Volume 7, Number 3
1004-423X/98/07030174-09

ACTA PHYSICA SINICA
(Overseas Edition)

March, 1998
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PREPARATION OF NONCLASSICAL STATES OF A CAVITY FIELD BASED ON A NONLINEAR JAYNES-CUMMINGS MODEL*

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(Received 15 April 1997; revised manuscript received 24 July 1997)

It is proven in this paper that some kinds of nonclassical quantum states of the cavity field, such as Schrödinger cat state (amplitude cat or phase cat), sub-Poissonian photon distribution and Fock state, etc., can be generated by conditional measurements on atoms in the two-photon Jaynes-Cummings model with a Kerr-like medium. The nonlinear constant plays the key modulation role in the preparation of the quantum states, which gives a new controllable parameter for further experimental researches in this field.

PACC: 4250

I. INTRODUCTION

In resent years the preparations of nonclassical states[1-6], in which certain observables exhibiting less fluctuations than in a coherent state, have aroused considerable interest. The sub-Poissonian photon statistics generated from the interaction of one atom with a single mode of a high-Q cavity has been demonstrated experimentally[7], but either Schrödinger cat state or Fock state has not been observed as yet.[8] However, most theoretical schemes for preparing the nonclassical states based on the fundamental Jaynes-Cummings model (JCM) have been suggested[8-14] in the light of two approaches[9], i.e., finding the appropriate Hamiltonian which transforms a given initial state to the desired final state via unitary time evolution, or making a measurement on one of two entangled quantum systems and obtaining the state of the other system by the corresponding state reduction (which is the so-called "entanglement-measurement" method). Gea-Banacloche[10] found that the field can evolve into Schrödinger cat state at the half-revival time of the population inversion; Garraway[8] acquired not only the Schrödinger cat state but also the Fock state of the cavity field with controllable photon-number and phase distributions by conditional measurements following two-photon resonant interactions; and Vogel[9] further gave the recipe for how to create an arbitrary field state in a single-mode resonator.

In this article we further study the role of a Kerr-like medium in the process of

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preparing nonclassical states based on the two-photon JCM. The systems actually include two nonlinear processes simultaneously: a two-level atom undergoing two-photon transition and an anhamonic oscillator representing the Kerr-like medium. Its dynamical behaviors have attracted much attention.[15-19] We find that the system can be used to prepare many kinds of nonclassical states according to the "entanglement-measurement" method[8,14,20-22], which is widely used in the quantum state engineering. After having measured the first atom, we can obtain the Schrödinger cat state as in Refs. [8] and [14]; furthermore, we can also obtain the photon-number distributions, such as Poisson, sub-Poisson, and sup-Poisson[23], by choosing some different Kerr-like nonlinear constants in our system; through many measurements on the atoms one by one, the field not only can get close to the Fock state but also to the Schrödinger cat state with different amplitudes but the same phase. In short, our results show that the Kerr-like medium can play an effective modulation role in the preparation of nonclassical states.

II. TWO-PHOTON JCM WITH A KERR-LIKE MEDIUM

The model under consideration in this paper consists of a single two-level atom undergoing two-photon transition at a time in a single-mode field surrounded by a nonlinear Kerr-like medium contained in an infinite-Q cavity. The effective Hamiltonian of the system in the rotating-wave approximation can be written as ($\hbar = c = 1$)[18]

$$H_{\text{eff}} = \omega_c a^{+} a + \omega_a S_z + \chi a^{+2} a^2 + g(a^{+2} S_- + a^2 S_+), \tag{1}$$

where a^{+} and a are the cavity annihilation and creation operators, respectively, S_z and $S_\pm$ are the atomic operators, g is the coupling constant between the atom and the field, χ gives the dispersive part of the third-order nonlinearity of the Kerr-like medium.

