Volume 5, Number 7
1004-423X/96/05070505-06

ACTA PHYSICA SINICA
(Overseas Edition)

July, 1996
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INFLUENCE OF DRIVING-FIELD PHASE FLUCTUATION ON ABSORPTION AND DISPERSION IN A SIMPLE THREE-LEVEL ATOMIC SYSTEM*

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(Received 23 August 1995)

The steady-state behavior of a simple three-level atomic system has been investigated by taking into account the effect of phase fluctuations in the driving field. For a monochromatic driving field, lasing with or without inversion can be established over a wide frequency range for the probe and driving fields. A large refractive index can also be generated even if the population inversion is positive. For a finite linewidth of the driving field, the refractive index enhancement tends to decrease. Furthermore, a behavior change from inversion laser to noninversion laser can occur as the driving field linewidth increases.

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I. INTRODUCTION

Optical coherence and quantum interference have resulted in many new optical effects such as lasing without population inversion[1-3], enhancement of the refractive index without absorption[4-6] and electromagnetically induced transparency.[7-9] The study in this area can lead to a better understanding of the role of atomic coherence and quantum interference in interaction between an atom and a field, and may predict new optical phenomena.

In our previous paper[10], we investigated the steady-state behavior in a simple three-level atomic system driven by a single pump field, and showed that the simple system could exhibit noninversion or inversion lasing between the lower excited state and the ground state, and that a large refractive index with zero absorption could also be generated. In all these studies[1-10], the driving field was taken as having a well-defined fixed phase, however this is merely an idealization. Generally, the phase of the driving field is fluctuating, and the phase fluctuation will lead to a finite linewidth. Dalton and Knight[11] have shown that the phase fluctuation of the driving field can substantially influence the optical properties of the driven atomic systems. Recently, Sultana and Zubairy[12] studied the effect of finite linewidth on the enhancement of refractive index and lasing without inversion in microwave schemes and found that both phenomena are adversely affected by the phase fluctuations of the driving field. We have investigated the effect of phase fluctuation on absorption and dispersion in a three-level atomic system with an excited doublet, and showed that the finite linewidth of the driving field would prevent the medium from being transparent.[13]

*Project supported by the National Natural Science Foundation of China.

In this paper, we investigate the steady-state behavior of a simple three-level atomic system by taking into account phase fluctuations. We find that, for a monochromatic driving field, the atomic system can exhibit lasing with or without inversion over a wide frequency range for the probe field and the driving field (within the frequency range of the driving field, the gain without inversion is larger than that with inversion), and that a large refractive index without absorption can also be generated even if the population inversion is positive. For a driving field with a finite linewidth, a change from inversion to noninversion lasing can occur as the driving-field linewidth increases, and the enhancement of the refractive index tends to be quenched because of the linewidth of the driving field.

II. STEADY-STATE SOLUTIONS

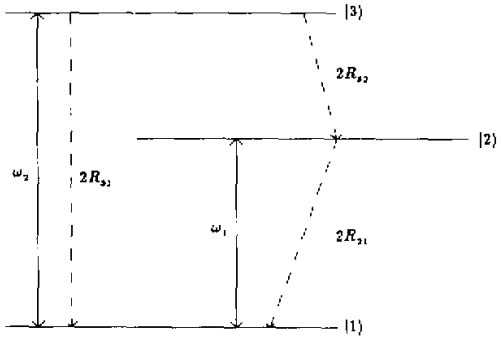

Fig. 1. A simple three-level atomic system. $\omega_1(\omega_2)$ is the frequency of a weak probe (a coherent driving) field, $2R_{ij}$ is the decay rate.

Consider a simple three-level atomic system with the ground state $|1\rangle$, the excited states $|2\rangle$ and $|3\rangle$ as illustrated in Fig. 1. A coherent pump field with Rabi frequency 2Ω drives the transition between states $|3\rangle$ and $|1\rangle$, and a weak probe field with Rabi frequency $2E$ is applied to the transition $|1\rangle \leftrightarrow |2\rangle$, with $2R_{ij}$ being the decay rate from $|i\rangle$ to $|j\rangle$. The interaction Hamiltonian for the atom and the two fields can be written as

$$H = [Ee^{-i\Delta_1(t)}|1\rangle\langle 2| + \Omega e^{-i\Delta_2(t)}|1\rangle\langle 3|] + \text{c.c.}\,, \tag{1}$$

where $\Delta_1(=\omega_{21}-\omega_1)$ is the frequency detuning of the probe field from the transition $|1\rangle \rightarrow |2\rangle$, $\Delta_2(=\omega_{31}-\omega_2)$ is the frequency detuning of the driving field from the transition $|1\rangle \rightarrow |3\rangle$, $\omega_1(\omega_2)$ is the frequency of the probe field (the driving field), and ω_{21} and ω_{31} are the transition frequencies from the ground state $|1\rangle$ to the excited states $|2\rangle$ and $|3\rangle$, respectively.

