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# EXPERIMENTAL STUDIES ON INTERMITTENT PHENOMENA IN A PLASMA*

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An intermittent process occurring in a low-temperature plasma discharge has been investigated. During a continuous scan of the discharge voltage, one kind of intermittency evolves from random upward bursts to a periodic oscillation, while the other evolves to chaos from random downward bursts. A critical exponent concerning the onset of the latter intermittency is quantitatively determined to be $\gamma = 0.912$.

**PACC:** 0547; 5235; 5280

## I. INTRODUCTION

Intermittency is an important nonlinear process and is closely associated with chaotic phenomena. Progress in the theory and experiments has revealed a lot of characteristics of it. For example, from three different ways how Floquet multiplier crosses the unit circle, Pomeau and Manneville defined three types of intermittency.[1] They are commonly characterized as: approximately periodic → chaos → approximately periodic → chaos → ···. Besides these types, Grebogi, *et al.*[2] discussed crisis-induced intermittencies which result from the collision between the attractor and an unstable periodic orbit. They are characterized as: chaos1 → chaos2 → chaos1 → chaos2 → ··· Recently, Platt, Spiegel, Tresser[3] discussed the mechanism of on-off intermittency in which the system stays in some constant-value state for a long time and then bursts into another state occasionally. These kinds of intermittency have been identified in plasmas,[4] laser,[5] and electronic circuits.[6,7] Recently, the crisis-induced intermittency by attractor-merging was observed in the plasma discharge in our device.[8] In this paper, we report two kinds of intermittency in the low-temperature plasma discharge, which seem not to belong to any kind of the intermittency mentioned above. One of them evolves from random upward bursts to a periodic oscillation, while the other evolves to chaos from random downward bursts. The law of the average duration between bursts vs discharge voltage and the correlation dimension have been obtained to quantitatively characterize the intermittency.

## II. EXPERIMENTAL ARRANGEMENT

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Our experimental device is a steady-state plasma device (SPD) which has been described previously.[8,9] In this kind of device, the plasma is produced by the electrons emitted from the hot tungsten filaments and accelerated in the electric field which is applied between the filaments and the chamber wall. The electric current $I_f$ in the filaments can be changed from tens to 150 A by changing the discharge voltage. As the anode, the chamber wall is grounded through a low-value resistance (0.05 $\Omega$). The pressure $P$ of the working gas (argon) is the second controllable parameter by using the feedback technique. Hundreds of small permanent magnets are stacked to the outside of the chamber wall to reduce the loss of the plasma resulting from diffusion. The third accessible parameter is the discharge voltage $V$ which is scanned from 0 to 36 V with a resolution of 0.01 V. The signals of the discharge current $I_d(t)$ and the floating potential ($V_p$) of several Langmuir probes at different positions are directly observed on an oscilloscope and sampled with a micro-computer, model No. PC286 (with a 12-bit A/D converter). An X-Y recorder is used to obtain the macroscopic $I$-$V$ characteristics. The typical plasma parameters are electron density $10^8$–$10^9$ cm$^{-3}$ and electron temperature 1–3 eV.

## III. EXPERIMENTAL RESULTS AND DISCUSSION

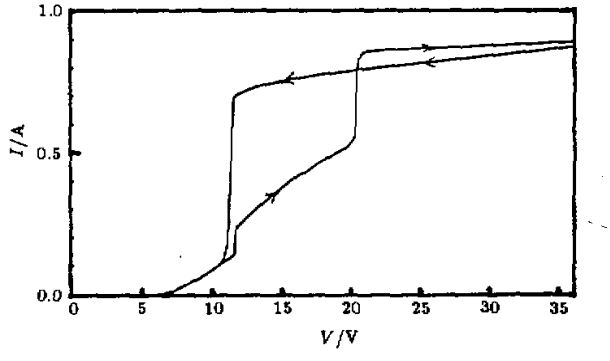
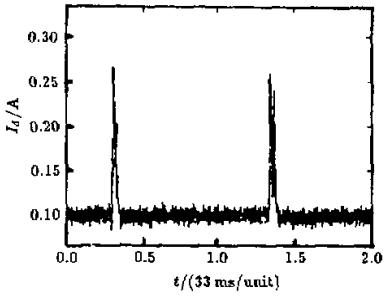


