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THEORETICAL AND EXPERIMENTAL INVESTIGATION ON ANOMALOUS HALL EFFECT OF HETEROGENOUS SEMICONDUCTOR SAMPLES

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Based on the theoretical results of inversion layers model and interpenetrating network model, we studied the anomalous Hall effect of heterogenous semiconductor samples, and obtained the following results: (1) the wider the sample energy gap, the higher the temperature at which anomalous Hall effect begins to appear; (2) the lower the sample resistivity, the higher the temperature at which anomalous Hall effect begins to appear, which is further verified in our experiment; (3) mobility is the main factor which determines what role p-n junction plays in sample Hall voltage; (4) for n-Ge sample with inversion layers structure, with the increase of p-type region, the Hall effect changes from normal n-type Hall effect to anomalous Hall effect of the single dip type, then to anomalous Hall effect of the second reversal type.

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I. INTRODUCTION

A large number experimental results show that high temperature heat treatment[1,2], high energy radiation[3,4], or other chemical or physical means[5-11] can invert the type of semiconductor sample partly or wholly. It is these inversion regions that result in the appearance of anomalous Hall effect (AHE). It merits attention for the temperature at which AHE begins to appear and the types of AHE are very different for different samples. Based on temperature dependence of Hall coefficient R, these phenomena of AHE can be divided into three types, namely "single peak", "single dip" and "second reversal". Based on the temperature at which AHE begins to appear, these phenomena of AHE can be divided into three situations, namely room temperature (300 K), low temperature (below 200 K) and very low temperature (below 100 K). To reveal the relations among these phenomena is not only an important theoretical issue but also is conducive to utilizing AHE or avoiding its appearance in experiment.

Now most of AHE theories for heterogenous samples are based on Petritz's two-layer model[12], such as Liu[5], Chen[6], Lou[9], Zemel[10] and so on. However the two-layer model has two disadvantages: (1) though it is effective to use it for qualitative interpretation, it is too much simplistic and particular for quantitative calculation, so after changing Petritz's two-layer model to three-layer model, Besikci *et al.*[11] got the calculation results which are in agreement with their experiment; (2) the two-layer model does not concern p-n junction which play a key role in sample Hall voltage for Ge and Si samples. The other AHE theories

for heterogenous samples are only directed against a sample or a experiment. So these theories can not interpret the experimental phenomena mentioned above.

In this paper, on the basis of inversion layers model and interpenetration network model, combining sample Hall coefficient with sample parameters, we studied the dependences of the type of AHE and the temperature at which AHE begins to appear on sample structure, sample energy gap and sample resistivity, and give the experimental verification for part of theoretical results.

II. THEORY

For the sample in which n-type regions coexist with p-type regions, the p-type regions may exist in two forms: one is that each p-type region is isolated by n-type regions, in this form p-n junction plays a key role in sample Hall voltage, inversion layers model is a particular structure of this form; the other one is that these p-type regions form an interpenetrating network structure.

For the sample with inversion layers structure, its Hall coefficient can be expressed as[13]

$$R = b\{2\varepsilon_{11} + [2\varepsilon_{21}/G_{21} + 2\varepsilon_{31}/G_{31} + \varepsilon_p/(G_p + 2G_M)]G_2\}/IB \,, \tag{1}$$

where b is the sample thickness along the direction of magnetic induction B, I is sample current intensity, $\varepsilon_{11}, \varepsilon_{21}, \varepsilon_{31}, \varepsilon_p$ and G_{21}, G_{31}, G_p refer to the Hall electromotive force and its inner resistivity for n-type regions and p-type region, respectively, G_M is the linear resistivity of p-n junction (the resistivity at very low voltage), which is expressed as

$$G_M = kT/(eSJ_s) \,, \tag{2}$$

where S is the area of p-n junction, and

$$J_s = eD_n n_p/L_n + eD_p p_n/L_p = (ekT/\tau)^{1/2}(\mu_n^{1/2} n_p + \mu_p^{1/2} p_n) \,, \tag{3}$$

where τ is the carrier lifetime, T is absolute temperature, k is Boltzmann constant, p_n and n_p are the hole concentration in n-type regions and the electron concentration in p-type regions, which can be expressed as

$$n_p = n_i^2/N_A \,; p_n = n_i^2/N_D \,, \tag{4}$$

where n_i is intrinsic carrier concentration, N_D and N_A are the donor concentration in n-type regions and the acceptor concentration in p-type region, respectively.

