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SUPERCONDUCTING JUNCTIONS IN THE CASE OF CORRELATED SYMMETRIC NOISES*

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We study two models for Josephson junctions, *i.e.*,a Gaussian white noise model and a symmetric dichotomous noises model. In this study we find that the correlated symmetric noises can produce a net voltage. This phenomenon is due to the correlation between the additive and the multiplicative noises. We show some characteristic features for this phenomenon: the net voltage is always negative and has a peak value when the strength of the additive noises varies for the former model(or when the strengths of the noises vary for the latter model). The results provide a theoretical foundation for reducing the net voltage caused by the correlated symmetric noises.

PACC: 0540; 7440; 7450

I. INTRODUCTION

A typical Josephson junction consists of a junction shunted by a resistance R and driven by a current $I(t)$. Such junctions are of interest not only due to their technological applications, but also due to their ability to exhibit (singly or in arrays) a number of important nonlinear phenomena, including phase locking, bifurcations, chaos, solitonic excitations, and pattern formation.[1] Josephson[2] showed that electron pairs can tunnel through a narrow insulating material between two superconductors. The current across the junction is given by $J_c = J_0 \sin\phi$, where ϕ is the phase difference of the superconducting order parameter across the junction and J_0 is a critical current. The evolution of the phase difference is described by the equation

$$\frac{\hbar}{2eR}\dot{\phi} + J_0 \sin\phi = I(t), \tag{1}$$

where $I(t)$ is a driving current.[3] The phase difference is related to the voltage through $V(t) = \frac{\hbar}{2e}\dot{\phi}$.

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In Ref.[4], Millonas and Chialvo showed that the asymmetry in the spectral properties of the noise (we call this noise the asymmetric noise) can lead to a fluctuation-induced net voltage

$$< V(t) >= \frac{\hbar}{2e} <\dot{\phi}> . \tag{2}$$

But the case studied by them is only the additive noise. In this case they said that any noise with a uniform distribution of phases, such as Gaussian noise, should be symmetric, and would not give rise to a net voltage.

In this paper we try to study the net voltage caused by environmental fluctuation. The environmental fluctuation is induced by the environmental perturbation, such as the perturbation of the electromagnetic field, the external vibration, the change of the external temperature, and so on. It is generally expressed as a multiplicative noise in the Langevin equation.[5−7] We find that in some circumstances symmetric noises can produce a net voltage.

II. A GAUSSIAN WHITE NOISE MODEL

In the case of environmental perturbations, such as the perturbation of the electromagnetic field, the external vibration, the change of the external temperature, *etc.*, the internal structure of the Josephson junction should change. In general, the change is very small. But when the environmental fluctuation becomes larger and larger, the change will become clearer and clearer. The change of the internal structure of the Josephson junction should make the critical current vary. We describe it via a stochastic parameter, *i.e.*, $J_0 + \sigma\xi_0(t)$, in which $\xi_0(t)$ is a stochastic force (the driving current $I(t) = \eta_0(t)$, which is a Gaussian white noise). Then Eq.(1) becomes

$$\frac{\hbar}{2eR}\dot{\phi} + (J_0 + \sigma\xi_0(t))\sin\phi = \eta_0(t), \tag{3}$$

where as an approximation we set $\xi_0(t)$ as Gaussian white noise. The statistical properties of $\xi_0(t)$ and $\eta_0(t)$ are $< \xi_0(t) >_f = < \eta_0(t) >_f = 0, < \xi_0(t)\xi_0(t') >_f = 2D'\delta(t-t')$, and $< \eta_0(t)\eta_0(t') >_f = 2D\delta(t-t')$($< >_f$ denotes averaging over the noises). And σ is a positive constant, $\eta_0(t)$ is the thermal noise, and $\xi_0(t)$ is the external noise. Equation (3) can be simplified as

