Volume 7, Number 10 ACTA PHYSICA SINICA October, 1998
1004-423X/98/07100744-07 (Overseas Edition) © 1998 Chin. Phys. Soc.

PERFORMANCE OF VERTICAL VIBRATION ISOLATION SYSTEM WITH GRAVITATIONAL POSITIVE FEEDBACK*

Zhao Peng-fei(赵鹏飞)[a),d)], Zhang Ping-hua(张平华)[b)],
Tang Jia-xiang(唐家祥)[c)], and Luo Jun(罗 俊)[a)]

[a)] *Department of Physics, Huazhong University of Science and Technology, Wuhan* 430074, *China*

[b)] *Institute of High Energy Physics, Academia Sinica, Beijing* 100080, *China*

[c)] *Department of Civil Engineering, Huazhong University of Science and Technology, Wuhan* 430074, *China*

[d)] *Department of Physics, Zhanjiang Normal College, Zhanjiang* 524048, *China*

(Received 5 February 1998)

A new model of vertical vibration isolation system is presented in the paper. It is principally constructed by mounting an additional mass on the simplest passive vertical isolator, which can give rise to gravitational positive feedback. By theoretical analyzing and numerical calculating, we come to a conclusion that the resonant frequency is decreased in the new system, which agrees with the result of the experiment.

PACC: 4630; 0480

I. INTRODUCTION

The detectors for predicted gravitational waves are required to achieve sensitivities in the region $h \sim 10^{-21}$ to $h \sim 10^{-22}$ or better over a range of frequencies f from about ten hertz to a few kilohertz in a bandwidth $\Delta f \approx f/2$, but the performance is limited by several noise sources such as seismic noise, suspension thermal noise, etc. At low frequency, the noise is dominated by seismic noise. To broaden the range of the detection of gravitational waves, the isolator with very low resonant frequency is required. Therefore the study on isolation system for detection of gravitational waves has been playing a very important role in the gravitational experiments.

So far, much progress has been made in the field of horizontal isolation system with ultra-low frequency. The folded pendulum with a resonant frequency of 17 mHz[1] and the conical pendulum with the resonant periods in excess of 20 s[2] have been developed successively. But it is still a problem how to develop a vertical isolation system with a very low resonant frequency. Lately a new idea on vertical vibration isolation was established[3], he concept of which is expressed as: a large scale vibration system can be operated by monitoring the relative displacement of the oscillator with respect to a quasi-fixed reference system by proper feedback control forces, and both the resonant frequency and the resonant peak of the vibration isolation system could be decreased effectively. In this method, constructing a fine

*Project supported by the National Natural Science Foundation of China (Grant No. 19425008).

quasi-fixed reference system is essential and crucial. At present, the horizontal isolators as stated above can be taken as excellent quasi-fixed references in the horizontal direction, but the vertical passive isolator with an ultra-low resonant frequency has not been achieved yet. Thus establishing a new kind of passive vertical isolator with a very low resonant frequency is urgent and significant.

Based on our initiative and conceptual idea on vertical vibration isolation associated with gravitational positive feedback[4] which has been applied to the gravimeters and seismometers as well as the torsion balance[5−7], we develop a new model of vertical passive isolation system. First of all, the system is theoretically discussed by analyzing the isolation ratio at different situations in detail. As a result, it was shown that the total spring coefficient (defined as equivalent spring coefficient) of the system was obviously decreased due to the generation of gravitational positive feedback. Then, a practical device was built, and the predicted decrease of the resonant frequency was demonstrated by experiment.

II. EXPERIMENTAL DESIGN

Figure 1 shows the design of the system, where M_P and K are the mass of load and the spring constant, respectively. At the point A, a hinge consisting of an interior ring and an external ring is connected with M_P. Either of the two declined rods, along which an additioinal mass m_1 or m_2 is placed at a distance S from the hinge A, is, respectively, supported by the point B_1 or B_2 of the cross frame and can slide freely. The cross frame is fixed on ground. The spring K suspended from the point O' is connected with the hinge A. For convenience in calculation, we assume that the radius of the hinge approaches zero. The load M_P can only move in vertical direction due to the symmetry of the system, and both of the ends of the declined rods can simultaneously rotate about the central axis of the hinge A.