Let $\phi(t)$ represent the phase fluctuation of the driving field, i.e.,

$$\Omega = \Omega_0 \exp(i\phi(t))\,. \tag{2}$$

The phase is characterized by the following random equation of motion[14,15]:

$$\dot{\phi}(t) = \mu(t) , \tag{3}$$

with a zero average, i.e., $< \mu(t) >= 0$. Here $\mu(t)$ is a δ-correlated Langevin-noise term, whose diffusion coefficient gives the linewidth $2R_{\rm L}$ of the driving field, i.e.,

$$< \mu(t)\mu(t') >= 2R_{\rm L}\delta(t - t') . \tag{4}$$

In this case, the density-matrix equations should be averaged over the randomly fluctuating phases, that is, the density-matrix elements ρ_{ii} and ρ_{ij} must be replaced by $< \rho_{ii} >$ and $< \rho_{ij} >$. According to the method in Refs. [13] and [16], the stochastic averaged density-matrix equations can be derived as follows:

$$< \dot{\rho}_{11} >= 2R_{31} < \rho_{33} > +2R_{21} < \rho_{22} > +i\Omega_0(< \rho_{31} > - < \rho_{13} >) , \tag{5}$$

$$< \dot{\rho}_{22} >= -2R_{21} < \rho_{22} > +2R_{32} < \rho_{33} > , \tag{6}$$

$$< \dot{\rho}_{13} >= -(R_{21} + R_{32} + R_{\rm L} + i\Delta_2) < \rho_{13} > +i\Omega_0(< \rho_{33} > - < \rho_{11} >) , \tag{7}$$

$$< \dot{\rho}_{12} >= -(R_{21} + i\Delta_1) < \rho_{12} > +iE(< \rho_{22} > - < \rho_{11} >) + i\Omega_0 < \rho_{32} > , \tag{8}$$

$$< \dot{\rho}_{33} >= -[R_{21} + R_{31} + R_{32} + R_{\rm L} + i(\Delta_2 - \Delta_1)] < \rho_{23} > +iE < \rho_{13} > -i\Omega_0 < \rho_{21} > . \tag{9}$$

Here we have ignored the terms including E in Eqs. (5), (6) and (8) (weak-field approximation). Setting all the time derivatives in Eqs. (5)–(9) equal to zero and performing simple mathematical manipulations, we obtain the steady-state solutions:

$$\begin{aligned} < \rho_{22} > - < \rho_{11} >= \frac{1}{D}[&\Omega_0^2(R_{31} + R_{32} + R_{\rm L})(R_{32} - R_{21}) \\ &-R_{21}(R_{31} + R_{32})[(R_{31} + R_{32} + R_{\rm L})^2 + \Delta_2^2]] , \end{aligned} \tag{10}$$

$$< \rho_{33} > - < \rho_{11} >= -\frac{R_{21}(R_{31} + R_{32})}{D}[(R_{31} + R_{32} + R_{\rm L})^2 + \Delta_2^2] , \tag{11}$$

$$\begin{aligned} < \rho_{12} >= \frac{iE}{M}&[(R_{31} + R_{32} + R_{\rm L})[(R_{21} + R_{31} + R_{32} + R_{\rm L}) - i(\Delta_2 - \Delta_1)] \\ &\times(< \rho_{22} > - < \rho_{11} >) - \Omega_0^2(< \rho_{33} > - < \rho_{11} >)] . \end{aligned} \tag{12}$$

In these equations, D and M are, respectively, defined by

$$D = \Omega_0^2(R_{31} + R_{32} + R_{\rm L})(2R_{21} + R_{32}) + R_{21}(R_{31} + R_{32})[(R_{31} + R_{32} + R_{\rm L})^2 + \Delta_2^2] , \tag{13}$$

$$\begin{aligned} M = (R_{31} + R_{32} + R_{\rm L})&[\Omega_0^2 + R_{21}(R_{21} + R_{31} + R_{32} + R_{\rm L}) \\ &+\Delta_1(\Delta_2 - \Delta_1) + i\Delta_1(2R_{21} + R_{31} + R_{32}) - i\Delta_2 R_{21}] . \end{aligned} \tag{14}$$

Equation (12) is our main result. For a monochromatic driving field ($R_L = 0$), it reduces to the result in Ref. [10] (Im$< \rho_{12} >$ and Re$< \rho_{12} >$ can easily be derived from Eq. (12)). From Eq. (12), we can see that, Re$< \rho_{12} >$ and Im$< \rho_{12} >$ depend on the atomic decay rates R_{ij}, the frequency detunings Δ_1 and Δ_2, and the Rabi frequency Ω_0 of the driving field as well as the linewidth R_L of it. In the next section, we will investigate the influence of the driving-field phase fluctuation on absorption and dispersion in an atomic system. In numerical calculations, we will choose dimensionless parameters (Ω_0/R_{21} and $R_{21} = 1$, etc.).