$$\dot{\phi} = -\omega_0\sin\phi - \xi(t)\sin\phi + \eta(t), \tag{4}$$

in which $\omega_0 = \frac{2eRJ_0}{\hbar}, \xi(t) = \frac{2e\sigma R}{\hbar}\xi_0(t)$, and $\eta(t) = \frac{2eR}{\hbar}\eta_0$. We assume that $\xi_0(t)$ and $\eta_0(t)$ are correlated,*i.e.*, $< \eta_0(t)\xi_0(t) >_f = 2\lambda\sqrt{DD'}\delta(t-t')$ $(0 \le \lambda \le 1)$. The Stratonovich interpretation of the stochastic differential equation(4) yields the Fokker-Planck equation[8,9]

$$\partial_t P(\phi,t) = -\partial_\phi A(\phi)P(\phi,t) + \partial_\phi^2 B(\phi)P(\phi,t)\,, \tag{5}$$

where $A(\phi) = -\omega_0\sin\phi - \widetilde{D'}\cos\phi(-\sin\phi + \lambda\sqrt{\widetilde{D}\,/\,\widetilde{D'}})$ and $B(\phi) = \widetilde{D}\,(1-\lambda^2) + \widetilde{D'}(-\sin\phi + \lambda\sqrt{\widetilde{D}\,/\,\widetilde{D'}})^2$ ($\widetilde{D} = \left(\frac{2eR}{\hbar}\right)^2 D, \widetilde{D'} = \left(\frac{2e\sigma R}{\hbar}\right)^2 D'$). Under the periodic boundary conditions,we

obtain the stationary solution of Eq.(5)[10,11]

$$P_{\rm s}(\phi) = N\frac{{\rm e}^{\Psi(\phi)}}{B(\phi)}\oint {\rm d}\phi' {\rm e}^{-\Psi(\phi')-\mu\theta(\phi-\phi')}, \tag{6}$$

where $\Psi(\phi) = \int_0^{\phi}\frac{A(\phi')}{B(\phi')}{\rm d}\phi', \mu = \omega_0\int_0^{2\pi}\frac{\sin\phi}{\widetilde{D}\,(1-\lambda^2)+\widetilde{D}'\,(-\sin\phi+\lambda\sqrt{\widetilde{D}\,/\,\widetilde{D}'})^2}{\rm d}\phi$, and $\theta(\phi-\phi')$ is the Heaviside step function. N is the normalization constant.

The net voltage is

$$<V(t)>=\frac{\hbar}{2e}<<\dot{\phi}>_f>=\frac{\hbar}{2e}<<-\omega_0\sin\phi-\xi(t)\sin\phi>_f> . \tag{7}$$

According to the Novikov theorem[12], we have (see Appendix A)

$$<\xi(t)\sin\phi>_f=\widetilde{D}'\,(-\sin\phi+\lambda\sqrt{\widetilde{D}\,/\,\widetilde{D}'})\cos\phi. \tag{8}$$

From Eqs.(7) and (8) we can obtain

$$\begin{aligned}<V>_{\rm s} &= \lim_{t\to\infty}\frac{1}{t}\int_0^t\frac{\hbar}{2e}<<\dot{\phi}\,(\phi,\tau)>_f>_{\phi}{\rm d}\tau=\frac{\hbar}{2e}<A(\phi)>_{\phi}\\ &=\frac{\hbar}{2e}\int_0^{2\pi}P_{\rm s}(\phi)A(\phi){\rm d}\phi=\frac{N\hbar}{2e}\oint{\rm d}\phi\frac{A(\phi){\rm e}^{\Psi(\phi)}}{B(\phi)}\oint{\rm d}\phi'{\rm e}^{-\Psi(\phi')-\mu\theta(\phi-\phi')}.\end{aligned} \tag{9}$$

From Eq.(9) we can see that when the additive and the multiplicative noises are uncorrelated ($\lambda=0$), the net voltage will be zero; if $\lambda\neq 0$, then the net voltage will not be zero. Below

