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BACKACTION EVADING MEASUREMENT AND QUANTUM STATE PREPARATION FOR PHONONS BY USING THE RAMAN-INDUCED KERR EFFECT*

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A scheme of quantum nondemolition measurement is proposed for probing the phonon inside a crystal by using the Raman-induced Kerr effect. The results of calculation show that under certain conditions either a backaction evading measurement or a quantum state preparation can be performed.

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I. INTRODUCTION

A quantum nondemolition (QND) measurement, in which an observable is measured without perturbing its free motion, was first proposed in order to overcome the quantum limit in the detection of gravitational waves.[1-3] It was then developed in quantum optics. A number of theoretical papers have proposed schemes for performing QND measurements[4-10], and some schemes have been demonstrated experimentally.[11-18] A practical QND measurement device is not ideal and its effectiveness for performing QND measurement can be described by quantitative criteria.[6]

In this paper we apply the principle of QND measurements to the phonon detection. The QND measurement of a phonon field is a new subject which is important in phonon physics and solid state physics. We propose a scheme for probing phonon inside a crystal using Raman-induced Kerr effect. Levenson *et al.*[19] have used the effect to carry out a QND measurement of one quadrature of the optical field. Their aim was measuring the properties of light and in their system the phonon was eliminated adiabatically because there was a resonant cavity of the light. However, our aim is to study the properties of the phonons. In our system, phonons are signals and cannot be eliminated because there is no cavity for optical mode resonance.

In Sec. II we describe our scheme. In Sec. III we establish the dynamics equations of the system and obtain the exact solutions of them. In Sec. IV we calculate the QND measurement effectiveness using the solutions and the quantitative criteria for QND measurement. In Sec. V we give the calculated results and describe the scheme that can be used to perform backaction evading (BAE) measurements and quantum state prepa-

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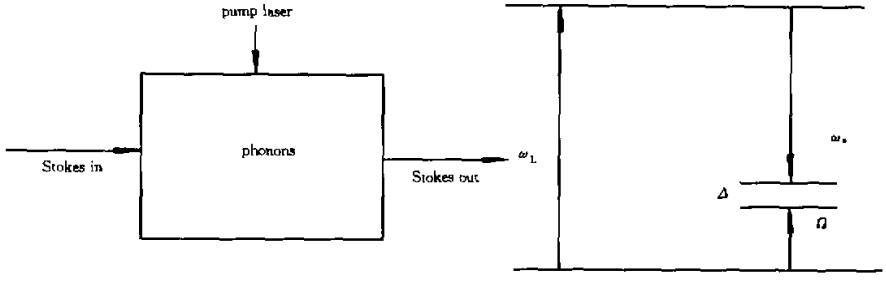
ration (QSE) under some conditions. In Sec. VI we give the conclusions.

II. THE SCHEME

The Raman-induced Kerr effect (RIKE)[19-21] is used to probe the phonons in a crystal. As shown in Fig. 1, when a Raman-active crystal is irradiated simultaneously by two exciting radiation sources, namely by a pump laser wave (frequency ω_L) and a linearly polarized Stokes wave (frequency ω_s), the nonlinear third-order interaction gives rise to a new Stokes wave that is polarized perpendicularly to the wave originally incident on the crystal. We wish to make a QND measurement of the phonons via detecting Stokes output, namely the phonon mode is the signal mode and the Stokes mode is the probe mode. We consider the measurement for the amplitude quadrature of the phonon mode by the phase quadrature of the Stokes mode.



(a) Diagram of QND measurement for the phonons using RIKE method.

(b) Level scheme where two potical modes are detuned by Δ from the Raman-active phonon mode.

Fig. 1. A scheme of QND measurement for phonos.

III. THEORETICAL ANALYSIS

In our analysis, we only consider the transverse optical (TO) phonons in the crystal because they have a stronger coupling with the light field. For the TO phonon field, we make a long wavelength approximation[22] that is sufficient for the analysis of the effects in which light interacts with the phonon mode since the wavelength of the light in the infrared radiation is always much greater than the lattice spacing. Thus TO phonon mode can be described as a simple harmonic oscillator. In addition, for the pump field we make a constant approximation, thus the pump field can be treated classically.

