Lambda\Lambda}$Be	13.67	0.41	13.09	0.43	${}^{11}_{\Lambda\Lambda}$B	18.11	0.29	17.95	0.31
${}^{10}_{\Lambda\Lambda}$Be	15.69	0.43	15.45	0.38	${}^{12}_{\Lambda\Lambda}$B	20.37	0.33	20.28	0.18
${}^{11}_{\Lambda\Lambda}$Be	18.42	0.18	18.02	0.26	${}^{13}_{\Lambda\Lambda}$B	22.14	0.26	22.62	0.28
${}^{12}_{\Lambda\Lambda}$Be	19.93	0.31	20.20	0.32	${}^{14}_{\Lambda\Lambda}$B	22.84	0.20	22.80	0.20
${}^{13}_{\Lambda\Lambda}$Be	20.93	0.23	20.81	0.13	${}^{15}_{\Lambda\Lambda}$B	23.45	0.23	23.18	0.18
${}^{14}_{\Lambda\Lambda}$Be	21.71	0.21	21.30	0.20					

theory, might be quite important for these light hypernuclei on increasing $\Delta B_{\Lambda\Lambda}$.

In Figs. 2 – 5, we plot the total (R_t), the proton(R_p), the neutron(R_n) and the Λ

(R_Λ) root-mean-square(rms) radii of single- and double-Λ light hypernuclei for elements He, Li, Be, and B, respectively. We can see from Fig.2 that the Λ radius rapidly decreases from nucleon number 3 to 4 and then slowly decreases. In the meantime, the neutron and the total radii reach the minimum values in $A = 5$. It is easy to understand that there exists a strong effect of shell in $N = 2$ which also affects the Λ particle strongly. This also indicates that ${}^{5}_{\Lambda}$He is more stable than ${}^{4}_{\Lambda}$He. For ${}^{5}_{\Lambda}$He (${}^{6}_{\Lambda\Lambda}$He) and ${}^{4}_{\Lambda}$He (${}^{5}_{\Lambda\Lambda}$He), the Λ radius is greater than the neutron and the proton radii. This means that one or two Λ particles are loosely bound with nucleons, and this kind of hypernuclei tends to form the structure of the Λ halo. However, with the increase of neutron number, the Λ radius gets smaller than the neutron radius. It seems that the Λ particle will be stable in neutron-rich hypernuclei. For double-Λ hypernuclei ${}_{\Lambda\Lambda}$He, the rms

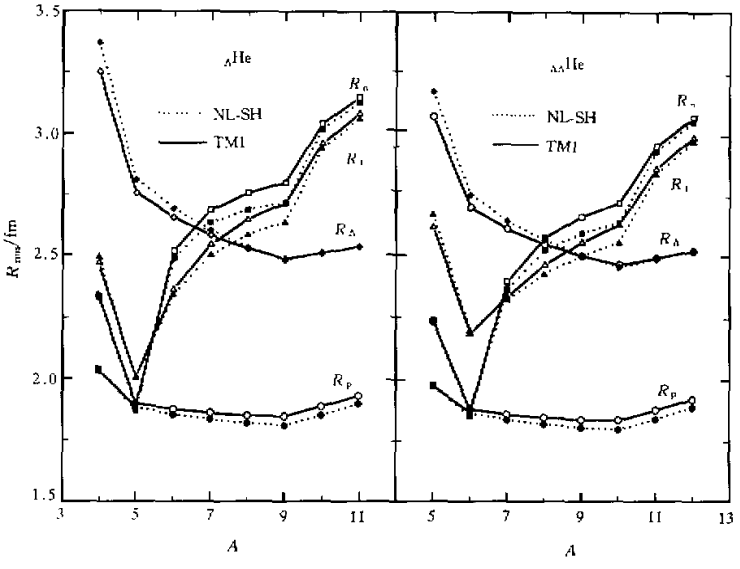

Fig.2. The total (R_t), the proton (R_p), the neutron (R_n) and the Λ (R_Λ) rms radii of single-and double-Λ light hypernuclei ${}_{\Lambda}$He and ${}_{\Lambda\Lambda}$He. The symbols (circle, square, diamond and triangle) denote the proton, the neutron, the Λ and the total rms radii, respectively.

radii have a similar trend to that in single-Λ hypernuclei ${}_{\Lambda}$He. The Λ and neutron radii are slightly smaller than that in ${}_{\Lambda}$He. It can be also explained that there exists an effect of shell for Λ particles in $L = 2$ as that for neutrons in $N = 2$. The only difference is that in the Λ particles the effect of shell is smaller than that in the neutrons because the Λ radius in double-Λ hypernuclei is slightly smaller than that in single-Λ ones.