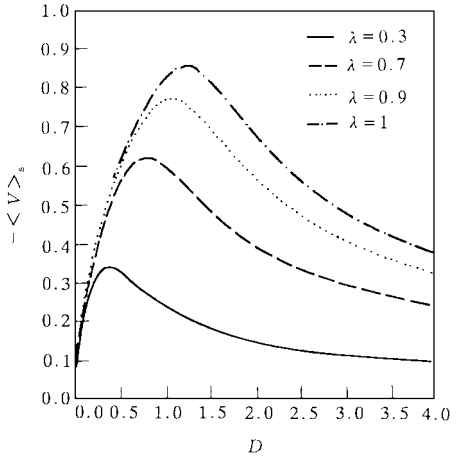

Fig.1. The mean negative voltage *vs* D for different values of λ. $D'=0.3$ and $\omega_0=1$ are fixed and λ =0.3, 0.7, 0.9, and 1.

we shall calculate $< V >_{\rm s}$ from Eq.(9). For convenience, we make the calculation in a dimensionless form, and set $\frac{\hbar}{2e}=J_0=\sigma=R=1$.

In Fig.1 we plot the negative net voltage versus the additive noise strength D for different values of λ. Here we set $D' = 0.3, \omega_0 = 1$, and λ =0.3, 0.7, 0.9, and 1. When

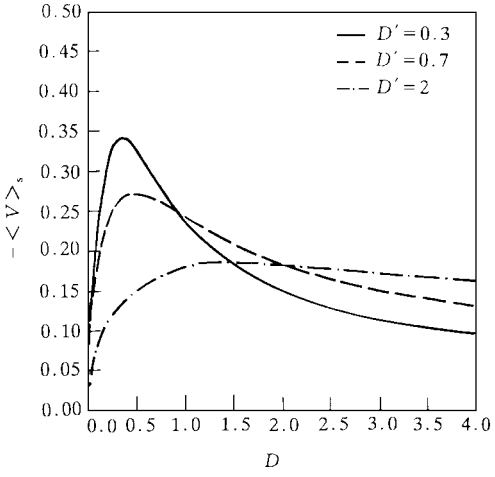

Fig.2. The mean negative voltage *vs* D for different values of D', with$\lambda = 0.3, \omega_0 = 1$, and D' =0.3, 0.7, and 2.

$\lambda = 1$, the stationary probability density $P_s(\phi)$ (see Eq.(6)) will be divergent at the points ϕ=arcsin$\sqrt{D/D'} + 2n\pi$(n=0,1,2,3,$\cdots$), so when $D < D'$ we cannot determine $< V >_s$ from Eq.(9); hence we only plot the curves in the case of $D > D'$. Firstly, the figure shows that the net voltage is always negative. In addition, the figure also shows: (a) In the curves a clear peak value appears; (b) with the increase of λ, the net voltage becomes more and more distinct and the peak value moves towards the right and increase. In Fig.2 we have represented the net voltage in the $-< V >_s$ versus D parameter plane with different multiplicative noise strength D'. From the figure we find that when the value of D' increases, the crest value descends and the curve becomes more and more even.

III. A DICHOTOMOUS NOISE MODEL

If in Eq.(3) $\xi_0(t)$ becomes a symmetric dichotomous noise (here we set the driving current $I(t) = a\xi_0(t)$), then we have

$$\dot{\phi} = -\omega_0 \sin\phi - \xi(t)\sin\phi + \widetilde{a}\ \xi(t), \tag{10}$$

where $\xi(t) = \frac{2e\sigma R}{\hbar}\xi_0(t), \omega_0 = \frac{2eRJ_0}{\hbar}$,and $\widetilde{a} = \frac{2eaR}{\hbar}$(Eq.(10) corresponds to Eq.(6) of Ref.[4]). The statistical properties of $\xi_0(t)$ are now changed into

$$<\xi_0(t)>_f = 0,\ <\xi_0(t)\xi_0(t')>_f = \frac{D}{\tau}\mathrm{e}^{-|t-t'|/\tau} = E^2\mathrm{e}^{-\lambda'|t-t'|}. \tag{11}$$

