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ELECTRON DIFFUSION IN A NON-UNIFORM ELECTRIC FIELD AND A UNIFORM MAGNETIC FIELD*

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A Monte Carlo simulation technique has been used to model the electron transport behavior, especially the electron diffusion motion, in the cathode fall region of a glow discharge under the influence of a non-uniform electric field and a transverse magnetic field perpendicular to the cathode sheath electric field. Three types of collisions (elastic, excitation and ionization) are taken into account in our model. The electron free flying time is determined by the electron-neutral atom collision frequency. We focus attention on the electron diffusion distance and velocity. The electron-neutral atom collision processes and the electron drift velocity are also studied. The results indicate that with the increase of the magnetic field the electron diffusion distance increases and the electron diffusion velocity decreases. The results also show that the collision processes are enhanced by the magnetic field, this is in agreement with the experimental result. However, the axial magnetic field does not affect the electron transport behavior.

PACC: 5265; 5280; 5240K; 5225

I. INTRODUCTION

Much effort has been paid on the research about the glow discharge of plasma with the more and more extensive application of glow discharge of plasma at low temperature. With the developing of laser technique especially the laser treatment and laser deposition films, the laser plasma transport process becomes an important subject in plasma science. In laser processing, dense plasma is produced near the center of a laser beam, as a result, the laser processes will be weakened as the laser energy is absorbed or reflected by the dense plasma. To solve the problem, one method is to reduce the laser plasma density in the center of a laser beam. We know that the charged particles may diffuse in a cross electric and magnetic field $\boldsymbol{E} \times \boldsymbol{B}$, this diffusion motion is useful for reducing the plasma density in the center of a laser beam.

Some experimental results have shown that in glow discharge plasma the electron transport behaviors can be controlled by a magnetic field. Fukumasa *et al.*[1] showed that magnetic field could be used to control the electron transport behaviors in discharge plasma.

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Heylen and Dargen[2] reported that the cathode sheath parameters were changed by a magnetic field. In addition, experimental results[2-7] indicated that other parameters of electron were also affected by a magnetic field. These results showed that magnetic field could affect the collision processes between particles, and as a result, the light output from the glow discharge and the plasma transport processes were changed. Bonin and Mason *et al.*[5] found that the fluorescence and ionization processes in glow discharge plasma were enhanced by a magnetic field. The results obtained by Heylen and Dargen[2] also showed that ionization number depended on the magnetic field. The radiation (or excitation) in plasma enhanced by a magnetic field was reported by Zhou and Wang.[4] These results are useful for the discharge plasma deposition processing and the laser plasma application. The enhancement of the fluorescence light can be used in lamp industry.

Theoretically, Monte Carlo simulation (MCS) is a tool to study the effect of a magnetic field on the electron transport behavior in discharge plasma.[8,9] Razdan *et al.*[8] investigated the discharge instability in a uniform magnetic field and a non-uniform electric field using MCS. Labahn *et al.*[9] applied the similar model to study the electron energy distribution in helium discharge. In these models, the null-collision technique was used to treat the collision process between electron and neutral atom. The major problem in using this method was the large computation cost.

In the present paper, the effects of a magnetic field on the electron transport behavior in discharge cathode region are studied using the MCS. A simple technique is used to treat the electron-neutral atom collision process, in which the electron free flying time is determined by the electron-neutral atom collision frequency. This technique can save computation time. We focus attention on the electron diffusion processes in a magnetic field. The electron-neutral atom collision processes are also investigated. The results are in agreement with the experimental results.

II. MONTE CARLO SIMULATION

The Monte Carlo simulation has been described in detail in Ref. [10], and here a brief description of the method is given. Every electron's track should be followed in studying the electron transport behaviors using the MCS, then the electron transport parameters will be obtained by summing over a large number of electrons. In fact, the electron density is about $10^9\,\mathrm{cm}^{-3}$ in discharge plasma, it is impossible to follow all the electrons' tracks. Generally, only a small part of electrons (2000–10000) is followed as test electrons. About 2500 electrons' tracks (this implies the accuracy about 2%) are followed in our model. According to the experimental result[11], electrons are emitted from the cathode with energy distributing between 4 to 12 eV, and we assume the initial electron energy is 5 eV.