Suppose that at the beginning the atom is in the excited state $|e\rangle$, and the cavity mode is prepared in the coherent state $|\alpha\rangle$, thus the initial state vector $|\Psi(0)\rangle_{\text{AF}}$ may take the form

$$|\Psi(0)\rangle_{\text{AF}} = \sum_{n=0}^{\infty} Q_n |n, e\rangle, \tag{2}$$

where $Q_n = \exp\left(-\frac{\alpha^2}{2}\right)\frac{\alpha^n}{n!}$ (α is a real number for simplicity). So the solution of Schrödinger equation with the initial condition (2) is [8]

$$|\Psi(t)\rangle_{\text{AF}} = \sum_{n=0}^{\infty} \exp\{-i[\omega_c(n+1) + \chi^2(n^2+n+1)]t\} \times [c_0(n)|n, e\rangle + d_0(n)|n+2, g\rangle], \tag{3}$$

and the coefficients $c_0(n)$ and $d_0(n)$ are

$$c_0(n) = Q_n\left[\cos(\beta_n t) + \frac{i\chi(2n+1)}{\beta_n}\sin(\beta_n t)\right], \tag{4}$$

$$d_0(n) = -iQ_n\left[\frac{g[(n+1)(n+2)]^{1/2}}{\beta_n}\sin(\beta_n t)\right]. \tag{5}$$

Here we assume $\omega_a = 2\omega_c$ (exact resonance condition), and the generalized Rabi frequency β_n is defined as

$$\beta_n^2 = g^2(n+1)(n+2) + \chi^2(2n+1)^2. \tag{6}$$

We shall use the form of the wave function (3) in the interaction picture for simplicity, that is

$$|\Psi(t)\rangle_{AF} = \sum_{n=0}^{\infty} [c_0(n)\,|e, n\rangle + d_0(n)\,|n+2, g\rangle], \tag{7}$$

where the coefficients $c_0(n)$ and $d_0(n)$ are given by Eqs. (4) and (5), respectively. It is shown from Eq. (7) that the interaction (1) between the atom and the field leads to the form of the atoms-field entanglement state.

Under the high field approximation ($n \sim \alpha^2 \approx \bar{n} \gg 1$), we can further make the following approximations:

$$\begin{aligned} \beta_n &\cong nb + c, \\ \frac{\chi(2n+1)}{\beta_n} &\cong \frac{2\chi}{B} \equiv X, \\ \frac{g\sqrt{(n+1)(n+2)}}{\beta_n} &\cong \frac{g}{b} \equiv Y, \end{aligned} \tag{8}$$

where $b = \sqrt{g^2 + 4\chi^2}$, $c = \frac{b}{2} + \frac{g^2}{b}$. (Here X is obviously the effective nonlinear constant). Then the coefficients $c_0(n)$ and $d_0(n)$ can be simplified to

$$c_0(n) = Q_n[\cos[(bn+c)t] + iX\sin[(bn+c)t]], \tag{9}$$

$$d_0(n) = -iQ_nY\sin[(bn+c)t]. \tag{10}$$

III. THE PREPARATION OF NONCLASSICAL STATES BY CONDITIONAL MEASUREMENTS ON EXCITED ATOMS

Similar to the method in Ref. [8], we make the conditional measurements chosen on a sequence of the atoms emerging in the excited state after the interaction. Each measurement in such a sequence is realizable with a high efficiency by the field ionization technique.[17]

A. First measurement

Supposing that the first atom and the field evolve into the entangled state $|\Psi(t_1)\rangle$ at time t_1, we can observe the excited state with probability

$$P_1 = |\langle e\,|\,\Psi(t_1)\rangle|^2 = \sum_{n=0}^{\infty} [X^2 + Y^2\cos^2[(bn+c)t_1]]. \tag{11}$$

Once we measure the atom in the excited state, the corresponding field will reduce the pure state

$$\Psi_f(t_1)\rangle = \sum_{n=0}^{\infty} c_1(n)\,|n\rangle, \tag{12}$$

where

$$c_1(n) = \frac{1}{\sqrt{P_1}} c_0(n)[\cos[(bn+c)t_1] + iX\sin[(bn+c)t_1]] \tag{13}$$

is a new coefficient for the field distribution. Here the photon-number distribution $|c_1(n)|^2$ is normalized to unity. According to Eq. (9), Eq. (12) can be further rewritten as

$$|\Psi_f(t_1)\rangle \frac{1}{2\sqrt{P_1}}\{(1+X)e^{iet_1}|\alpha e^{ibt_1}\rangle + (1-X)e^{-iet_1}|\alpha e^{-ibt_1}\rangle\}. \quad (14)$$

It is clear that due to the measurement on the first atom emerging in the excited state, the field has evolved into the macroscopic quantum superposition state consisting of two coherent states with the same amplitude but opposite phases, that is, an amplitude cat state. If $X=0$, Eq.(14) has the same result as that in Ref.[8]. $X\neq 0$ means that the weight factors in the component parts of state (14) are different, thus the properties of the field are modulated by the nonlinear constant X, which can lead to the generation of some new nonclassical states.