Under the above approximations, the interaction Hamiltonian of the system can be written as[19,20,22]

$$H' = \hbar R E_L \exp(-i\omega_L t) a^+ + a_s^+ + \hbar R^* E_L^* \exp(i\omega_L t) a\, a_s, \tag{1}$$

and the total Hamiltonian as

$$H = \hbar\Omega a^+ a + \hbar\omega_s a_s^+ a_s + \hbar R E_L \exp(-i\omega_L t) a_s^+ a^+ + \hbar R^* E_L^* \exp(i\omega_L t) a\, a_s, \tag{2}$$

where $a(a^-)$ and $a_s(a_s^+$ are the annihilation (creation) operators of the phonons and the Stokes mode respectively. The symbol Ω denotes the frequency of the phonon mode, and R is the coupling constant between the Stokes mode and the phonon mode; E_L is the classical amplitude of the pumping field.

In the following we just consider the case of no losses in phonon mode and the Stokes mode, which is reasonable for the crystal at very low temperature and the system having no resonant cavity of Stokes mode.

According to Heisenberg equation of motion, we have

$$\frac{da}{dt} = -i\Omega a - iRE_L \exp(-i\omega_L t)a_s^+,$$
$$\frac{da_s}{dt} = -i\omega_s a_s - iRE_L \exp(-i\omega_L t)a^+. \tag{3}$$

Define a detuning parameter Δ as

$$\Delta = \omega_L - \omega_s - \Omega, \tag{4}$$

which can be changed by controlling the frequency $\omega_L$or ω_s. In the following we will see that it is the detuning Δ that plays an important part in the measurement of the phonons in our system.

We make a rotating transformation

$$a = A\exp[-i(\Omega + \Delta)t],$$
$$a_s = A_s \exp(-i\omega_s t). \tag{5}$$

Then we have, from Eqs. (3) - (5),

$$\frac{dA}{dt} = i\Delta A - iRE_L A_s^+, \tag{6a}$$
$$\frac{dA_s}{dt} = -iRE_L A^+. \tag{6b}$$

Let E_L be real and use the notation $\beta \equiv |RE_L|$, we obtain

$$\frac{dA}{dt} = i\Delta A - i\beta A_s^+, \tag{7a}$$
$$\frac{dA_s}{dt} = -i\beta A^+. \tag{7b}$$

We define the amplitude quadrature X, and the phase quadrature Y of the phonon mode as

$$X = A^+ + A, \quad Y = i(A^+ - A). \tag{8a}$$

Similarly, for the Stokes mode, we define

$$X_s = A_s^+ + A_s, \quad Y_s = i(A_s^+ - A_s). \tag{8b}$$

Then we have the following equations,

$$\frac{dX}{dt} = -\Delta Y - \beta Y_s, \tag{9a}$$
$$\frac{dY}{dt} = \Delta X - \beta X_s, \tag{9b}$$
$$\frac{dX_s}{dt} = -\beta Y, \tag{9c}$$

$$\frac{dY_s}{dt} = -\beta X. \tag{9d}$$

We derive the solutions of Eqs. (9) in different regions of the value Δ (see Appendix A).

In order to evaluate the QND effectiveness of our system, we use three quantitative criteria[23]: (i) signal-equivalent input noise N_s^{eq}; (ii) measurement-equivalent input noise N_m^{eq}; (iii) conditional variance V.

If the product of the signal-and measurement-equivalent noises is less than unity, that is,

$$N_s^{eq} N_m^{eq} < 1, \tag{10}$$

the system can be used to perform a backaction evading (BAE) measurement; if the conditional variance is less than unity, that is,

$$V < 1, \tag{11}$$

the system can be used to make a quantum state preparation.

A true QND device will be defined as a BAE measurement yielding also quantum state preparation, that is, both inequalities (10) and (11) must be satisfied. If the product of the equivalent noises equal to unity and the conditional variance also equal to unity, then the device will be in the classical limit of QND measurement; if both the product and the conditional variance are greater than unity, the device will be in the classical domain.