Here $\xi_0(t)$ has two values $\pm E(E > 0)$. And $1/2\tau = \lambda'/2$ is the transition probability of $\xi_0(t)$ from E to$-E$, and is also for that from $-E$ to E. From Eq.(10) we can obtain its stochastic

Liouville equation[13]

$$\partial_t \rho(\phi,t) = \partial_\phi[-\omega_0 \sin\phi - \sin\phi\xi(t) + \widetilde{a}\ \xi(t)]\rho(\phi,t). \tag{12}$$

The probability density $P(\phi,t) = <\rho(\phi,t)>_f$, so from Eq.(12) we have

$$\partial_t P(\phi,t) = \partial_\phi \omega_0 \sin\phi P(\phi,t) + \partial_\phi(\sin\phi - \widetilde{a})P_1(\phi,t), \tag{13}$$

where $P_1(\phi,t) = <\xi(t)\rho(\phi,t)>_f$. From the Shapino-Loginov differential formula[14] we obtain

$$\partial_t P_1(\phi,t) = \omega_0 \partial_\phi \sin\phi P_1(\phi,t) + \widetilde{E}^2\ \partial_\phi(\sin\phi - \widetilde{a})P(\phi,t) - \lambda' P_1(\phi,t), \tag{14}$$

in which $\widetilde{E} = \frac{2e\sigma R}{\hbar}E$.

The formal solution of Eq.(14) is

$$P_1(\phi,t) = \int_0^t \mathrm{e}^{-\widehat{A}(t-\tau)}[\widetilde{E}^2\ \partial_\phi(\sin\phi - \widetilde{a})P(\phi,\tau)]\mathrm{d}\tau, \tag{15}$$

where $\widehat{A} = \lambda' - \omega_0 \partial_\phi \sin\phi$. Substituting Eq.(15) into Eq.(13) yields the probability density equation for Eq.(10)

$$\partial_t P(\phi,t) = \partial_\phi \omega_0 \sin\phi P(\phi,t) + \partial_\phi(\sin\phi - \widetilde{a}) \int_0^t \mathrm{e}^{-\widehat{A}(t-\tau)} \widehat{E}^2 \partial_\phi(\sin\phi - \widetilde{a})P(\phi,\tau)\mathrm{d}\tau. \tag{16}$$

In the case of $\lambda' \gg \omega_0$, the integral part of the right hand side of Eq.(16) mainly stems from the values near $t = r$, hence we take $P(x,\tau) \approx P(x,t)$ as an approximation. Then Eq.(16) becomes

$$\partial_t P(\phi,t) = \partial_\phi \omega_0 \sin\phi P(\phi,t) + \partial_\phi(\sin\phi - \widetilde{a})\widehat{A}^{-1}\widehat{E}^2 \partial_\phi(\sin\phi - \widetilde{a})P(\phi,t). \tag{17}$$

Considering the condition $\lambda' \gg \omega_0$, we obtain the approximate Fokker-Planck equation

$$\partial_t P(\phi,t) = -\partial_\phi A_1(\phi)P(\phi,t) + \partial_\phi^2 B_1(\phi)P(\phi,t), \tag{18}$$

where $A_1(\phi) = -\omega_0 \sin\phi - \frac{\widetilde{E}^2}{\lambda'}\cos\phi(\widetilde{a} - \sin\phi)$ and $B_1(\phi) = \frac{\widetilde{E}^2}{\lambda'}(\widetilde{a} - \sin\phi)^2$. Under the periodic boundary conditions, the stationary solution of Eq.(18) is

$$P_{\mathrm{s}1}(\phi) = N_1 \frac{\mathrm{e}^{\Psi_1(\phi)}}{B_1(\phi)} \oint \mathrm{d}\phi' \mathrm{e}^{-\Psi_1(\phi') - \mu_1 \theta(\phi - \phi')}, \tag{19}$$

where $\Psi_1(\phi) = \int \frac{A_1(\phi')}{B_1(\phi')}\mathrm{d}\phi'$, $\mu_1 = \frac{\lambda'\omega_0}{E^2}\int_0^{2\pi} \frac{\sin\phi \mathrm{d}\phi}{(\widetilde{a} - \sin\phi)^2}$, and N_1 is the normalization constant. From Eq.(18) we can find that when $|\ a\ | \leq 1$, the stationary state of the system will be divergent at the points $\phi = \arcsin a + 2\pi n\,(n = \pm1, \pm2, \pm3, \ldots)$. Below we only study the case when $|\ a\ | > 1$.