Here we make some simple analyses about how the photon-number distribution of state (14) is modulated by the nonlinear constant X. According to Eq.(14), the mean values $<\hat{n}>$ and $<\hat{n}^2>$ are given by

$$\begin{aligned}<\hat{n}(t_1,X)> &\equiv \langle\Psi_f(t_1,X)|a^+a|\Psi_f(t_1,X)\rangle \\ &= \frac{\alpha^2}{2P_1}\{1+X^2+Y^2\exp[-2\alpha^2\sin^2(bt_1)] \\ &\quad \times\cos[2(c+b)t_1+\alpha^2\sin(2bt_1)]\}, \quad (15)\end{aligned}$$

$$\begin{aligned}<\hat{n}^2(t_1,X)> &\equiv <\hat{n}(t_1,X)> + \langle\Psi_f(t_1,X)|a^{+2}a^2|\Psi_f(t_1,X)\rangle \\ &= <\hat{n}(t_1,X)> + \frac{\alpha^4}{2P_1}\{1+X^2+Y^2\exp[-2\alpha^2\sin^2(bt_1)] \\ &\quad \times\cos[2(b+c)t_1+\alpha^2\cos(2bt_1)]\}, \quad (16)\end{aligned}$$

Furthermore, the normalized uncertainty σ of the photon number in state (14)

$$\sigma = \left[\frac{<\hat{n}^2>-<\hat{n}>^2}{<\hat{n}>}\right]^{1/2} \equiv \frac{\Delta\hat{n}}{\sqrt{<\hat{n}>}} \quad (17)$$

is also obtained. It is shown that the normalized uncertainty σ of the photon number in our system not only depends on the interaction time but also are closely related to the nonlinear constant X. The changes of σ with the interaction time and the nonlinear constant X are illustrated in Figs.1 and 2, respectively.

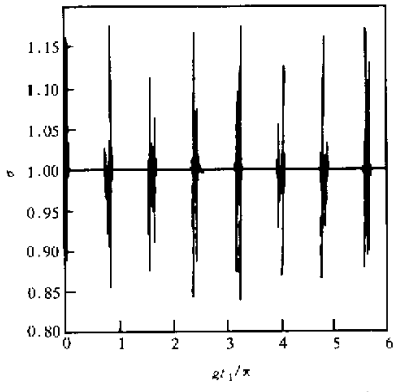

Fig.1. Normalized photon-number uncertainty σ as a function of the dimensionless interaction time t, with $X=0.6$, $\alpha=6$.

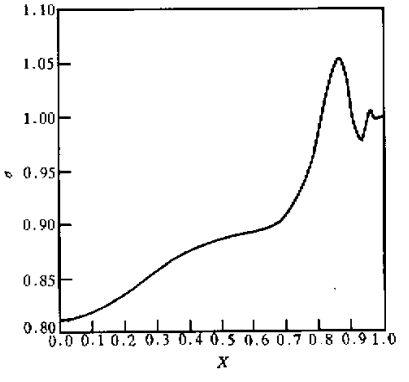

Fig.2. Normalized photon-number uncertainty σ as a function of the dimensionless nonlinear constant X, with $t_1=0.02\pi$, $\alpha=6$.

Figure 1 shows that the normalized photon-number uncertainty σ still oscillates rapidly and periodically with the interaction time, but reveals some fluctuations at the peaks due to the influence of the Kerr-like medium; and Fig.2 shows that any photon-distributions, i.e., sub-Poisson $\sigma < 1$, Poisson $\sigma = 1$, and sup-Poisson $\sigma > 1$, can be produced by choosing some suitable nonlinear constants X. Comparing the above two figures, we can see that nonlinear constant X can be controlled more easily than the interaction time for realizing the nonclassical distributions, which is one of the advantages for the generation of the Schrödinger cat states in our system.