III. INFLUENCE OF DRIVING-FIELD PHASE FLUCTUATION ON ABSORPTION AND DISPERSION

In Ref. [10], we found that for $R_{32} < R_{21}$, an atomic system could establish lasing without inversion between the lower excited state and the ground state, and the same atomic system could generate a large refractive index with zero absorption. We also showed that for $R_{32} > R_{21}$, the atomic system could establish lasing with atomic population inversion. Here we will study, for $R_{32} > R_{21}$, the influence of the driving-field phase fluctuation on absorption and dispersion in the atomic system

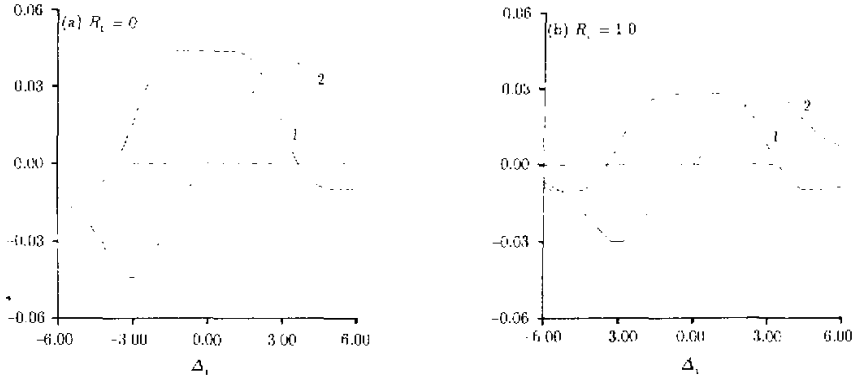

Fig. 2. Plots of Im$< \rho_{12} > /E$ (curve 1) and Re$< \rho_{12} > /E$ (curve 2) vs. Δ_1 for given R_L. The dashed line corresponds to Im$< \rho_{12} >=$ Re $< \rho_{12} >= 0$. Values of parameters are $\Delta_2 = 0$, $R_{31} = 0.2$, $R_{32} = 1.4$ and $\Omega_0 = 3.0$.

1. The driving-field frequency detuning Δ_2 equals zero. In this case, from Eq. (8) we can see that the population difference $< \rho_{22} > - < \rho_{11} >$ has no relation to the frequency detunings. Setting the atomic decay rates $R_{32} = 1.4, R_{31} = 0.2$, and the Rabi frequency $\Omega_0 = 3.0$, we obtain the plots of Im$< \rho_{12} > /E$ (curve 1) and Re$< \rho_{12} > /E$ (curve 2) versus Δ_1 for $R_L = 0$ in Fig. 2(a) and $R_L = 1.0$ in Fig. 2(b). In these figures, the dashed lines correspond to Im$< \rho_{12} >=$ Re $< \rho_{12} >= 0$. We can see from Figs. 2(a) and 2(b) that Im$< \rho_{12} >> 0$ holds true in a wide detuning range, therefore lasing can be established in both cases. However, for a monochromatic driving field ($R_L = 0$), $< \rho_{22} > - < \rho_{11} >= 0.031$ as obtained from Eq. (8), and the atomic system establishes a conventional inversion laser;

for a given finite linewidth, e.g., $R_L = 1.0$, $< \rho_{22} > - < \rho_{11} >= -0.015$, and lasing without inversion occurs. Thus a change from lasing with inversion to that without inversion can take place when a monochromatic driving field develops a finite linewidth.

Scully *et al.*[4–6] have shown that for $< \rho_{22} > - < \rho_{11} >< 0$, it was possible to obtain a large refractive index with zero absorption. From Fig. 2(a), where $< \rho_{22} > - < \rho_{11} >> 0$, we find that around $\Delta_1 \simeq 3.65$, Im$< \rho_{12} >$ vanishes while Re$< \rho_{12} >$ ($\simeq 0.40$) reaches its maximum. Thus we can say that even if the population inversion is positive, the atomic system can generate a large refractive index due to the atomic coherence and quantum interference effects. For a finite linewidth of the driving field (e.g., $R_L = 1.0$ in Fig. 2(b)), the effect of the refractive index enhancement is weaker (Re$< \rho_{12} >\sim 0.30$).