In our model, the magnetic field is perpendicular to the cathode sheath electric field. Three types of collisions (elastic, excitation and ionization collision) are taken into account. The electron free flying time is determined by the electron-neutral atom collision frequency. The electrons reaching the edge of the cathode region are assumed to be absorbed by the electrode. Some electrons which return to the cathode surface due to the scattering will be reflected by the cathode. The reflection coefficient is 0 or 0.3 in this model. Since the gap between electrodes is smaller than the electrode radial, we use the one-dimensional coordinate and three-dimensional velocity model. In a magnetic field, the effect of the magnetic field will cause variation of the cathode parameters[2], but we still use the experimental data given by Den Hartog[12] in which the magnetic field was not applied because of the lack of experimental data in a magnetic field. The cathode sheath potential is 173 V, the sheath thickness is 0.382 cm, and the neutral atom density is $11.2 \times 10^{16}\,\mathrm{cm}^{-3}$. The magnetic field ranges from 0 to 0.08 T.

A. Electron motion

The magnetic field vector and the coordinates are depicted in Fig. 1. The electric field as a function of the distance from the cathode is described by Den Hartog[12]

$$E(z) = \frac{2V_c}{d_c}\left(1 - \frac{z}{d_c}\right), \tag{1}$$

where z is the electron position along z-axis, V_c and d_c are the cathode sheath potential and thickness, respectively. The uniform magnetic field $\boldsymbol{B}$ is in the $+y$ direction. In an electromagnetic field, the total force acting on an electron is given by

$$\boldsymbol{F} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}), \tag{2}$$

where q is the electron charge, $\boldsymbol{B}$ is the magnetic field intensity, and $\boldsymbol{v}$ is the electron velocity. The equations of an electron motion are

$$\mathrm{d}v_z/\mathrm{d}t = [qE(z) - qv_yB]/m_e\,, \quad \mathrm{d}v_y/\mathrm{d}t = qv_zB/m_e\,, \quad \mathrm{d}v_x/\mathrm{d}t = 0\,, \tag{3}$$

where v_z, v_y and v_x are the electron velocity components along z-, y- and x-axis, respectively, and m_e is the electron mass.

B. Collision processes

The variations of the velocity vector before and after collision are shown in Fig. 2, in which v_0 and v_1 are the electron velocity before and after collision, θ_0 and θ_1 are the angles between the velocity and the z-axis before and after collision, and γ and ϕ are the scattering and azimuth angles in the collision process.

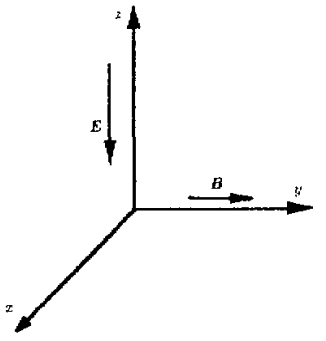

Fig. 1. The magnetic field vector and coordinates.

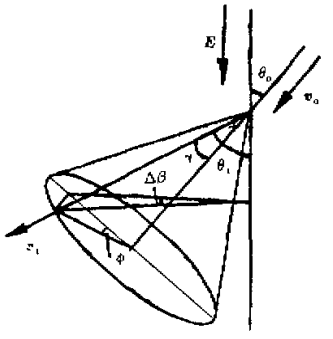

Fig. 2. The variations of the velocity vector before and after collision. The symbols v_0 and v_1 stand for the electron velocity vector before and after collision, θ_0 and θ_1 are the angles between the velocity and the z-axis before and after collision, and γ and ϕ are the scattering and azimuth angles in the collision process, respectively; β is the angle of the projection of the velocity with respect to the x-axis, $\Delta\beta$ is the variation of the angle in the collision process.