For calculating N_s^{eq}, N_m^{eq} and V, we first define the correlation quantity of operators A and B,

$$W(A,B) = < AB > - < A >< B >. \tag{12}$$

We assume that the initial states of TO phonons and the Stokes field are coherent states. In this case, N_s^{eq}, N_m^{eq} and V can be expressed as [23]

$$N_s^{eq} = \frac{|W(X(0),X(0))|^2}{|W(X(0),X(t))|^2} W(X(t),X(t)) - W(X(0),X(0)), \tag{13}$$

$$N_m^{eq} = \frac{|W(X(0),X(0))|^2}{|W(X(0),Y_s(t))|^2} W(Y_s(t),Y_s(t)) - W(X(0),X(0)), \tag{14}$$

$$V = W(X(t),X(t)) - \frac{|W(X(t),Y_s(t))|^2}{W(Y_s(t),Y_s(t))}. \tag{15}$$

Then, according to the solutions (A1) - (A22) and Eqs. (13) - (15), we obtain N_s^{eq}, N_m^{eq} and V for different regions of the value Δ (see Appendix B).

IV. RESULTS AND DISCUSSION

From Eqs. (B3) and (B4) we can see that for $\Delta = 0$ the system lies in the classical limit of BAE measurement. From Eqs. (B22) and (B23), we also see that for $\Delta = 2\beta$, the conditional variance is greater than unity and the product of the equivalent noises equal to unity, which indicates that it is not possible to perform a quantum state

preparation and the system lies in the classical limit of BAE measurement.

According to Eqs. (B5) – (B19), scaling all the variables by β (letting $\beta=1$), we make the plots for $0<\Delta<2$. Figure 2 shows the product of equivalent noises $N_s^{eq}N_m^{eq}$ versus the interaction time t of the TO phonons with the Stokes field for different values of detuning. For small detuning values ($\Delta=0.2, 0.6, 1.0, 1.4$), the product approaches unity. For large detuning values ($\Delta=1.5, 1.7, 1.9$), the product can be less then unity, have a periodic change as the time increases at a fixed detuning value and decrease as the detuning value increases. This shows that the closer the detuning value approaches 2, the greater the possibility for making BAE measurements. On the other hand, Fig. 3 shows the plot of the conditional variance $V^{\#}$ (the symbol # means making data conversion as $10\log_{10}(V)$), as a function of the interaction time for different detuning values. In this plot, the dotted line represents $V^{\#}=0$, which implies that the conditional variance is equal to unity, and $V^{\#}<0$ means $0<V<1$. Therefore, from this plot we see that the variance can be less than unity, if the time is short for a fixed detuning value; but it can increase rapidly if the time is long for all the detuning values. In addition, the variance increases as the detuning value increases, so the quantum state preparation will be benefited by smaller detuning values.

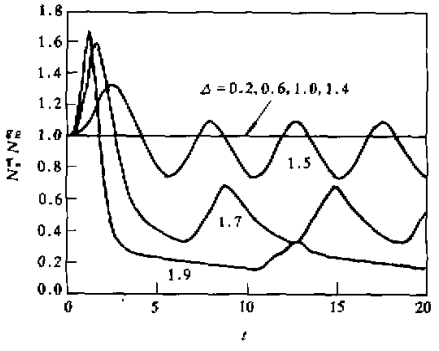


Fig. 2. Product of equivalent noises versus time t (inunit of rad/β) for different detuning values ($0<\Delta<2$).

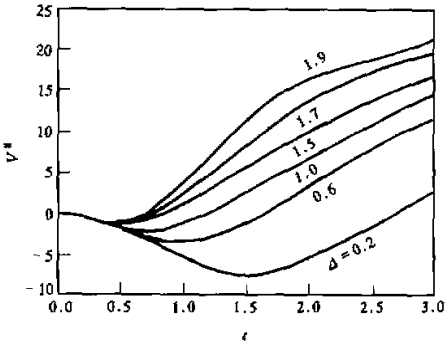


Fig. 3. Conditional variance $V^{\#}$ versus time for a range of detuning values ($0<\Delta<2$).

For $\Delta>2\beta$, also letting $\beta=1$, we have plotted the results of Eqs. (B24) – (B28) and Eq. (15) ad in Figs. 4 and 5. Figure 4 gives the product of the equivalent noises $N_s^{eq}N^{eq\#}$ (symbol # means making data conversion as $10\log_{10}(N_s^{eq}N_m^{eq})$) versus the interaction time for three fixed detuning values ($\Delta=2.1, 2.6$, and 3.6). For small detuning values, it is possible that $N_s^{eq}N_m^{eq\#}<0$. This means that the product can be less than unity for some intervals of time; but for large detuning values, it is always greater than unity. Figure 5 shows the conditional variance versus time for the above range of detuning values. We see that the variance can be less than unity for these de-

tuning values if the time is short, and the minimum of the variance decreases as the detuning value increases.