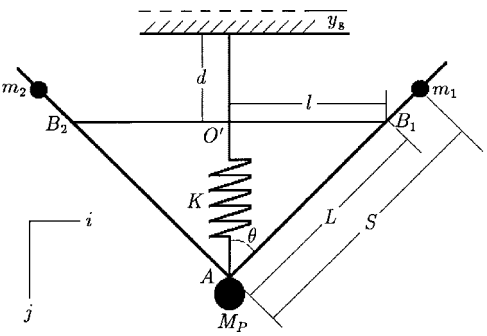

Fig.1. The vertical vibration isolation system with gravitational positive feedback.

The origin of coordinates is located at the point which is the static position of the load M_P. The position vectors of M_P, m_1 and m_2 are, respectively, expressed as following:

$$\boldsymbol{r}_P = u\boldsymbol{j} \ ,$$

$$\boldsymbol{r}_1 = \frac{Sl}{\sqrt{l^2+(y_0+u-y_g)^2}}\boldsymbol{i} + \left[\frac{S(y_0+u-y_g)}{\sqrt{l^2+(y_0+u-y_g)^2}} - u\right]\boldsymbol{j} \ ,$$

$$\boldsymbol{r}_2 = -\frac{Sl}{\sqrt{l^2+(y_0+u-y_g)^2}}\boldsymbol{i} + \left[\frac{S(y_0+u-y_g)}{\sqrt{l^2+(y_0+u-y_g)^2}} - u\right]\boldsymbol{j} \ ,$$

where y_0 is the static coordinate of M_P, u represents the displacement of the load M_P around the origin, and y_g represents the motion of the ground. In general case, the condition

$|u|$, $|y_{\mathrm{g}}| \ll \sqrt{l^2+y_0^2}$ is satisfied, thus the Lagrangian function of the system can be expressed as following:

$$\begin{aligned} L =& \left[\frac{1}{2}M_P + m\frac{S^2y_0^2l^2}{\alpha^6} + m - 2mS\left(\frac{1}{\alpha} - \frac{y_0^2}{\alpha^3}\right) + mS^2\left(\frac{1}{\alpha} - \frac{y_0^2}{\alpha^3}\right)^2\right]\dot{u}^2 + \left[-2m\frac{S^2y^2l^2}{\alpha^6}\right. \\ &\left. + 2mS\left(\frac{1}{\alpha} - \frac{y_0^2}{\alpha^3}\right) - 2mS^2\left(\frac{1}{\alpha} - \frac{y_0^2}{\alpha^3}\right)^2\right]\dot{u}\dot{y}_{\mathrm{g}} + \left[m\frac{S^2y_0^2l^2}{\alpha^6} + mS^2\left(\frac{1}{\alpha} - \frac{y_0^2}{\alpha^3}\right)^2\right]\dot{y}_{\mathrm{g}}^2 \\ &+ M_P gu - 2mg\left[\frac{Sy_0}{\alpha} - \left(\frac{S}{\alpha} - \frac{Sy_0^2}{\alpha^3}\right)(u-y_{\mathrm{g}}) - \frac{3y_0S}{2\alpha^3}\left(\frac{y_0}{\alpha^2}-1\right)(u-y_{\mathrm{g}})^2 - u\right] \\ &- \frac{1}{2}K(y_0 - y_0' + u - y_{\mathrm{g}}) \,, \end{aligned} \tag{1}$$

where $\alpha = \sqrt{l^2+y_0^2}$, y_0' is the initial length of the spring without load, g is the gravity acceleration.

The generalized force exerted on the system can be expressed as a total damping force $-C(\dot{u}-\dot{y}_{\mathrm{g}})$, contributed by all of the non-conservation forces such as the friction between the rings of the hinge, the sliding friction between the declined rods and the supporting points, the damping force of the current air, the structural dissipation force of the spring, etc., and C represents the equivalent damping coefficient which is determined by the experiment.

From the equations of force balance and torque balance, we get the following formula:

$$K(y_0 - y_0') = M_P g + 2mg - \frac{2mgSl^2}{\alpha^3} \,, \tag{2}$$

and from the generalized Lagrangian equation, we can obtain the equation of motion of the system as following:

$$M_{\mathrm{E}}\ddot{u} + C\dot{u} + K_{\mathrm{E}}u = M_I\ddot{y}_{\mathrm{g}} + C\dot{y}_{\mathrm{g}} + K_{\mathrm{E}}y_{\mathrm{g}} \,, \tag{3}$$

where

$$M_{\mathrm{E}} = M_P + 2m + \frac{2mS^2l^2}{\alpha^4}\left(1 - \frac{2\alpha}{S}\right) \,, \tag{4}$$