The electron-neutral atom collision frequency is

$$\nu_t(\varepsilon) = N_g \sigma_t(\varepsilon)(2\varepsilon/m_e)^{1/2} , \tag{4}$$

where N_g is the neutral atom density, and $\sigma_t(\varepsilon)$ is the total collision cross section, ε is the electron energy. The probability of collision in a time step Δt is given by

$$p_c = 1 - e^{-\nu_t(\varepsilon)\Delta t} . \tag{5}$$

When Δt is very small, the variation of electron energy ε is also small. By comparing p_c with R_1 (a uniform random number), for $p_c > R_1$, the electron collides with a neutral atom

in the step Δt. Then electron flying time can be calculated by

$$\tau = -\frac{1}{\nu_t(\varepsilon)} \ln(1 - R_1) \,. \tag{6}$$

For $p_c < R_1$, the probability p_c is recalculated in the next step Δt and R_1 is replaced by $R_1 - p_c$.

The new direction of electron motion after collision is obtained with

$$\theta_1 = \arccos(\cos\theta_0 \cos\gamma + \sin\theta_0 \sin\gamma \cos\phi) \,. \tag{7}$$

The angle β of the projection of the electron velocity with respect to the x-axis is given by

$$\beta_1 = \beta_0 \pm \arcsin[\sin\gamma \sin(\pi - \phi)/\sin\theta_1] \,, \tag{8}$$

where β_0 and β_1 are the angle β before and after collision. The signs "$\pm$" in Eq. (8) are determined by the value of ϕ.

III. RESULTS AND DISCUSSION

The electron diffusion and collision processes are studied by the MCS described above. The test electrons (2500) are emitted from the center of the cathode. The intensity of the magnetic field ranges from 0 to 0.08 T.

A. Electron mean transport time

The electron mean transport time Δt in the cathode sheath region is an important parameter in the study of the electron transport behaviors. In this paper, $\overline{\Delta t}$ represents the time that the electron takes in the region $[z_i, z_{i+1}]$. Figure 3 shows the results. It is clear from Fig. 3 that for the larger values of magnetic field, the electron mean transport time $\overline{\Delta t}$ becomes longer. We also find that the effect of magnetic field is stronger in the latter parts of the cathode sheath region than that near cathode. It is worthy to notice that Δt is a linear function of the position z.

There are two forces acting on the electron in an electromagnetic field, Lorentz and electric field forces; and the electron's track is a spiral. If Δz is small, the electric field can be considered to be constant, so the electron gyroradius can be written as

$$r = m_e v_\perp / eB \,, \tag{9}$$

where e is the electron charge and $v_\perp$ is the electron velocity perpendicular to the magnetic field. It is almost independent of the magnetic field.

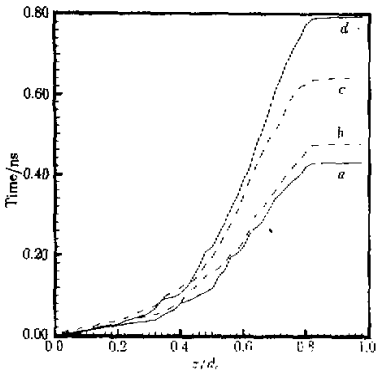

Fig. 3. The electron mean transport time in the cathode sheath region. The symbols $a - d$ correspond to different values of B : $0, 0.02, 0.05$ and $0.08\,\mathrm{T}$, respectively.

From Eq. (9), we can find that the electron gyroradius r decreases with increasing magnetic field. This causes an increase of λ/r (λ is the electron mean free path). The electron's track, in this case, changes greatly. It is equivalent to the increase of the electron mean free path, in other words, the electrons are trapped in the cathode sheath region, therefore $\overline{\Delta t}$ increases. On the other hand, with larger values of electric field, $v_\perp$ and the electron drift velocity become larger, and the electron gyroradius r is correspondingly larger; as a result, the variation of the electron trajectory is small near the cathode, and $\overline{\Delta t}$ becomes short, so $\overline{\Delta t}$ increases linearly with the distance from the cathode. The effect of the magnetic field is weaker near the cathode due to the strong electric field.

We note that $\overline{\Delta t}$ is almost constant near the edge of the cathode sheath region ($z/d_\mathrm{c} > 0.85$). This is because the electrons are in thermal motion in this region where the electric field is very weak.