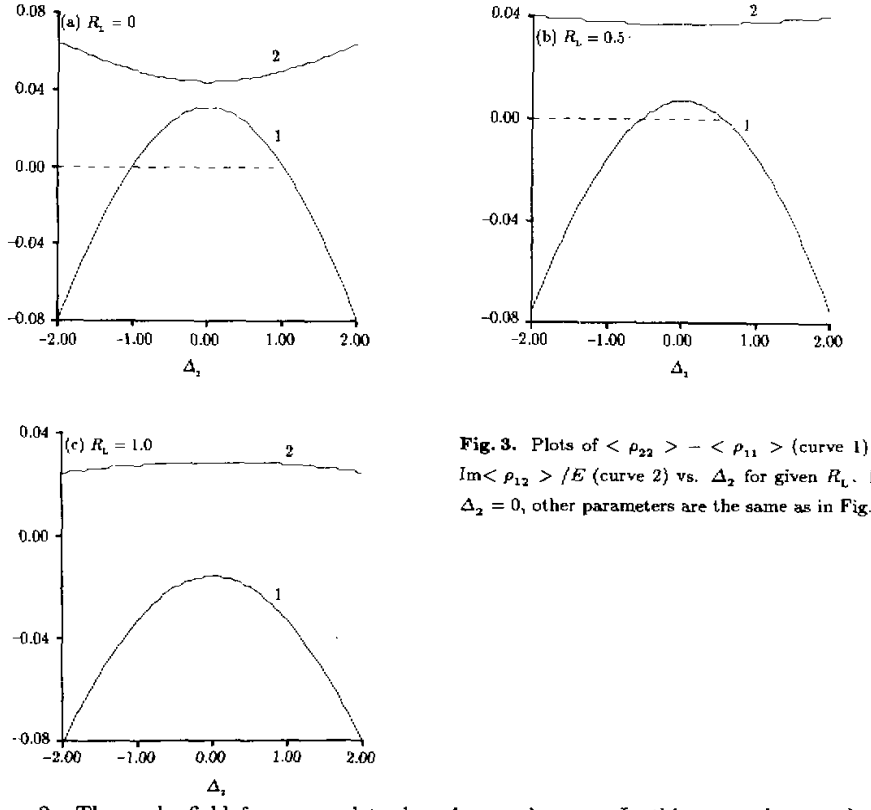

Fig. 3. Plots of $< \rho_{22} > - < \rho_{11} >$ (curve 1) and Im$< \rho_{12} > /E$ (curve 2) vs. Δ_2 for given R_L. Here $\Delta_2 = 0$, other parameters are the same as in Fig. 2.

2. The probe-field frequency detuning Δ_1 equals zero. In this case, the population difference $< \rho_{22} > - < \rho_{11} >$ is closely related to the driving-field frequency detuning Δ_2. Using the same atomic parameters and Rabi frequency of the driving field, we obtain the plots of $< \rho_{22} > - < \rho_{11} >$ (curve 1) and Im$< \rho_{12} > /E$ (curve 2) versus Δ_2 for given R_L as shown in Fig. 3(a) ($R_L = 0$), Figs. 3(b) ($R_L = 0.5$) and 3(c) ($R_L = 1.0$). We can see

from Fig. 3(a) that, for $R_L = 0$, in the region of $|\Delta_1| \leq 1.0$, $< \rho_{22} > - < \rho_{11} >> 0$ and $\mathrm{Im}< \rho_{12} >> 0$. The atomic system can establish a conventional inversion laser. While in the region of $1.0 \leq |\Delta_1| \leq 7.95$, $< \rho_{22} > - < \rho_{11} >< 0$ and $\mathrm{Im}< \rho_{22} >> 0$. The atomic system can establish lasing without inversion. Furthermore, the gain without inversion is larger than that with inversion. For a finite linewidth of the driving field, the region of inversion laser tends to decrease, as shown in Fig. 3(b), and to disappear in Fig. 3(c). Here lasing with inversion changes into lasing without inversion as the linewidth of the driving field increases.

IV. SUMMARY

In this paper, we investigate the effects of phase fluctuation in the driving field on the steady-state behavior of a simple three-level atomic system. We find that:

1. $\Delta_2 = 0$. For a monochromatic driving field, the atomic system can establish lasing with inversion in a wide frequency range of the probe field. The same atomic system can also generate a large refractive index without absorption even if the population inversion is positive. For a finite linewidth of the driving field, the refractive index tends to decrease, and a change from inversion laser to noninversion laser can occur as the driving-field linewidth increases.

2. $\Delta_1 = 0$. For a monochromatic driving field, the atomic system can establish lasing with or without inversion in a wide frequency range of driving field. For a finite linewidth of the driving field, the region of inversion laser tends to decrease and finally disappear as the linewidth increases, indicating a change from inversion laser to noninversion laser.

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