$$M_I = \frac{2mS^2l^2}{\alpha^4}\left(1 - \frac{\alpha}{S}\right) \,, \tag{5}$$

$$K_{\mathrm{E}} = K - \frac{6mgSy_0l^2}{\alpha^5} \,, \tag{6}$$

where M_{E}, which includes not only the mass of the load but also the additional mass m used to produce gravitational positive feedback, is defined as the equivalent mass of the system; M_I, which represents the coupling degree between the ground and the system, is defined as the coupling mass. Clearly, the bigger, the M_I, the larger the coupling degree. K_{E}, which consists of the spring constant K and the equivalent negative gravitational spring coefficient $K_{\mathrm{G}} = -6mgSy_0l^2/\alpha^5$ due to the gravitational positive feedback, is defined as the equivalent spring coefficient of the system. Obviously, the inequality of $K_{\mathrm{E}} < K$ is always kept because K_{G} is always negative, which leads to the decrease of the resonant frequency of the new system. The physical reason can be explained as following: if the ground goes down, the

spring is compressed, simultaneously the angle of θ between the declined rod and the spring becomes larger, the additional force which is attributed by m and ultimately exerted on the spring will decrease in the vertical direction, and will result in the further compression of the spring. Otherwise if the ground goes up, the increase of the additional force will make the spring stretched further, and this means that the spring becomes softer due to the gravitational positive feedback.

The angular resonant frequency of the new system is expressed as $\omega_{\rm E} = \sqrt{(K + K_{\rm G})/M_{\rm E}}$. From the formula (4), it can be seen that the condition $M_{\rm E} > M_P$ is easily satisfied by adjusting related parameters, and then $\omega_{\rm E} < \sqrt{K/M_{\rm E}} < \sqrt{K/M_P}$. It means that the resonant frequency $\omega_{\rm E}$ is decreased compared with that of the traditional passive vertical isolation system. If we make the value of $-K_{\rm G}$ to approach K, the resonant frequency $\omega_{\rm E}$ would be much lower than that of traditional isolator. Of course, when $-K_{\rm G}$ is reduced to a certain degree, we have to consider the non-linear problems of the spring as well as the stability of the system, which are beyond the study in the paper.

In order to take a full advantage of the gravitational negative spring coefficient, we should choose appropriate parameters to make the $-K_{\rm G}$ to be maximum. If $l = \sqrt{2/3}y_0$ is chosen for a fixed y_0, which corresponds to the angle $\theta = 43.6^\circ$, $-K_{\rm G}$ has a maximum value as following:

$$(-K_{\rm G})_{\max} = \frac{24}{25}\sqrt{\frac{3}{5}}\frac{mgS}{l^2} \ . \tag{7}$$

Ⅲ. ISOLATION RATIO

Compared with the equation of the motion of traditional passive vertical isolation system, there are two differences in the Eq.(2). First, the extra term with M_I will compromise the isolation ratio because the ground acceleration is directly coupled in the system. Second, $K_{\rm E} < K$ means that the perturbance from ground is reduced and the resonant frequency is decreased in the new system.

Now we put a Laplace transformation on the Eq.(2) and get the formula of the isolation ratio as following:

$$\eta = \sqrt{\frac{(K_{\rm E} - M_I\omega^2)^2 + C^2\omega^2}{(K_{\rm E} - M_{\rm E}\omega^2)^2 + C^2\omega^2}} \ .$$

The effect caused by M_I can be discussed in the two different cases: i.e., $M_I \neq 0$ and $M_I = 0$.

A. $M_I \neq 0$

If $M_I > 0$, the curve 1 shown in Fig.2 shows that the isolation ratio η reaches a minimum at 1.32 Hz with the increase of frequency after reaching the maximum at 0.92 Hz, and then approaches a constant 0.48 ultimately. The physical reason is that, once the ground vibrates, the vibration acceleration is directly coupled into the system through the points B_1 and B_2 of the system, i.e., the extra force $M_I\ddot{y}_{\rm g}$ is exerted on the system. Because the term of $M_I\ddot{y}_{\rm g} = -M_I\omega^2 y_{\rm g}$ is proportional to the ω^2 and the phase between $M_I\ddot{y}_{\rm g}$ and $K_{\rm E}y_{\rm g}$ always

keeps in reverse, the two forces cancel out each other at a certain frequency f_2, thus there is a minimum of isolation ratio η. Certainly there is also a maximum at the resonant frequency f_1. f_1 and f_2 are given as following:

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{K_{\mathrm{E}}}{M_{\mathrm{E}}}}\frac{\left[1-\sqrt{\left(\frac{M_{\mathrm{E}}-M_I}{M_{\mathrm{E}}+M_I}\right)^2+\frac{2C^2}{K_{\mathrm{E}}(M_I+M_{\mathrm{E}})}}\right]^{1/2}}{\left(\frac{2M_I}{M_I+M_{\mathrm{E}}}-\frac{C^2}{2K_{\mathrm{E}}M_{\mathrm{E}}}\right)^{1/2}},$$

$$f_2 = \frac{1}{2\pi}\sqrt{\frac{K_{\mathrm{E}}}{M_I}}\frac{\left[1+\sqrt{\left(\frac{M_{\mathrm{E}}-M_I}{M_{\mathrm{E}}+M_I}\right)^2+\frac{2C^2}{K_{\mathrm{E}}(M_I+M_{\mathrm{E}})}}\right]^{1/2}}{\left(\frac{2M_{\mathrm{E}}}{M_I+M_{\mathrm{E}}}-\frac{C^2}{2K_{\mathrm{E}}M_I}\right)^{1/2}}.$$

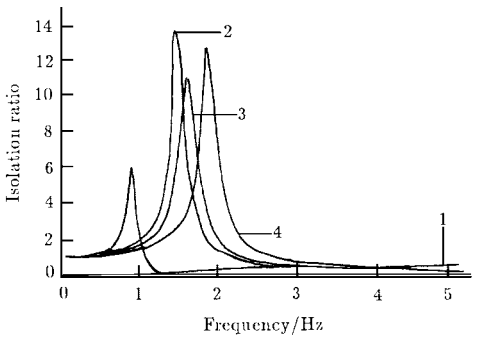

Fig.2. The theoretical isolation ratio curves of the isolation system with the parameters K=388.11 N/m, M_P=1 kg, m=1.5 kg and C=2.60 kg/s. And the four curves are, respectively, corresponding to the different values of the parameters $l, S, M_I, M_{\mathrm{E}}$ and K_{E} given in the Table 1.

Table 1. The parameters of the isolation ratio curves.

	Curve 1	Curve 2	Curve 3	Curve 4
l/m	0.1555	0.1852	0.1802	0
S/m	0.4916	0.5238	0.2849	0
M_I/kg	2.400	−0.1920	0	0
M_{E}/kg	5.000	3.616	2.800	1
K_{E}/(N/m)	165.8	313.4	292.2	388.11

If $M_I < 0$, there is only a maximum of the isolation ratio η shown in the Curve 2. The physical reason is that $M_I\ddot{y}_{\mathrm{g}}$ and $K_{\mathrm{E}}y_{\mathrm{g}}$ are in the same phase. As the frequency ω increases, the isolation ratio η increases and approaches a constant $|M_I|/M_{\mathrm{E}}$ ultimately.

B. M_I=0

From the formula (4), we can see that the condition of $M_I = 0$, which means the ground

vibration acceleration would not be directly coupled into the new system, can be satisfied by mounting the additional masses on the proper positions of the declined rods.

Compared with the Curve 4 of the traditional passive vertical isolation system, the Curve 3 shows that both of the resonant frequency and the resonant peak of the new system with $M_I = 0$ are decreased. Because of the characteristics of $K_{\rm E} < K$ the damping factor identical with that of the traditional isolator will lead to the decrease of the quality factor Q in the new system. Hence the new system with $M_I = 0$ can be used to isolate the test mass from ground vibration in the vertical direction.

IV. EXPERIMENTAL RESULT

Based on the theoretical analyzing, a practical device was built. In order to reduce the friction possibly, two ball bearings, were, respectively, mounted on the points B_1 and B_2 as shown in Fig.1, and the other two ball bearings were used as hinge connected with the spring and the declined rods.