We also make the comparison between the photon-distribution P_n and the phase distribution $P(\theta)$ of state (14), where the phase distribution $P(\theta)$ is given by Pegg-Barnett formalism[24]

$$F_1(\theta) = \frac{1}{2\pi} \sum_{n=0}^{\infty} c_1(n) e^{-in\theta} \Big|^2, \tag{18}$$

and the results are shown in Figs.3 and 4. Sup-Poisson and sub-Poisson statistics are illustrated in Fig.3(a) and Fig.4(a) respectively. the distance between the two peaks in Fig.3(b) shows the relative phase $2b_1 t = \dfrac{2gt_1}{\sqrt{1-X^2}}$ between the superposition parts of state (14). We can see that the two peaks in Fig.3(b) lost symmetry and those in Fig. 4(b) overlap due to the influence of nonlinear constant X in the weight factors of the state (14) and the Rabi frequency (6).

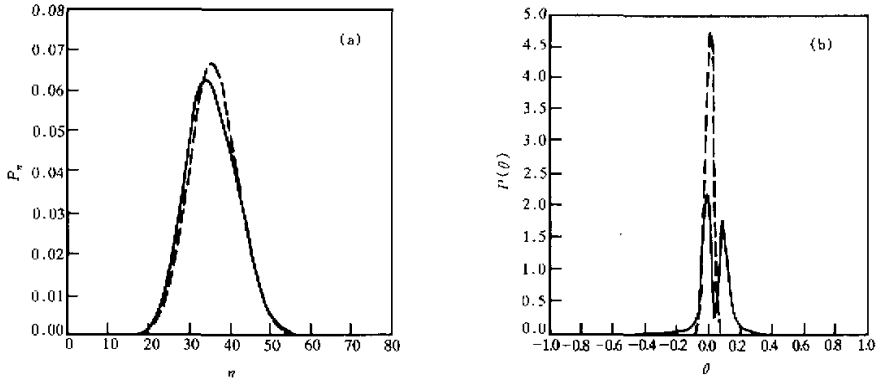

Fig.3. (a) The photon-number distribution P_n and (b) the phase distribution $P(\theta)$ of the field after a conditional measurement on the first atom, for $\alpha = 6$, $t_1 = 0.02\pi$, $X = 0.85$. (P_n and $P(\theta)$ of the initial coherent state with $\alpha = 6$ are plotted by the dashed lines for comparison, respectively).

B. Many measurements

Now let the second atom enter into the cavity after the first one leaves and interact

with the field in the same way. Once the atom in the excited state is measured, the cavity field reduces another pure state. Repeating the procedure, after k excited atoms have been measured, the resulting state of the cavity field is

$$|\Psi_f(t_k, X)\rangle = \sum_{n=0}^{\infty} c_k(n)\,|n\rangle, \tag{19}$$

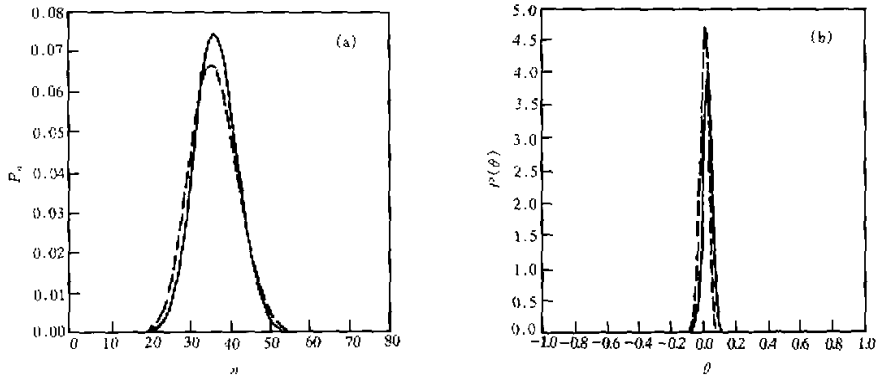

Fig. 4. (a) The photon-number distribution P_n and (b) the phase distribution $P(\theta)$ of the field after a conditional measurement on the first atom, for $\alpha = 6$, $t_j = 0.02\pi$, $X = 0.6$. (P_n and $P(\theta)$ of the initial coherent state with $\alpha = 6$ are plotted by the dashed lines for comparison, respectively).