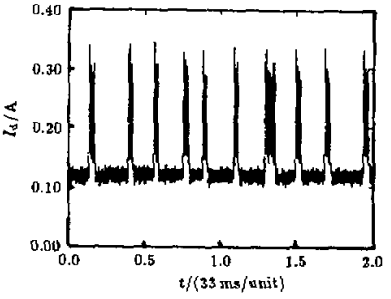

**Fig. 1.** The $I$-$V$ characteristic curve plotted under the condition $P$=0.13 Pa, $I_f$=130 A, $V$=0–36 V. The intermittency appears in the regions $V$=10–12 V and $V$=18–20 V respectively, which correspond to the regions before the second and third jumps of the discharge current $I$.

The experiments are performed in the following way. For given values of $P$ and $I_f$, the discharge voltage $V$ is scanned from 0 to 36 V. The intermittency is most frequently observed under the condition $I_f$=130 A, $P$=0.13 Pa. Figure 1 shows a typical multi-stability, or hysteresis diagram in the $I$-$V$ characteristics of the plasma under this condition. The oscillating and chaotic phenomena always appear in the region before the discharge current $I$ jumps to a higher state. Therefore, the multi-stability, in which there are more than one

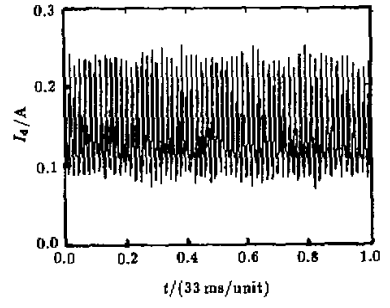
upward jumps in the hysteresis diagram, will result in several discrete oscillating regions, and just according to a simple curve of the $I$-$V$ characteristics, we are able to find the region where the oscillation and chaos happen. Rich phenomena and interesting results in different parameter spaces are researched in progress. In this paper we focus our attention on two regions in which there are evident jumps of the discharge current $I$ and, meanwhile, a larger ratio of signal to noise is assured.




(a) $V = 9.74$ V.




(b) $V = 10.55$ V.




(c) $V = 11.75$ V.

**Fig. 2.** The evolution of the upward bursts with the change of the discharge voltage $V$; $I_f$=130 A, $P$=0.13 Pa.

There is no observable oscillation until the discharge voltage $V$ is raised to about 6 V. Oscillation with very small amplitude appears in the region of the first hysteresis between ~6–7 V. The oscillation stops when the discharge voltage $V$ is raised to about 7 V (corresponding to the first and smaller upward jump in the $I$-$V$ characteristic curve). At $V$ ~9.74 V, the upward intermittency appears (Fig. 2(a)) and the bursts occur more frequently and with more spikes (Fig. 2(b)) as the discharge voltage $V$ is raised. Then they are linked together to form a seemingly periodic oscillation as the discharge voltage $V$ is raised to about 11.75 V

(Fig. 2(c)). The fast fourier transform (FFT) algorithm of the signals (Fig. 3) proves that it is approximately a periodic oscillation with the main frequency being 3.5 kHz. The region from upward bursts to the periodic oscillation is located before the second observable upward jump on the diagram of multi-stability. After this visible upward jump, the signal becomes random with higher frequencies ($f > 5$ kHz) and much smaller amplitude.