To examine the decrease of the resonant frequency of the new system and verify the effect of gravitational positive feedback, we have finished two experiments. At first, we detected the response of the new system by stimulating the load in the vertical direction directly and obtained the response curve as shown in Fig.3. And then, under the same condition that the extended length of the spring was kept as that in the first experiment, we measured the response of the traditional mass-spring isolator and obtained the response

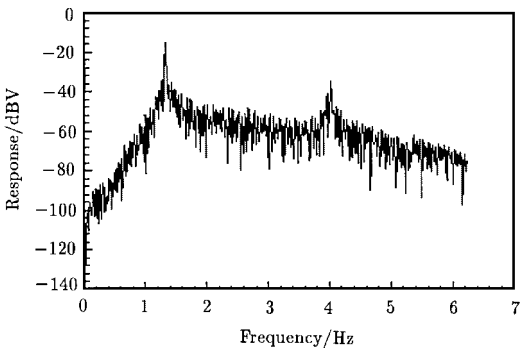

Fig.3. The response measured in the interval 0–6.25 Hz for the excited new isolator with parameters: the initial length of the spring $y_0 = 0.1650 \pm 0.0005$ m, the extended length of the spring $\Delta y_0 = 0.1190 \pm 0.0005$ m, $M_P = 1.400 \pm 0.001$ kg, $m = 1.306 \pm 0.001$ kg, $K = 326 \pm 4$ N/m, $S = 0.2920 \pm 0.0005$ m, $l = 0.2700 \pm 0.0005$ m.

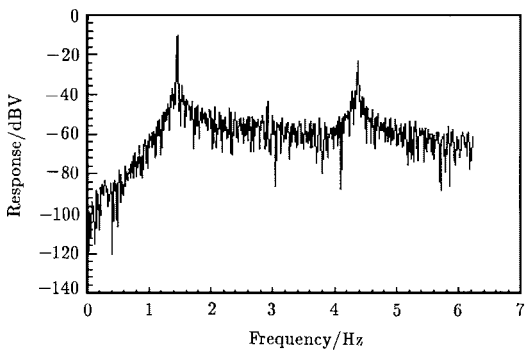

Fig.4. The response measured in the interval 0–6.25 Hz for the excited simple mass-spring isolator with parameters $M = 3.870 \pm 0.001$ kg, $K = 326 \pm 4$ N/m, $\Delta y_0 = 0.1190 \pm 0.0005$ m, $y_0 = 0.1650 \pm 0.0005$ m.

curve as shown in Fig.4. It is clear that the resonant frequency of 1.34 Hz of the new system is lower than that of 1.46 Hz of the traditional one, i.e., the resonant frequency of the new system was decreased by the gravitational positive feedback, which agreed with the theoretical predictions. The third harmonic frequency at 4.02 Hz in Fig.3 and that at 4.38 Hz in Fig.4 arose due to the non-linear vibration of the system.

However, without extra strong excitation, the above effect was not noticed due to the friction, especially the static friction. The problem of the friction at the supporting point must be considered further for practical use in the gravitation wave experiment. In addition, we cannot take the friction to be the damping force if $\pi M\rho\omega^2 < 4F_0$[8], where M, ρ, ω and F_0 are the equivalent mass, the amplitude of the ground vibration, the frequency of the ground vibration and the magnitude of the friction, respectively. It is well-known that the amplitude of the seismic noise is approximately expressed as $x_s = 10^{-6}\,\nu^{-2}/\mathrm{m(Hz)}^{1/2}$, where ν is the frequency. We can reckon up that the ratio of $4F_0$ to $\pi M\rho\omega^2$ is in the order of 10^3 for our new system, which meets the condition of $\pi M\rho\omega^2 < 4F_0$.

V. CONCLUSIONS

We have shown that the vibration isolation system with gravitational positive feedback can be used to provide a decreased resonant frequency which is lower than that of a traditional isolator.

ACKNOWLEDGMENTS

We are grateful to Prof. Tang Meng-xi and Prof. Zhang Yuan-zhong for helpful discussion.

REFERENCES

[1] D. G. Blair *et al., Phys. Lett.*, **A193**(1994), 223.
[2] J. Winterflood, D. G. Blair, *Phys. Lett.*, **A222**(1996), 141.
[3] Luo Jun, Wu Cai-zhang, *Science in China(Series A)*, **40**(1997), 1216.
[4] Zhao Peng-fei, Luo Jun, Zhang Ping-hua, *Chin. Phys. Lett.*, **13**(1996), 568.
[5] A. B. Bryan, *Geophysics*, **2**(1937), 301.
[6] P. R. Heyl, *Bureau of Standards Journal of Research*, **5**(1930), 1224.
[7] Zhang Ping-hua, R. D. Newman, *Chin. Phys. Lett.*, **9**(1992), 397.
[8] J. P. Den Hartog, *Trans. Amer. Soc. Mech. Eng.*, **53**(1931), 107.
[9] R. D. Fabbro *et al., Rev. Sci. Instrum.*, **59**(1988), 292.