As for ${}_{\Lambda}$Li(${}_{\Lambda\Lambda}$Li), ${}_{\Lambda}$Be(${}_{\Lambda\Lambda}$Be) and ${}_{\Lambda}$B(${}_{\Lambda\Lambda}$B), there is a similar discussion to that for ${}_{\Lambda}$He(${}_{\Lambda\Lambda}$He). But in these hypernuclei, the proton radius has a large change which is

not same as that in the ${}_{\Lambda}$He(${}_{\Lambda\Lambda}$He) case, in which the proton radius is almost a constant because $Z=2$ is a close shell. In the case of ${}_{\Lambda}$Be(${}_{\Lambda\Lambda}$Be) and ${}_{\Lambda}$B(${}_{\Lambda\Lambda}$B), the proton and the Λ radii are larger than the neutron radii in the proton-rich side. This indicates that there may exist two halo structures, the proton and the Λ halos. From Figs. 2−5, we also can see the difference between the two RMF parameter sets TM1 and NL-SH is very small. It follows that the results of single-Λ and double-Λ hypernuclei are insensitive to the nucleon parameters.

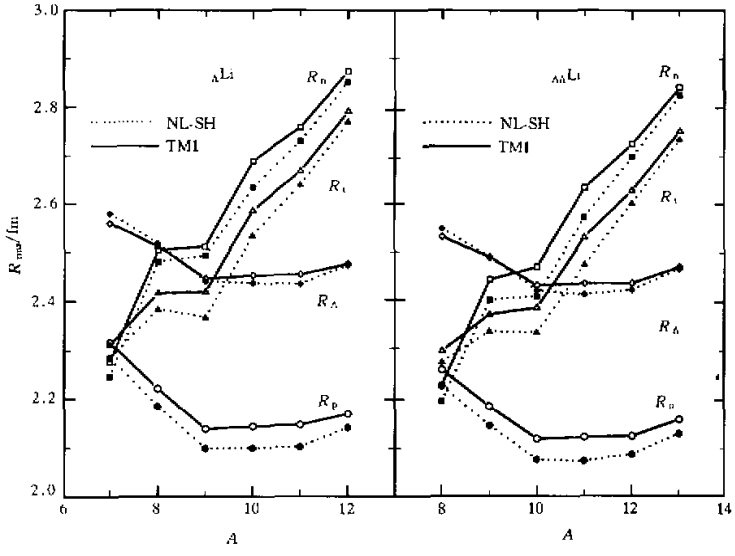

Fig.3. Same as Fig.2, but for ${}_{\Lambda}$Li and ${}_{\Lambda\Lambda}$Li hypernuclei.

In the following work, we will investigate multi-Λ systems for light hypernuclei built out of a spherical nuclear core with a neutron closed shell $N=8$ (^{11}Li, ^{12}Be, ^{13}B and ^{16}O) to which we add n Λ particles. It is well known that the pairing correlations are very important in nucleon-nucleon interactions. But for hyperons, whether the pairing correlations are important or not is unknown since the hyperon-hyperon interactions are more complicated. We treat the Λ particles for two cases: case(a), the pairing correlations are not taken into account; case (b), the pairing correlations are taken into account using the constant-force approach $G_L=10.0/(11+L)$. In Fig.6, we plot the lambda separation energy ($E_{SP}=B({}_{n\Lambda}Z)-B({}_{n-1\Lambda}Z)$) against the lambda numbers. It is obvious that when the pairing correlations are taken into account the odd-even effect in lambda separation energy is emerged. There occurs a dramatic decrease of lambda separation energy in 3 Λ's. This can be explained as a shell effect in $n_\Lambda=2$ and the 3rd Λ has to go into the $1p_{3/2}$ level, which is extremely weakly bound for Λ particles. It is also the cause why the multi-Λ hypernuclei are not observed up to now

for $n_\Lambda > 2$.