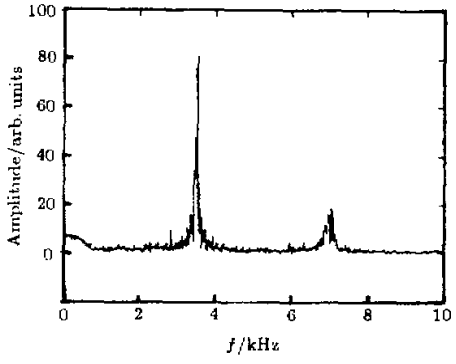



**Fig. 3.** FFT of the signal near the end of the upward burst. $V$=11.75 V.

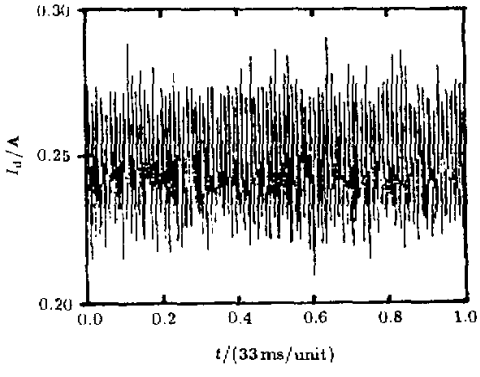
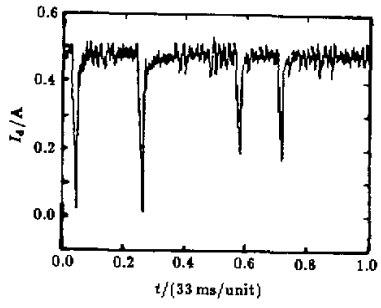


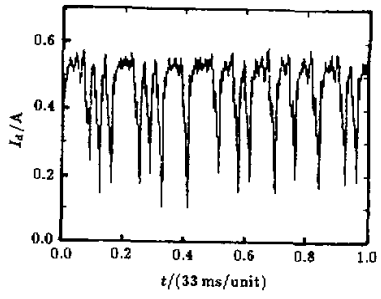

**Fig. 4.** The periodic-like oscillation replaced by the random oscillation with quite high frequency ($f > 5$ kHz) and small amplitude, for $V > 11.75$ V.

The random downward intermittency appears (Fig. 5(a)) as the discharge voltage $V$ is raised to about 18.40 V. Just as the phenomena happened in the upward intermittency, the bursts occur more frequently (Figs. 5(b) and 5(c)) as the discharge voltage $V$ is further raised. This region is located before the biggest jump, or the third observable jump on the diagram of multi-stability. Finally, at $V \sim 20$ V, the discharge current $I$ jumps to the highest state on the diagram of multi-stability, and after these jumps, the phenomena of

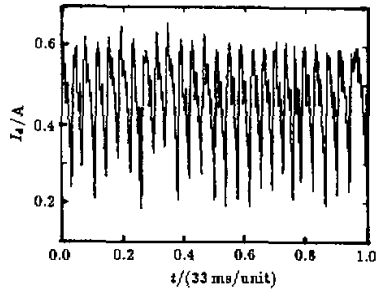
faint glow discharge replaces that of dark discharge, and the oscillation stops.




(a) $V$=18.41 V.




(b) $V$=18.85 V.

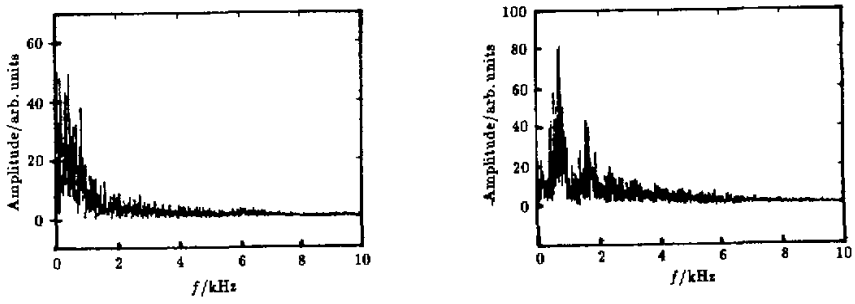



(c) $V$=19.32 V.

**Fig.5.** Evolution of the downward bursts with the changes of the discharge voltage $V$. $I_f$=130 A, $P$=0.13 Pa.

From the FFT of the signals at the beginning and the end of the downward intermittency (Figs. 6(a) and 6(b)), we can see clearly that they possess the characteristics of chaotic oscillation. Furthermore, according to the theory proposed by Packard *et al.*[10], that the strange attractors can be characterized from a single experimental signal, phase-portraits of $(I_d(t), I_d(t+T))$ are drawn to describe the characteristic of the intermittency. The delaytime $T$ is chosen by computing the average mutual information versus time using information theory.[11,12] In Fig. 7, we show an example of the average mutual information $I_m$ as a function of $T$. The best delaytime $T$ should be selected when the first minimum of $I_m$ appears. Figures 8(a) and 8(b) give the phase-portraits at the beginning and the end of the intermittency respectively (both with the delaytime $T = 5 \times \Delta t$, and $\Delta t = 33\,\mu$s is the time between two succeeding sampling data points). We will coin '$BI$' and '$EI$' to represent 'beginning of the intermittency' and 'end of the intermittency' for simplicity. From these