B. Mean number of electron-neutral atom collisions

Our results indicate that the variations of the number of collisions are obvious in different magnetic fields and at different positions in the cathode sheath region. At the beginning of this section, we assume that all test electrons (N_e) can reach the negative glow region, and the cathode reflection coefficient R is zero. Table 1 lists the mean number of collisions and the total transport time per test electron in the cathode sheath region. It can be found that with increasing magnetic field the number of elastic and metastable excitation collisions increases. The number of excitation and ionization collisions also increases with the magnetic field when $B < 0.05\,\mathrm{T}$, but they would decrease when $B > 0.05\,\mathrm{T}$. The total transport time has a similar tendency. These results are not in agreement with the experimental results, which show that the number of collisions increases with the magnetic field. From Table 1 we can find that N_r (the number of electrons which return to the cathode surface due to scattering) rapidly increases with the magnetic field. This result implies that with increasing magnetic field the number of electrons which actually reaches the negative glow region (N_n) decreases, so the calculation method used above is invalid. We recalculate these parameters using N_n instead of N_e (the total number of test electrons), the results are shown in Table 2 where the reflection coefficient is 0.3 and the number of test electrons is 2000. It is obvious

that the number of collisions increases with increasing magnetic field. From Table 2, we also note that N_n increases with the magnetic field when $B < 0.05$ T and decreases when $B > 0.05$ T. The number of back-scattered electrons N_f also increases with the magnetic field.

Table 1. The mean number of collisions and the electron transport time calculated using the total number of test electrons ($R = 0$), N_e is the total test electron number, N_r is the number of electrons which return to the cathode due to collision (or scattering).

Magnetic field B/T	Mean number of collision				N_r	Mean trans. time/ns
	Elastic	Metastable	Excitation	Ionization		
0	129.72	0.33	2.82	3.24	254	212
0.0200	170.10	0.39	3.32	3.51	286	222
0.0282	213.70	0.48	3.87	3.76	318	237
0.0500	340.24	0.63	4.12	3.98	438	250
0.0707	449.35	0.71	3.84	3.26	724	276
0.0800	514.60	0.72	3.79	3.28	780	271
0.1128	545.31	0.67	2.57	1.90	1209	219

Table 2. The mean number of collisions and the electron transport time calculated using N_n instead of N_e ($R = 0.3$, N_e is 2000), N_f is the number of back-scattered electrons, N_n is the number of electrons which actually reach the negative glow region.

Magnetic field B/T	Mean number of collision				N_n	N_f	Mean trans. time/ns
	Elastic	Metastable	Excitation	Ionization			
0	158.52	0.40	3.45	3.95	4080	98	215
0.0200	166.12	0.45	3.85	4.00	4335	116	233
0.0282	233.10	0.55	4.20	4.10	4584	144	255
0.0500	359.75	0.65	4.35	4.20	4729	180	277
0.0707	586.90	0.90	5.05	4.25	3828	290	299
0.0800	683.40	0.95	5.15	4.35	3765	308	322
0.1128	1260.00	1.55	5.95	4.40	2164	478	352

C. Electron mean diffusion velocity along $E \times B$

The electron mean diffusion velocity along $\boldsymbol{E} \times \boldsymbol{B}$ is given by

$$\overline{v}_x(z) = \int f(z,\varepsilon) v_x(z) \varepsilon^{1/2} \mathrm{d}\varepsilon / \int f(z,\varepsilon) \varepsilon^{1/2} \mathrm{d}\varepsilon \ , \tag{10}$$

where v_x is the electron velocity along x-axis. The $\overline{v}_x(z)$ is in $+x$ direction because the magnetic field is along $+y$-axis. The electron distribution function is $f(z,\varepsilon)$. In an electromagnetic field, the electron velocity perpendicular to the magnetic field is calculated by

$$\mathrm{d}\overline{v}_\perp / \mathrm{d}t = q(\boldsymbol{E} + \boldsymbol{v}_\perp \times \boldsymbol{B})/m_e \ . \tag{11}$$