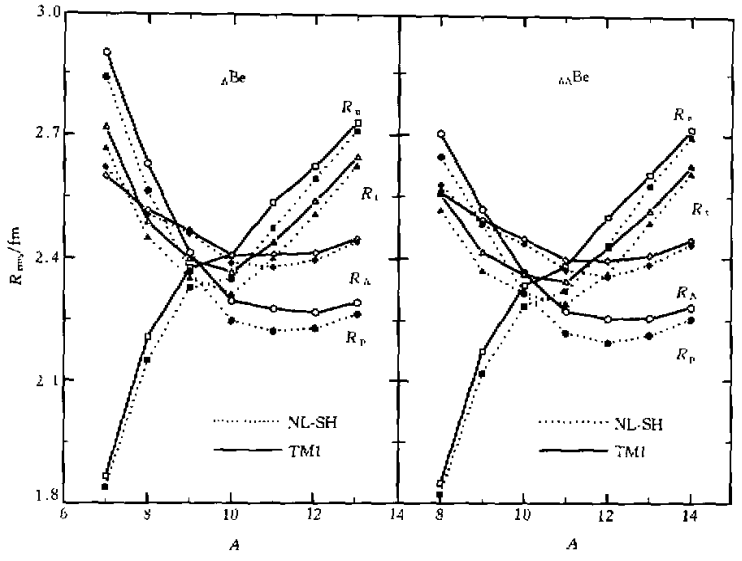

Fig.4. Same as Fig.2, but for $_{\Lambda}$Be and $_{\Lambda\Lambda}$Be hypernuclei.

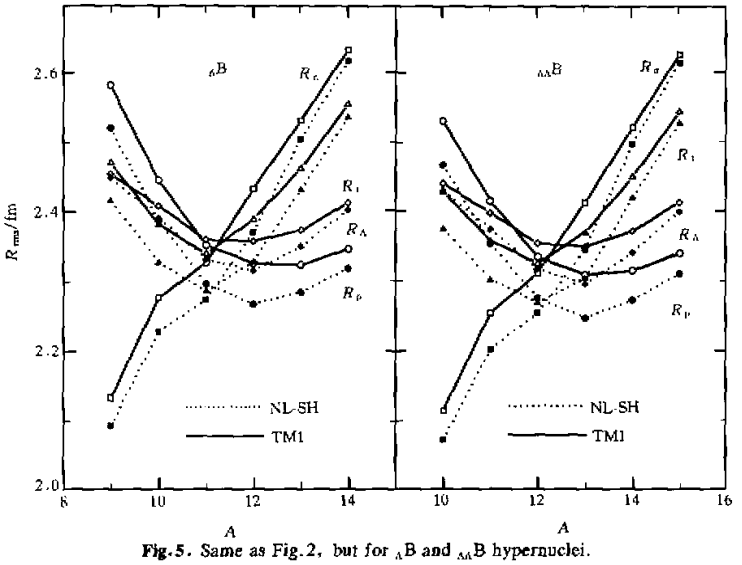

Fig.5. Same as Fig.2, but for $_{\Lambda}$B and $_{\Lambda\Lambda}$B hypernuclei.

In Fig. 7, we show the rms radius as a function of the number of Λ particles added. The influence of a growing amount of lambda particles on the total, the pro-

ton, the neutron and the Λ rms radii in multi-Λ hypernuclei is illustrated for ${}^{nA+11}_{nA}$Li, ${}^{nA+12}_{nA}$Be, ${}^{nA+13}_{nA}$B and ${}^{nA+16}_{nA}$O in the figure. While the Λ radius increases greatly with