We give a possible explanation for the above results. Because the interaction between the phonon mode and the photon mode is weakened when Δ is large, the correlation of $X(0)$ and $X(t)$ increases as the detuning Δ increases, while the correlation of $X(0)$ and $Y_s(t)$ decreases ad Δ increases. Similarly, the correlations are related to the interaction time, so BAE measurement can be realized only when the detuning and time are suitably chosen. On the other hand, the ability of QSP comes from the correlation of $X(t)$ and $Y_s(t)$, which is also affected by both the detuning and the time, therefore QSP becomes possible only at certain values of them.

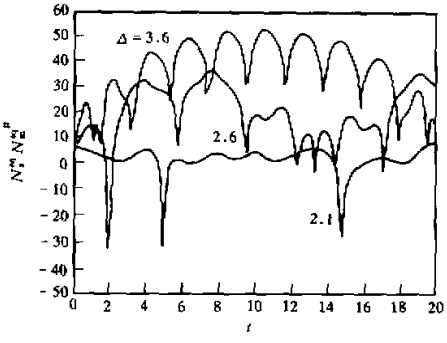


Fig. 4. Product of equivalent noises $N_s^{ss}N_m^{ss\#}$ versus time for $\Delta = 2.1, 2.6, 3.6 (\Delta > 2)$.

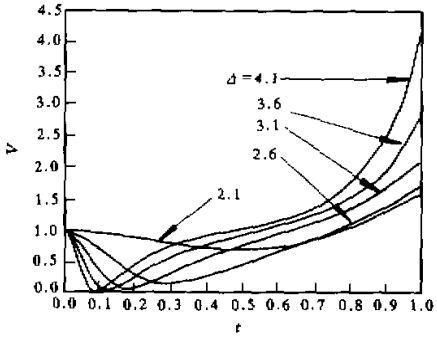


Fig. 5. Conditional variance versus time for a range of detuning values ($\Delta > 2$).

V. CONCLUSIONS

It is possible to perform either the BAE measurement or QSP for the TO phonons inside a crystal using RIKE method. In our system, the signal mode is the phonon mode, the initial state of which is a coherent state; the probe mode is the coherent Stokes field, and the amplitude quadrature of the phonon mode is measured by the phase quadrature of the Stokes mode. The exact solution of the system can be obtained; the solution without detuning is similar to that of a nondegenerate optical parametric amplifier (NOPA), and in this case the system is in the classical limit of BAE measurements. Fortunately, the detuning parameter of the system plays a key role in the measurement for the TO phonons, which is different from the NOP system. If a detuning value Δ is chosen near to 2β, a BAE measurement can be realized for some fixed values of interaction time. If a detuning value Δ is chosen except 2β, a quantum state preparation can be performed for a short time, especially for the detunning value near to zero.

APPENDIX A

In this appendix we give the solutions of Eqs. (9) in different regions of the value Δ:

(1) $\Delta=0$,

$$X(t) = \cosh(\beta t)X(0) - \sinh(\beta t)Y_s(0), \tag{A1}$$

$$Y_s(t) = \cosh(\beta t)Y_s(0) - \sinh(\beta t)X(0), \tag{A2}$$

where $X(0)$ and $Y_s(0)$ are the initial values of X and Y_s, respectively, $X(t)$ and $Y_s(t)$ are the values of X and Y_s corresponding to the interaction time t of the phonon and the Stokes field.