where

$$c_k(n) = \frac{c_0(n)}{\sqrt{P_k}} \prod_{j=1}^{k} |\cos[(bn + c)t_j] + iX\sin[(bn + c)t_j]|. \tag{20}$$

The probability to have k excited atoms is

$$P_k = \sum_{n=0}^{\infty} |c_0(n)|^2 \prod_{j=1}^{k} |\cos^2[(bn + c)t_j] + X^2\sin^2[(bn + c)t_j]|. \tag{21}$$

The photon-number distribution $|c_k(n)|^2$ still satisfies the normalization condition:

$$\sum_{n=0}^{\infty} |c_k(n)|^2 = 1. \tag{22}$$

By fixing interaction time, we compute the photon-number distribution P_n and the phase distribution $P(\theta)$ of state (19) with some different values of the nonlinear constant X. The results are shown in Figs. 5 and 6. (Here the unit of θ is π.)

Figure 6 shows that through many conditional measurements on the excited atoms, the field has evolved close to the phase-cat state, which is the superposition of the coherent states with the same amplitude but opposite phases. Comparing Fig. 4 with Fig. 5, we can see that owing to the same measurement procedure, the field can get close to the Fock state from the amplitude-cat state (14) for another suitable value of X. Of course, if we choose other values of X and t_j, the system can yield other kinds of nonclassical states.