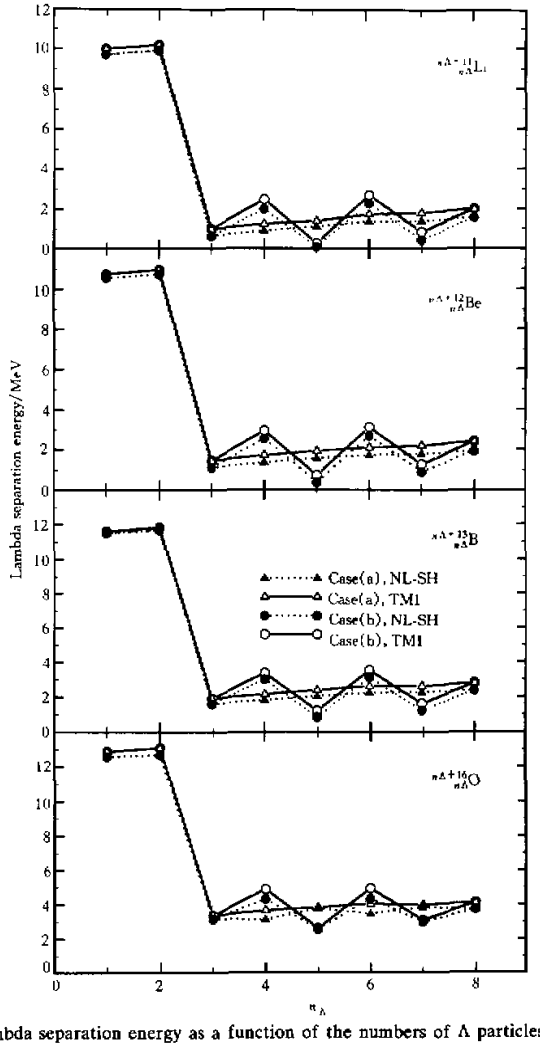

Fig.6. The Lambda separation energy as a function of the numbers of Λ particles. Case (a), the pairing correlations are not taken into account for Λ particles; case(b), the pairing correlations are taken into account using the constant-force approach $G_L = 10.0/(11 + L)$ and L is the Λ number.

the number of Λ hyperons added, the neutron and the proton radii are almost constant. Consequently, the changes of the total radius of the system are only affected by the Λ numbers, the total radius decreases first and turns back to an increase after more than 2Λ particles are added. We also can see from Fig. 7 that with increasing

number of the proton(from Li to O), the proton radius increases and the neutron radius decreases. Meantime, the increasing ratio of Λ radius also decreases. This can be explained by the fact that the core of ^{16}O is more stable than that of ^{11}Li and thus the hypernuclei formed with ^{16}O core are more stable than that with ^{11}Li core. It has been found that ^{11}Li is a nucleus with a neutron halo. The Λ radius is larger than the neutron radius in $n_\Lambda > 2$ and there may exist a neutron halo and a Λ halo in multi-Λ hypernuclei ${}^{n\Lambda+11}_{n\Lambda}$Li. We also find that the Λ radius is smaller than the proton and the neutron radii when one or two Λ particles are added. This is according to that in Fig.6, in which Lambda separation energy in 3Λ numbers has a dramatic decrease.

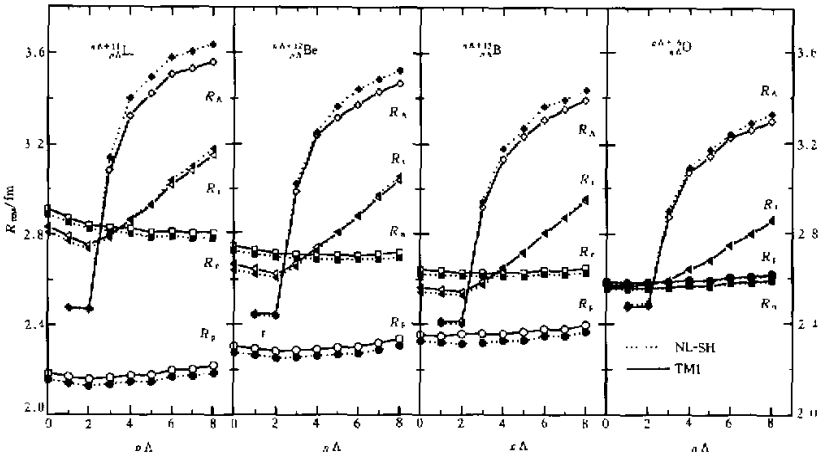

Fig.7. The total (R_t), the proton (R_p), the neutron (R_n) and the Λ (R_Λ) rms radii in hypernuclei ${}^{n\Lambda+11}_{n\Lambda}$Li, ${}^{n\Lambda+12}_{n\Lambda}$Be, ${}^{n\Lambda+11}_{n\Lambda}$B, and ${}^{n\Lambda+11}_{n\Lambda}$O as a function of the number of Λ hyperons added (n_Λ). The symbols (circle, square, diamond and triangle)denote the proton, the neutron, the Λ, and the total rms radii, respectively.