(2) $0<\Delta<2\beta$,

$$X(t) = e^{ut}(c_1\cos vt + c_2\sin vt) + e^{-ut}(c_3\cos vt + c_4\sin vt), \tag{A3}$$

$$\begin{aligned} Y_s(t) = {} & Y_s(0) - e^{ut}[c_1\cos(vt-\varphi) + c_2\sin(vt-\varphi)] \\ & + e^{-ut}[c_3\cos(vt+\varphi) + c_4\sin(vt+\varphi)] \\ & + (c_1 - c_3)\cos\varphi - (c_2 + c_4)\sin\varphi, \end{aligned} \tag{A4}$$

where

$$u = \beta\cos\varphi, \quad v = \beta\sin\varphi. \tag{A5}$$

$$\tan 2\varphi = \frac{\Delta\sqrt{4\beta^2 - \Delta^2}}{2\beta^2 - \Delta^2}. \tag{A6}$$

Here the constants c_1, c_2, c_3, and c_4 are give by the initial values $X(0)$, $Y(0)$, $X_s(0)$ and $Y_s(0)$:

$$c_1 = \frac{1}{2}X(0) - \frac{\Delta}{2\beta\cos\varphi}(1 - m_1\sin\varphi)Y(0) - \frac{1}{2\cos\varphi}(1 - m_2\sin\varphi)Y_s(0), \tag{A7}$$

$$c_2 = \frac{\beta^2 - \Delta^2 - \beta^2\cos 2\varphi}{2\beta^2\sin 2\varphi}X(0) + \frac{\Delta}{2\beta\sin 2\varphi}X_s(0) - \frac{\Delta}{2\beta}m_1 Y(0) - \frac{1}{2}m_2 Y_s(0), \tag{A8}$$

$$c_c = \frac{1}{2}X(0) + \frac{\Delta}{2\beta\cos\varphi}(1 - m_1\sin\varphi)Y(0) + \frac{1}{2\cos\varphi}(1 - m_2\sin\varphi)Y_s(0), \tag{A9}$$

$$c_4 = -\frac{\beta^2 - \Delta^2 - \beta^2\cos 2\varphi}{2\beta^2\sin 2\varphi}X(0) + \frac{\Delta}{2\beta\sin 2\varphi}X_s(0) - \frac{\Delta}{2\beta}m_1 Y(0) - \frac{1}{2}m_2 Y_s(0), \tag{A10}$$

where

$$m_1 = \frac{2\beta^2 - \Delta^2 - \dfrac{\beta^2\cos 3\varphi}{\cos\varphi}}{\beta^2(\sin 3\varphi - \tan\varphi\cos 3\varphi)}, \tag{A11}$$

$$m_2 = \frac{\beta^2 - \Delta^2 - \dfrac{\beta^2\cos 3\varphi}{\cos\varphi}}{\beta^2(\sin 3\varphi - \tan\varphi\cos 3\varphi)}. \tag{A12}$$

(3) $\Delta=2\beta$,

$$\begin{aligned} X(t) = {} & (\cos\beta t - \beta t\sin\beta t)X(0) - \frac{\Delta}{2}\left(\frac{\sin\beta t}{\beta} + t\cos\beta t\right)Y(0) \\ & + \frac{\Delta}{2}t\sin\beta t\, X_s(0) - \beta t\cos\beta t\, Y_s(0), \end{aligned} \tag{A13}$$

$$Y_s(t) = -\beta t\cos\beta t X(0) + \beta t\sin\beta t Y(0) - (\sin\beta t - \beta t\cos\beta t) X_s(0) + (\cos\beta t + \beta t\sin\beta t) Y_s(0). \tag{A14}$$

(4) $\Delta > 2\beta$,

$$X(t) = (b_1 \cos\omega_1 t + b_3 \cos\omega_2 t) X(0) + (b_2 \sin\omega_1 t + b_4 \sin\omega_2 t) Y(0) + (b_1' \cos\omega_1 t + b_3' \cos\omega_2 t) X_s(0) + (b_2' \sin\omega_1 t + b_4' \sin\omega_2 t) Y_s(0), \tag{A15}$$

$$Y_s(t) = -\beta\left(\frac{b_1 \sin\omega_1 t}{\omega_1} + \frac{b_3 \sin\omega_2 t}{\omega_2}\right) X(0) - \beta\left(\frac{b_2(1-\cos\omega_1 t}{\omega_1} + \frac{b_4(1-\cos\omega_2 t)}{\omega_2}\right) Y(0) - \beta\left(\frac{b_1' \sin\omega_1 t}{\omega_1} + \frac{b_3' \sin\omega_2 t}{\omega_2}\right) X_s(0) + \left[1 - \beta\left(\frac{b_2'(1-\cos\omega_1 t)}{\omega_1} + \frac{b_4'(1-\cos\omega_2 t)}{\omega_2}\right)\right] Y_s(0), \tag{A16}$$