The net voltage is

$$<V(t)> = \frac{\hbar}{2e} <<\dot{\phi}\ (\phi,t)>_f> = \frac{\hbar}{2e} << -\omega_0 \sin\phi - \xi(t)\sin\phi >_f> . \tag{20}$$

Using the Shapino-Loginov differential formula[14], we have

$$<\sin\phi\xi(t)>_t=\sum_{k=1}^{\infty}\frac{(-1)^k\ \widetilde{E}^2\ \omega_0^{k-1}}{(2\lambda)^k}\cos 2^{k-1}\phi(1-\sin 2^{k-1}\phi). \tag{21}$$

Thus Eq.(20) becomes

$$<V>_s=\frac{\hbar}{2e}\left(-\omega_0<\sin\phi>_\phi+\sum_{k=1}^{\infty}\frac{(-1)^k\ \widetilde{E}^2\ \omega_0^{k-1}}{(2\lambda)^k}<\cos 2^{k-1}\phi(1-\sin 2^{k-1}\phi)>\phi\right)$$

$$=-\frac{\hbar\omega_0}{2e}\int_0^{2\pi}P_{s1}(\phi)\sin\phi\mathrm{d}\phi+\frac{\hbar}{2e}\sum_{k=1}^{\infty}\frac{(-1)^k\ \widetilde{E}^2\ \omega_0^{k-1}}{(2\lambda)^k}\int_0^{2\pi}P_{s1}(\phi)\cos 2^{k-1}\phi(1-\sin 2^{k-1}\phi)\mathrm{d}\phi. \tag{22}$$

In Eq.(10) we have made the multiplicative noise and the additive noise the same one, which is for the convenience of theoretical calculations. If in Eq.(10) the multiplicative noise and the additive noise are not the same, then Eq.(10) becomes

$$\dot{\phi}=-\omega_0\sin\phi-\xi(t)\sin\phi+\widetilde{a}\ \eta(t), \tag{23}$$

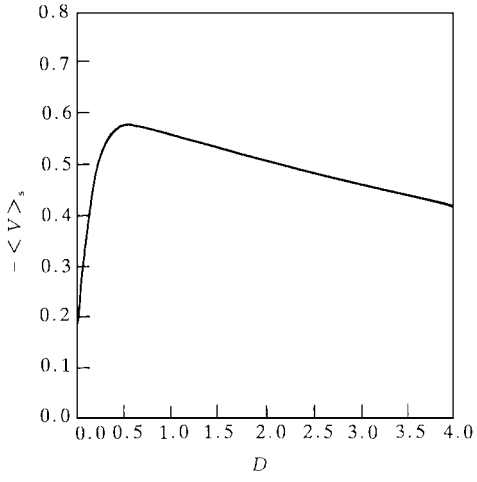

Fig.3. The mean negative voltage *vs* D. $\lambda'=100, \omega_0=1$, and $a=2$.

where $\xi(t)$ and $\eta(t)$ have the values $\pm\widetilde{E}$ and $\pm\widetilde{E}'$ ($\widetilde{E}\neq\widetilde{E}'$, and $\widetilde{E},\widetilde{E}'>0$),respectively. We set their statistical properties to be $<\xi(t)>_f=<\eta(t)>_f=0$, $<\xi(t)\xi(t')>_f=\widetilde{E}^2\ \mathrm{e}^{-\lambda'|t-t'|}$, $<\eta(t)\eta(t')>_f=\widetilde{E'^2}\ \mathrm{e}^{-\lambda|t-t'|}$, and $<\xi(t)\eta(t')>_f=\alpha\sqrt{\widetilde{E}\widetilde{E}'}\mathrm{e}^{-\lambda'|t-t'|}$ $(0\leq\alpha\leq 1)$. The transition probabilities for $\xi(t)$ from $\widetilde{E}$ to $-\widetilde{E}$ and that from $-\widetilde{E}$ to $\widetilde{E}$ are $\lambda'/2$; those for $\eta(t)$ from $\widetilde{E}'$ to $-\widetilde{E}'$ and from $-\widetilde{E}'$to $\widetilde{E}'$ are also $\lambda'/2$. It is clear that Eq.(10) is a special case for Eq.(23) in the case of $\alpha=1$ and $\widetilde{E}=\widetilde{E}'$. Further study shows that in the symmetric