In other cases, we substitute Λ particles for neutrons and keep a constant value in A. In Fig.8, we show the rms radius and the binding energy per nucleon for ${}^{16}_{n\Lambda}O$ as a function of the number of Λ particles substituting. The binding energy has a rapid decrease after more than 2Λ substituting. This again indicates that the 3rd Λ particle is very weakly bound and the observation of these hypernuclei which have more than 2Λ particles is very difficult. There occurs a dramatic changes in the radii at 6Λ's. This is explained by that the 7th Λ particle has to go into the $1p_{1/2}$ level and an extremely weakly bound state has formed. From Figs.6－8, we also see the difference between two RMF sets TM1 and NL-SH is very small for the multi-Λ hypernuclei. It is concluded from the above results that the properties of ground state of hypernuclei are insensitive to the nucleon parameters.

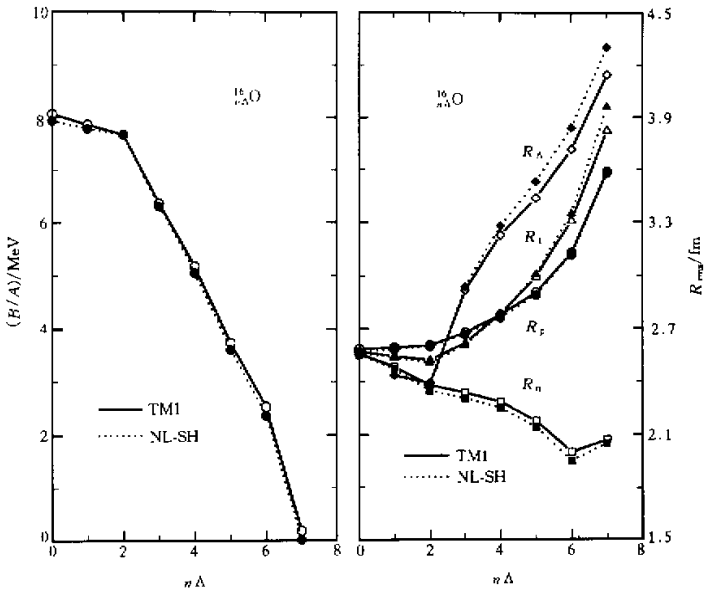

Fig.8. On the left of the plot, the binding energy per nucleon of ${}^{16}_{n\Lambda}O$ is shown as a function of the number of substituting Λ particles. On the right the total (R_t), the proton (R_p), the neutron (R_n), and the Λ (R_Λ) rms radii are shown for ${}^{16}_{n\Lambda}O$ as functions of the number of substituting Λ particles. The symbols (circle, square, diamond, and triangle) denote the poton, the neutron, the Λ, and the total rms radii, respectively.

IV. CONCLUSION

In the present work, we have studied the properties of light hypernuclei within the RMF approximation with new RMF parameter sets TM1 and NL-SH. Our results indicate that the properties of hypernuclei are insensitive to the parameters of effective nucleon-nucleon interactions. The results of Λ single-particle energies, single- and double-Λ binding energies agree well with the experimental data except $\Delta B_{\Lambda\Lambda}$. In the multi-Λ hypernuclei, we take into account the pairing correlations for Λ hyperon and find that this leads to the even-odd effect in the lambda separation energy. The Λ shell effect in $n_\Lambda = 2$ is so strong that the 3rd Λ hyperon has a weakly bound state in the $1p_{3/2}$ level. This may be the cause for which it is more difficult to observe 3Λ or multi-Λ hypernuclei than to observe 2Λ ones.

It is clear that the RMF model used in the present work has succeded in reproducing the ground state properties of light hypernuclei. But there also exist some problems such as hyperon-hyperon interactions needed to solve. And further study is underway.

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