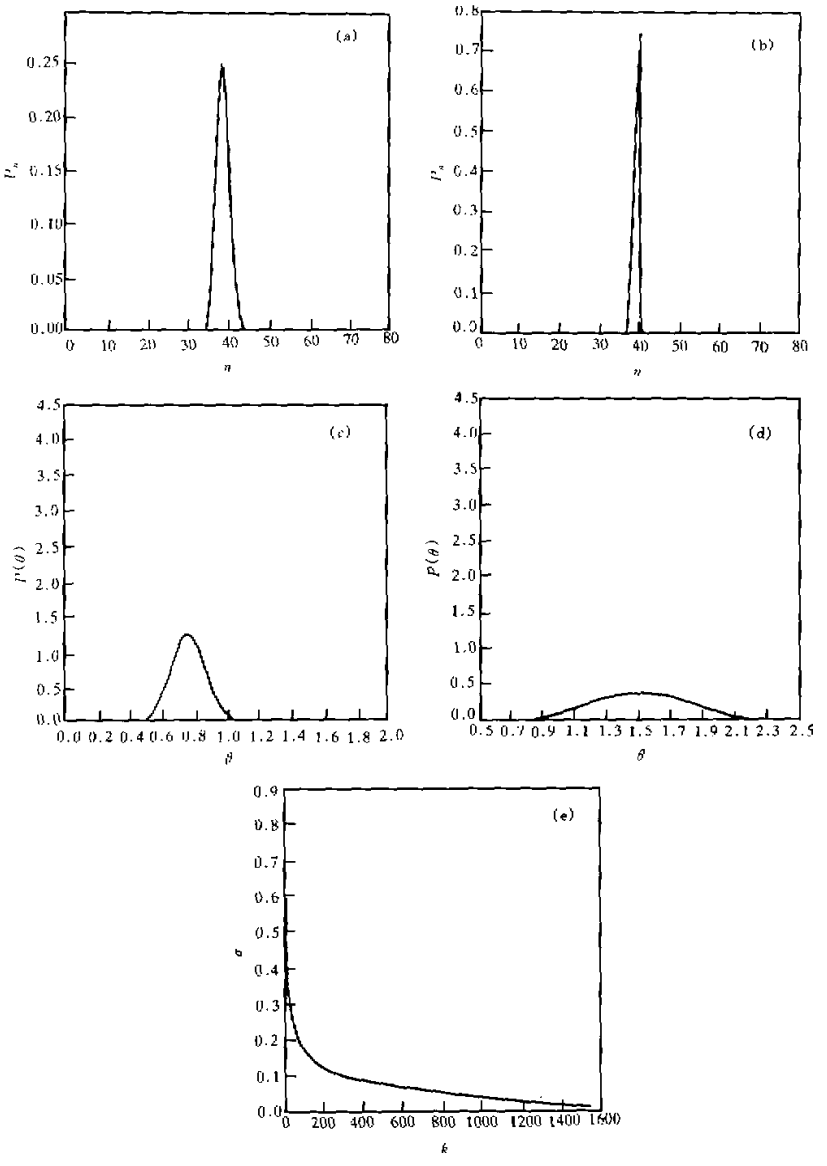

Fig. 5. The photon-number distribution, the phase distribution, and normalized photon-number uncertainty σ of the field following sequences of excited-atom conditional measurements for fixed interaction time $t_j = 0.02\pi$, with $\alpha = 6$, $X = 0.6$. (a) The photon-number distribution after $k = 50$ excited atoms, (b) the photon-number distribution after $k = 500$ excited atoms, (c) the phase distribution after $k = 50$ excited atoms, (d) the phase distribution after $k = 500$ excited atoms, and (e) normalized photon-number uncertainty as a function of the number of measured excited atoms.

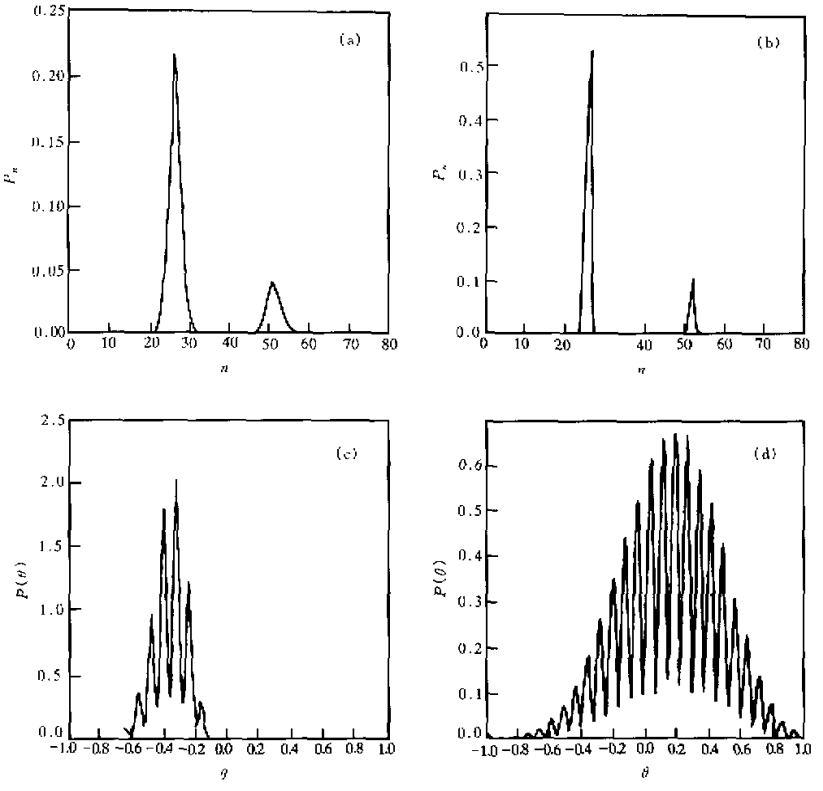

Fig. 6. The photon-number distribution and the phase distribution of the field following sequences of excited-atom conditional measurements for fixed interaction time $t_f = 0.02\pi$, with $\alpha = 6$, $X = 0.85$. (a) The photon-number distribution after $k = 50$ excited atoms, (b) the photon-number distribution after $k = 500$ excited atoms, (c) the phase distribution after $k = 50$ excited atoms, (d) the phase distribution after $k = 500$ excited atoms.

IV. CONCLUSION

The preparation of nonclassical states with controllable ways is still an important and interesting topic in the quantum optics. Our results show: (1) the nonlinear JCM based on the cavity quantum electrodynamic principle is a useful scheme; (2) more types of nonclassical states can be produced by using the "entanglement-measurement" method than the pure unitary process; (3) the mechanics of producing nonclassical fields is the same as that in Ref. [14], that is, conditional measurements result in repeated splitting of the phase-space field distribution and interferences between the split parts; however, it is the introduction of the nonlinear constant χ (i.e., X),

which modulates the weight factors of the field and the generalized Rabi frequency effectively, that makes the resulting states of the cavity field rather different from those in Ref. [14]. In a word, different nonclassical states can be generated in our system by choosing some suitable nonlinear constants X, which provides a new easy controllable parameter for further experimental researches.

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