where

$$\omega_1^2 = \frac{1}{2}(\Delta^2 - 2\beta^2 - \Delta\sqrt{\Delta^2 - 4\beta^2}), \tag{A17}$$

$$\omega_2^2 = \frac{1}{2}(\Delta^2 - 2\beta^2 + \Delta\sqrt{\Delta^2 - 4\beta^2}), \tag{A18}$$

$$b_1 = -b_2 = \frac{\beta^2 - \Delta^2 + \omega_2^2}{\omega_2^2 - \omega_1^2}, \tag{A19}$$

$$b_3 = -b_4 = \frac{\Delta^2 - \beta^2 - \omega_1^2}{\omega_2^2 - \omega_1^2}, \tag{A20}$$

$$b_1' = -b_3' = \frac{\beta\Delta}{\omega_2^2 - \omega_2^2}, \tag{A21}$$

$$b_2' = -b_4' = \frac{\beta(\Delta^2 - \beta^2 - \omega_2^2)}{\omega_1(\omega_2^2 - \omega_1^2)}. \tag{A22}$$

APPENDIX B

In this appendix we obtain the $N_s^{eq}N_m^{eq}$ and V in different regions of Δ from Eqs. (A1) – (A22) and Eqs. (13) – (15):

(1) $\Delta = 0$

$$N_s^{eq} = \frac{\sinh^2(\beta t)}{\cosh^2(\beta t)} < 1, \tag{B1}$$

$$N_m^{eq} = \frac{\cosh^2(\beta t)}{\sinh^2(\beta t)} > 1, \tag{B2}$$

$$V = (\cosh^2(\beta t) + \sinh^2(\beta t))^{-1} < 1, \tag{B3}$$

and obviously

$$N_s^{eq}N_m^{eq} = 1. \tag{B4}$$

(2) $0 < \Delta < 2\beta$

$$N_s^{eq} = \frac{A_3^2 + A_4^2}{A_1^2 + A_2^2}, \tag{B5}$$

$$N_m^{eq} = \frac{B_3^2 + B_4^2}{B_1^2 + B_2^2}, \tag{B6}$$

$$V = A_1^2 + A_2^2 + A_3^2 + A_4^2 + \frac{A_1B_1 + A_2B_2 + A_3B_3 + A_4B_4}{B_1^2 + B_2^2 + B_3^2 + B_4^2}, \tag{B7}$$

$$A_1 = \cosh(ut)\cos vt + \frac{\beta^2 - \Delta^2 - \beta^2\cos 2\varphi}{\beta^2 \sin 2\varphi}\sinh(ut)\sin vt, \tag{B8}$$

$$A_2 = -\frac{\Delta}{\beta\cos\varphi}(1 - m_1\sin\varphi)\sinh(ut)\cos vt - \frac{\Delta}{\beta}m_1\cosh(ut)\sin vt, \tag{B9}$$

$$A_3 = \frac{\Delta}{\beta\sin 2\varphi}\sinh(ut)\sin vt, \tag{B10}$$

$$A_4 = -\frac{1 - m_2\sin\varphi}{\cos\varphi}\sinh(ut)\cos vt - m_2\cosh(ut)\sin vt. \tag{B11}$$

$$B_1 = n_1 + \frac{\beta^2 - \Delta^2 - \beta^2\cos 2\varphi}{\beta^2\sin 2\varphi}n_4, \tag{B12}$$

$$B_2 = -\frac{\Delta}{\beta\cos\varphi}(1 - m_1\sin\varphi)n_3 - \frac{\Delta}{\beta}m_1n_2 + \frac{2\Delta}{\beta}(1 - 2m_1\sin\varphi), \tag{B13}$$

$$B_3 = -\frac{\Delta}{\beta\sin 2\varphi}n_4, \tag{B14}$$

$$B_4 = -4m_2\sin\varphi - \frac{1}{\cos\varphi}1 - m_2\sin\varphi)n_3 - m_2n_2, \tag{B15}$$