dichotomous noise case, only when the multiplicative noise and the additive noise have a correlation, is there the net voltage.

The curve of the negative net voltage for Eq.(10) is plotted in Fig.3, for which we only take the preceding four terms in the expansion of Eq.(22). Similarly, we calculate in dimensionless form, and set $\hbar/2e = J_0 = \sigma = R = 1$. Figure 3 shows that the net voltage is negative and has a crest value.

IV. DISCUSSION

From the above examples, we find that the net voltage caused by the correlated symmetric noises is always negative. We should manipulate the noises according to the above theoretical results to reduce the net voltage to the lowest degree. In the case of definite environmental perturbation, for the first example, we should appropriately raise or drop the temperature to make the net voltage depart from the peak value (in Eq.(3) the internal thermal noise strength $D \propto k_{\mathrm{B}}T$, where k_{B} is the Boltzmann constant and T is the temperature); for the second example, we should decrease or increase the intensity of the driving current so as to make the net voltage leave the peak value. In the case of defined temperature or driving current, we should take measures to reduce or eliminate the environmental perturbation in order to make the net voltage lower.

Since the change of the environmental perturbation is extremely complicated, under certain circumstances, there is the correlation between the multiplicative and the additive noises. So in many circumstances the net voltage reported in this paper exists.

In Ref.[4], the authors reported that the symmetric noise cannot induce the net voltage in Josephson junctions. By the study of the above two examples, we find that if in the system there is a correlation between additive and multiplicative noises, then the symmetric noise can also produce a net voltage in Josephson junctions. The energy in response to the net voltage originates from a part of the noise energy, which is determined by the correlation between the additive and the multiplicative noises. When the noises in Eq.(3) are O-U noises or symmetric shot noises, as long as there exists a correlation between the additive and the multiplicative noises, there occurs the same phenomenon as reported in this paper. The keys to the symmetric noises-caused net voltage are: (a) the additive and multiplicative noises should exist simultaneously; (b) there should be a correlation between the additive and multiplicative noises. Here the correlation between them makes the probabilities of the fluctuations to the right and to the left of the potential barrier different, so that a net voltage arises.

For the above two examples we only consider the case of correlated symmetric noises. As for the case of correlated asymmetric noises, we shall report it afterwards.

APPENDIX A

Equation (4) can be transformed into the following form[8,9] :

$$\dot{\phi} = -\omega_0 \sin\phi + (-\sin\phi + \lambda\sqrt{\widetilde{D}\widetilde{D'}})\xi(t) + \eta'(t), \qquad (\mathrm{A}\cdot 1)$$

where $\eta'(t) = \eta(t) - (\lambda\sqrt{\widetilde{D} \,/\, \widetilde{D'}})\xi(t)$. The statistical properties of $\eta'(t)$ are $<\eta'(t)>= 0$, and $<\eta'(t)\eta'(t')>= 2\,\widetilde{D}\,(1-\lambda^2)\delta(t-t')$. Here the noises $\xi(t)$ and $\eta'(t)$ are no longer correlated.

From Eq.(A.1) and the Novikov theorem[12] we can get

$$<\xi(t)\sin\phi>_t = \widetilde{D'}\,(-\sin\phi + \lambda\sqrt{\widetilde{D} \,/\, \widetilde{D'}})\cos\phi. \qquad (\mathrm{A}\cdot 2)$$

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