The parameter $v_\perp$ consists of the circle velocity v_c and the diffusion velocity v_d:

$$v_\perp = v_c + v_d \ . \tag{12}$$

As Δz is small, we can assume the electric field is constant in Δz, then v_c is given by

$$\mathrm{d}\boldsymbol{v}_c/\mathrm{d}t = q(\boldsymbol{v}_c \times \boldsymbol{B})/m_e \ . \tag{13}$$

The electron diffusion velocity along $\boldsymbol{E} \times \boldsymbol{B}$ is obtained by

$$\boldsymbol{v}_d = \boldsymbol{E} \times \boldsymbol{B}/B^2 \ . \tag{14}$$

Averaging over one circle, the electron cycle motion velocity does not affect the electron mean diffusion velocity, so we focus on the electron diffusion velocity. We rewrite the Eq. (14) as

$$v_{dx} = E_z/B_y \ . \tag{15}$$

Figure 4 shows the variations of the electron mean diffusion velocity (EDV) in different magnetic fields. The EDV is uniformly distributed around zero at 0 T, but the EDVs show a noticeable shift towards positive value at $B > 0$ T. This is due to the result of the induced $\boldsymbol{E} \times \boldsymbol{B}$ electron drift. The EDVs decrease with increasing magnetic field due to $v_{dx} \propto B^{-1}$[Eq. (15)]. The law $v_{dx} \propto B^{-1}$ appears to be invalid near the cathode due to the strong electric field. We also find that the EDVs decrease with the distance from the cathode due to $v_{dx} \propto E_z$. Our model indicates that the electron mean diffusion velocities are not affected by the axial magnetic field.

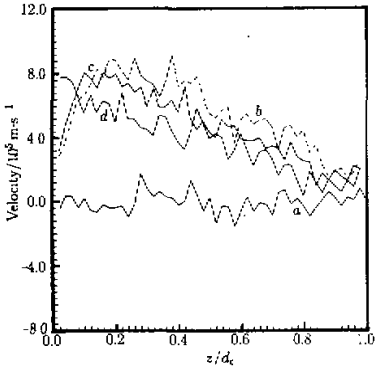

Fig. 4. The variations of the electron mean diffusion velocity along $+x$-axis in different magnetic fields. Conditions are the same as in Fig. 3.

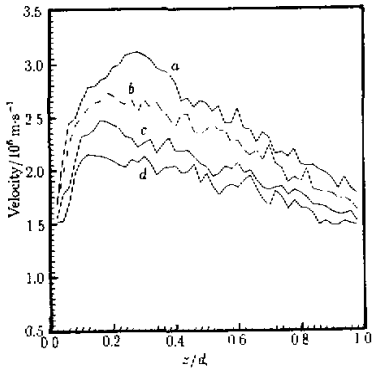

Fig. 5. The variations of the electron mean drift velocity in different magnetic fields. Conditions are the same as in Fig. 3.

D. Electron mean drift velocity

The electron mean drift velocity is obtained by

$$\overline{v}_z(z) = \int f(z,\varepsilon)v_z(z)\varepsilon^{1/2}\mathrm{d}\varepsilon / \int f(z,\varepsilon)\varepsilon^{1/2}\mathrm{d}\varepsilon \ . \tag{16}$$

The results are shown in Fig. 5. It can be seen from Fig. 5 that the electron mean drift velocities decrease with the increase of the magnetic field, this is the result of the loss of

the electron energy especially the forward kinetic energy and the increase of the number of collisions in the magnetic field. Figure 5 illustrates that the electron mean drift velocity is small near the cathode, with the increase of distance it shows a large rise and reaches maximum value near $z/d_c = 0.25$. This is because the electrons emitted from the cathode with smaller energy are accelerated by the electric field, their forward kinetic energies greatly increase before the first excitation collision occurs. Near $z/d_c = 0.25$, the first excitation collision occurs, therefore the electron forward kinetic energies decrease. This result also implies that near the cathode the electron energies obtained from the electric field are larger than that of loss in the collisions, so the electrons are in non-equilibrium state in this region. It is noteworthy that with increasing magnetic field, the maximum values of the electron mean drift velocity shift to the cathode. This is because the number of collisions increases. In the latter part of the cathode sheath region ($z/d_c > 0.4$) the electron mean drift velocities decrease linearly with the distance from the cathode. This result indicates that the electrons are approximately in an equilibrium state in this region.