$$n_1 = e^{ut}\cos(vt - \varphi) - e^{ut}\cos(vt + \varphi), \tag{B16}$$

$$n_2 = e^{ut}\sin(vt - \varphi) - e^{ut}\sin(vt + \varphi), \tag{B17}$$

$$n_3 = e^{ut}\cos(vt - \varphi) + e^{-ut}\cos(vt + \varphi), \tag{B18}$$

$$n_4 = e^{ut}\sin(vt - \varphi) + e^{-ut}\sin(vt + \varphi). \tag{B19}$$

(3) $\Delta = 2\beta$

$$N_s^{eq} = \frac{\beta^2t^2}{1 + \beta^2t^2} < 1, \tag{B20}$$

$$N_m^{eq} = \frac{1 + \beta^2t^2}{\beta^2t^2} > 1, \tag{B21}$$

$$V = \frac{1 + 4\beta^4t^4}{1 + \beta^2t^2} > 1, \tag{B22}$$

and

$$N_s^{eq}N_m^{eq} = 1. \tag{B23}$$

(4) $\Delta > \beta$

$$N_s^{eq} = \left[1 + \frac{\Delta^2 - 4\beta^2}{4\beta^2}\frac{1}{\sin^2\sqrt{\Delta^2 - 4\beta^2}t}\right]^{-1}, \tag{B24}$$

$$N_m^{eq} = \left|\beta^2\left(\frac{b_1\sin\omega_1t}{\omega_1} + \frac{b_3\sin\omega_2t}{\omega_2}\right)^2 + \left[1 - \beta\left(\frac{b_2(1 - \cos\omega_1t)}{\omega_1} + \frac{b_4(1 - \cos\omega_2t)}{\omega_2}\right)\right]^2\right| \Big/ \left[\beta^2\left(\frac{b_1\sin\omega_1t}{\omega_1} + \frac{b_3\sin\omega_2t}{\omega_2}\right)^2 + \beta^2\left(\frac{b_2(1 - \cos\omega_1t}{\omega_1} + \frac{b_4(1 - \cos\omega_2t}{\omega_2}\right)^2\right], \tag{B25}$$

and V can be obtained from Eq. (15), where

$$W(X)+t),X(t)) = (b_1\cos\omega_1 t + b_3\cos\omega_2 t)^2 + (b_2\sin\omega_1 t + b_4\sin\omega_2 t)^2 + (b_1'\cos\omega_1 t + b_3'\cos\omega_w t)^2 + (b_2'\sin\omega_1 t + b_4'\sin\omega_2 t)^2, \quad \text{(B26)}$$

$$W(Y_s(t), Y_s(t)) = \beta^2\left(\frac{b_1\sin\omega_1 t}{\omega_1} + \frac{b_3\sin\omega_2 t}{\omega_2}\right) + \beta^2\left(\frac{b_2(1-\cos\omega_1 t)}{\omega_1} + \frac{b_4(1-\cos\omega_2 t)}{\omega_2}\right)^2 + \beta^2\left(\frac{b_1'\sin\omega_1 t}{\omega_1} + \frac{b_3'\sin\omega_2 t}{\omega_2}\right)^2 + \left[1-\beta\left(\frac{b_2'(1-\cos\omega_1 t)}{\omega_1} + \frac{b_4'(1-\cos\omega_2 t)}{\omega_2}\right)\right]^2, \quad \text{(B27)}$$

$$W(X(t), Y_s(t)) = -\beta(b_1\cos\omega_1 t + b_3\cos\omega_2 t)\left(\frac{b_1\sin\omega_1 t}{\omega_1} + \frac{b_3\sin\omega_2 t}{\omega_2}\right) - \beta(b_2\sin\omega_1 t + b_4\sin\omega_2 t)\left(\frac{b_2(1-\cos\omega_1 t)}{\omega_1} + \frac{b_4(1-\cos\omega_2 t)}{\omega_2}\right) - \beta(b_1'\cos\omega_1 t + b_3'\cos\omega_2 t)\left(\frac{b_1'\sin\omega_1 t}{\omega_1} + \frac{b_3'\sin\omega_2 t}{\omega_2}\right) + (b_2'\sin\omega_1 t + b_4'\sin\omega_2 t)\left[1-\beta\left(\frac{b_2'(1-\cos\omega_1 t)}{\omega_1} + \frac{b_4'(1-\cos\omega_2 t)}{\omega_2}\right)\right]. \quad \text{(B28)}$$

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