phase-portraits, we can see clearly that when the discharge voltage $V$ is slightly larger than $V_c$, the orbit on the attractor typically spends long stretches of time in the old region to which the attractor was confined before ($V < V_c$). At the end of one of these stretches, the orbit suddenly bursts out of the old region and bounces around in a new region, and then it returns to the old region for long stretches followed by another burst into the new region, and the processes are repeated. As the discharge voltage $V$ is increased, the attractor follows more and more frequently the new branches in which the trajectories get denser and denser. Finally, these new branches smear out together with the old one (Fig. 8(b)). Experiments and investigations show that the average lifetime $< \tau >$, i.e. the average time in the old region of attractor, decreases with the increase of the discharge voltage $V$ in the regime of the intermittency.




(a) $V$=18.41 V. (b) $V$=19.32 V.

**Fig. 6.** FFT of the intermittency.

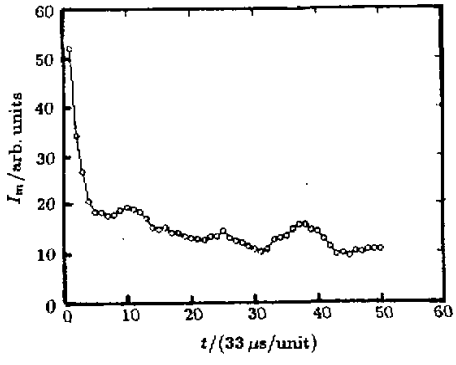
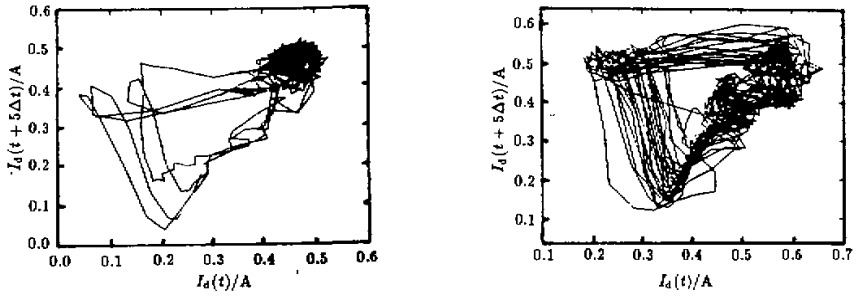



**Fig. 7.** The average mutual information $I_m$ vs delaytime $T$.

In general, the average lifetime $< \tau >$ as a function of the difference of the discharge voltage $\Delta V = V - V_c$ $(V > V_c)$ obeys the power law near the onset of the intermittency:

$$< \tau > \propto (\Delta V)^{-\gamma} , \tag{1}$$

$< \tau >$ can be obtained by direct measurements of the duration between bursts in the signals and then we get their averages. The critical value $V_c$ is determined by optimum method which




(a) $V$=18.43 V. (b) $V$=19.32 V.

**Fig. 8.** The evolution of the phase-portrait $I(t) - I(t + 5\Delta t)$ with the changes of the $V$.

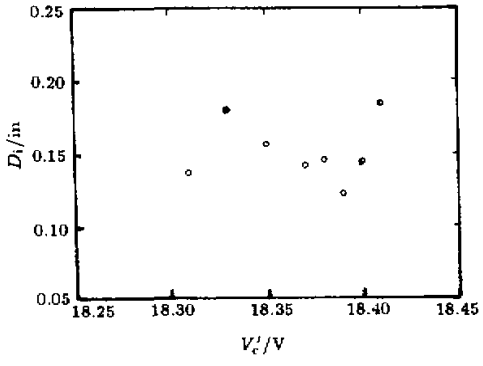



**Fig. 9.** Mean distances $D_i$ for each supposed critical discharge voltage $V_c'$. The minimum point is at $V$ = 18.39 V.

is described as follows: first, from the direct observation, we can read the $V_0$ at the moment the intermittency appears; secondly, around $V_0$ every 0.02 V difference is selected to be the supposed $V_c$ (noted as $V_c'$), and several log$< \tau >$-log$\Delta V$ lines can be drawn by means of the least square method. The mean distances $D_i$ between dots and the line are calculated and compared for every case to look for the smallest one to be the true $V_c$. In this experiment, the