E. Diffusion distance of an electron along $E \times B$

The electron diffusion in an electromagnetic field is useful for the plasma applications in the film technique. In the laser treatment, it may cause a decrease of the center plasma density in the center of the laser and results in a decrease of the reflection and absorption of the laser energy, therefore the effectiveness of the laser treatment can be increased.

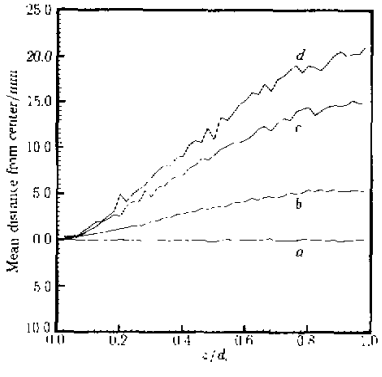

Fig. 6. The variations of the electron diffusion distance along $\vec{E} \times \vec{B}$ in different magnetic fields. Conditions are the same as in Fig. 4.

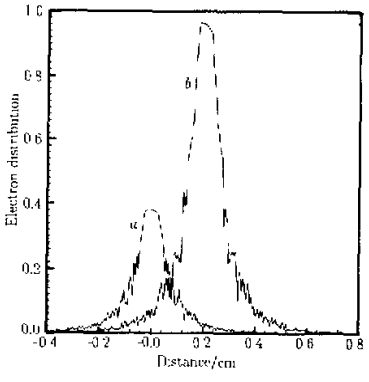

Fig. 7. The distribution of the electron as a function of the position x at the edge of the cathode sheath region (a: $B = 0$ T; b: $B = 0.08$ T).

The electron diffusion distance along $\boldsymbol{E} \times \boldsymbol{B}$ is calculated by

$$\overline{x}_x(z) = \int f(z,\varepsilon) x_x(z) \varepsilon^{1/2} \mathrm{d}\varepsilon / \int f(z,\varepsilon) \varepsilon^{1/2} \mathrm{d}\varepsilon \,, \tag{17}$$

where x is the electron position along x-axis.

Figure 6 shows the electron diffusion distance in different magnetic fields. From this

figure we note that $\overline{x}(z)$ is uniformly distributed around zero at 0 T. This is contrast to that at $B \neq 0$ T, which shows a noticeable shift towards a positive value and increases with the magnetic field. Our calculation also shows that $\overline{x}(z)$ is not affected by the axial magnetic field.

Figure 7 shows the distribution of the number of electrons in x direction at the boundary of the cathode sheath region. One can conclude from Fig. 7 that the electrons shift to a positive value along x-axis due to the induction of the $\boldsymbol{E} \times \boldsymbol{B}$.

IV. CONCLUSION

The MCS technique is used to study the electron diffusion motion in the helium glow discharge cathode sheath under the influence of a magnetic field perpendicular to the cathode sheath electric field. The computational technique used in the present simulation is different from the earlier technique where the electron-neutral atom collision processes were treated by the null-collision technique. Instead, the collision processe are now treated by the electron-neutral atom collision frequency. Such a method can save the computation cost and the accuracy is not affected.

Our calculation results indicate that the electron diffusion distances increase and the electron diffusion velocities decrease with increasing magnetic field. The axial magnetic field does not affect the electron transport behaviors. The results also show that the collisions between the electron and the neutral atom are enhanced by the magnetic field, the agreement with the experimental results is satisfactory.

The electron diffusion in an electromagnetic field is useful for the plasma applications in film technique. In the laser treatment, it may cause a decrease of the center plasma density in the center of the laser and results in a decrease of the reflection and absorption of the laser energy, therefore the effectiveness of the laser treatment can be increased.

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