$V_0$ is 18.41 V. We select six values of $V_c$, and find the minimum point is at the $V_c' = 18.39$ V. To be more accurate in determining $V_c$, $V_c' = 18.38$ V and 18.40 V are selected and $D_i$ are calculated respectively. Totally eight mean distances $D_i$ vs $V_c'$ are calculated and compared, as in Fig. 9. Finally, we determine the $V_c$=18.39 V to be the most reasonable one. This is in agreement with the perception that $V_c$ should be slightly smaller than $V_0$. Here 16000 points have been acquired, and tens of lifetimes are measured and their averages are calculated for each signal to minimize the accidental error. The experimental results are shown in Fig. 10 and within the experimental error, the dots are in good agreement with the line drawn by means of the least square method. Figure 10 gives the value of $\gamma = 0.912$.

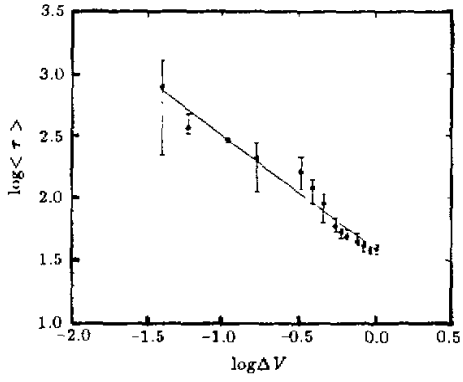



**Fig. 10.** The logarithmic plot of the average lifetime $< \tau >$ vs $\Delta V$. The equation of the line is $\log < \tau >= -0.912 \log \Delta V + 1.584$.

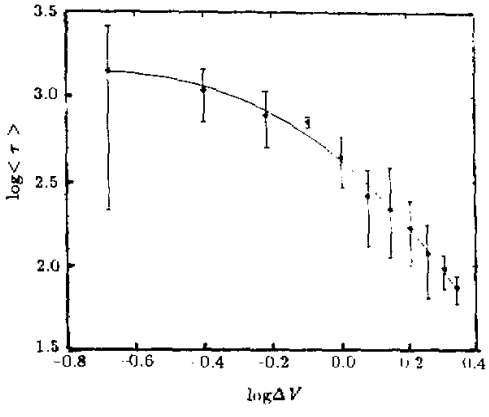



**Fig. 11.** The logarithmic plot of the lifetime $< \tau' >$ vs $\Delta V$. The equation of the line is $\log < \tau' >= -0.38(\log \Delta V)^3 - 1.57(\log \Delta V)^2 - 1.65 \log \Delta V + 2.63$.

The law of lifetime $< \tau' >$ vs $\Delta V$ on upward intermittency is also obtained with the same method as the above and is shown in Fig. 11. It can be fitted by the curve $\log < \tau' >= -0.38(\log\Delta V)^3 - 1.57(\log\Delta V)^2 - 1.65\log\Delta V + 2.63$. The mechanism study and the more accurate measurement on these two kinds of intermittency are in progress.

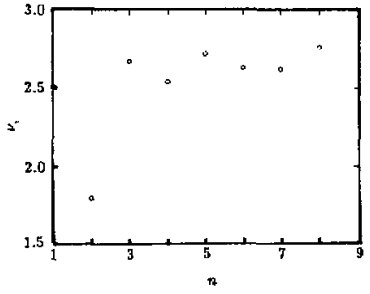
The correlation dimensions of the experimental signals are computed at $BI$ and $EI$ respectively with the method proposed by Grassberg and Procaccia[13] to investigate if there is any difference after the intermittency has developed. The correlation sum $C(r)$ is defined as

$$C(r) = \lim_{N\to\infty} \sum_{i=1,j=1,i\neq j}^{N} \theta(r - |\mathbf{r}_i - \mathbf{r}_j|) , \tag{2}$$

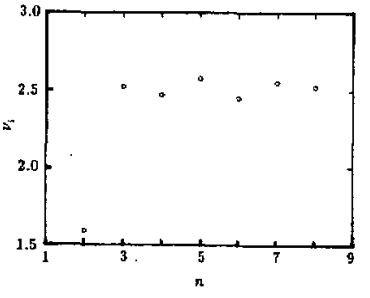
where $\theta(x)$ is the Heaviside function. The summation $\sum_{i=1,j=1,i\neq j}^{N} \theta(r - |\mathbf{r}_i - \mathbf{r}_j|)$ represents the number of points in the $n$-dimensional ball in the phase space which is constructed by the time delay method. For an intermediate region of $r$, $C(r)$ should scale like

$$C(r) \propto r^{\nu} , \tag{3}$$

where the exponent $\nu$ is the correlation dimension.




(a) At the beginning of the intermittency. $V = 18.43$ V.

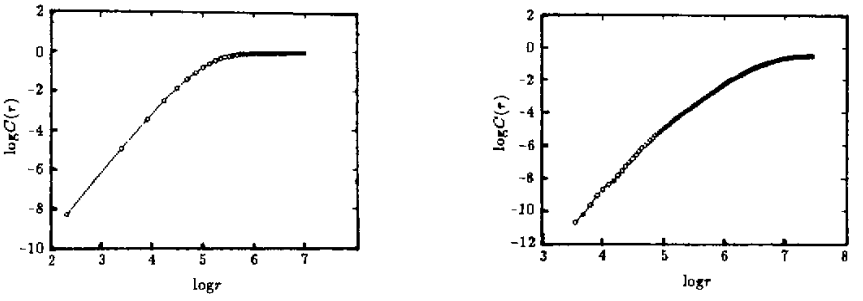



(b) Before the end of the intermittency. $V = 19.32$ V.

**Fig. 12.** The pointwise dimension $\nu_i$ vs embedding dimension $n$.

The delaytime $T$ is chosen by the principle used in phase-portrait restoring mentioned above, five $\Delta t$ are used at both $BI$ and $EI$. Embedding dimensions $n$=2–8 are tried to find the suitable case by calculating the pointwise dimension since it can give almost the same value as the correlation dimension but save a lot of computing time. Totally 15000 points are used to compute the pointwise dimension to satisfy the condition that the number of points should be large enough. The pointwise dimensions are calculated by randomly choosing a single value of subscript $i$ and then summing over only all the values with subscript $j$ in Eq. (2). In the log-log plot of $C_i(r)$ vs $r$, the slope of the linear region gives the pointwise

dimension. The fact that the pointwise dimension is saturated at $n \sim 3$ (Figs. 12(a) and 12(b)) at $BI$ and $EI$ makes us sure that $n = 4$ is enough to be the supposed embedding dimension to calculate the correlation dimension for both cases.




(a) $V = 18.43\,\mathrm{V}$. (b) $V = 19.32\,\mathrm{V}$.

**Fig. 13.** The log-log plot of $C(r)$ vs $r$.

With the above choices of delaytime and embedding dimension, we finally calculate the correlation dimension at $BI$ and $EI$ respectively. Figures 13(a) and 13(b) give the curves of $\log C(r)$ vs $\log r$. The slope of the two log-log curves of $C(r)$ vs $r$ give the values of $\nu_1 = 2.73 \pm 0.05$ and $\nu_2 = 2.78 \pm 0.06$. This means that the chaotic attractors in our system should be described by autonomous differential equations with at least three independent variables. The difference between the two correlation dimensions is not large enough to describe the development of chaotic attractor in the intermittent phenomena for the 'undriven' plasma in our experiment.

## IV.CONCLUSIONS

In summary, we have discovered two kinds of intermittent phenomena under the given condition. One kind of intermittency evolves from random upward bursts to an approximately periodic oscillation, while the other evolves to a chaotic oscillation from random downward bursts. FFTs, phase-portraits, critical exponent $\gamma$, together with correlation dimension are obtained to characterize the latter intermittent phenomena.

Up to date, in addition to the crisis-induced intermittency by attractor-merging,[8] we have discovered and studied three kinds of intermittent phenomena in the SPD. As the different kinds of intermittency occur in different parameter spaces, further work about the intermittent phenomena is going on in the SPD and it will be reported successively.

## ACKNOWLEGEMENTS

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team, especially from Professor Yuan Ding-pu and Professor Li Zhi-min for their technical help.

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