For the sample with interpenetration network structure, as a particular form, its Hall coefficient R can be expressed as[14]

$$R = (R_n\sigma_n + \alpha R_p\sigma_p)/(\sigma_n + \alpha\sigma_p) \,, \tag{5}$$

where R_n, R_p and σ_n, σ_p refer to the Hall coefficients and conductivity for n-type regions and p-type regions, respectively, α is the ration of the volume of p-type region to the total

volume of all n-type regions, R_p can be expressed as

$$R_p = (p_p\mu_p^2 - n_p\mu_n^2)/[q(p_p\mu_p + n_p\mu_n)^2] , \tag{6}$$

where μ_p and μ_n are the mobilities of hole and electron, respectively, n_p is the minority carriers concentration in p-type region, which is shown in Eq. (4).

A. The effect of the width of sample energy gap and sample resistivity on AHE

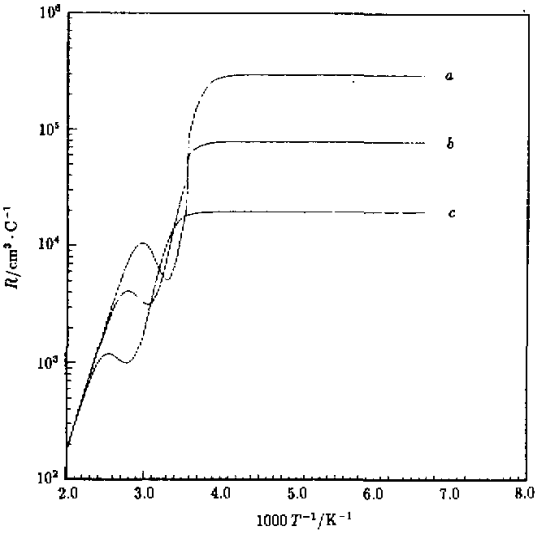

Fig. 1. Calculation results for the donor concentration and accepter concentration dependence of the temperature at which AHE begins to appear for n-Ge sample with AHE of single dip. Curve *a*: $N_A = 2.5\times10^{14}\,cm^{-3}$, $N_D = 4.5\times10^{13}\,cm^{-3}$; curve *b*: $N_A = 7.5\times10^{14}\,cm^{-3}$, $N_D = 1.7\times10^{14}\,cm^{-3}$; curve *c*: $N_A = 3.0\times10^{15}\,cm^{-3}$, $N_D = 6.8\times10^{14}\,cm^{-3}$.

For the sample with inversion layers structure, from Eq. (1) we can find that only if G_M is low enough to match the inner resistivity of n-type region or p-type region, then ε_p begins to affect sample Hall coefficient R, and AHE begins to appear. For the samples made of same material (same energy gap), the intrinsic carrier concentration n_i is constant if temperature remains unchanged. From Eqs. (3), (4), we can obtain the following results: the lower donor concentration N_D is, the higher minority carriers concentration p_n will be, so the lower linear resistivity of p-n junction G_M and the lower temperature at which AHE begins to appear, which are shown in Fig. 1. For the different material samples, the narrower sample energy gap is, the higher intrinsic carriers concentration n_i, so the lower temperature at which AHE begins to appear.

For the sample with interpenetration network structure, the appearance of AHE is due

to the sign of Hall coefficient R_p of p-type region changes from positive to negative with temperature increasing. Form Eqs. (6), (4), we can obtain that the lower accepter concentration N_A is, the higher minority carriers concentration of p-type region n_p will be, so the lower temperature at which R_p changes its sign and the lower temperature at which AHE begins to appear; the narrower energy gap is, the higher intrinsic carrier concentration n_i, the lower temperature at which R_p changes its sign and the lower temperature at which AHE begins to appear.

In summary, we can obtain: the higher the sample resistivity (the lower donor concentration or accepter concentration) is, the lower the temperature at which AHE begins to appear; the wider sample energy gap is, the lower temperature at which AHE begins to appear.

For n-Ge sample, the resistivity of Konorova's sample[2] is $40\Omega \cdot$ cm before high temperature heat treatment, the temperature at which AHE begins to appear is 200 K; the resistivity of Xing's sample[1] is $3.2\Omega \cdot$ cm before high temperature heat treatment, so the temperature at which AHE begins to appear is 300 K; the width of energy gap at temperature of absolute zero for $Hg_{1-x}Cd_xTe$ samples can be obtained from empirical Eq. (7).

$$E_{g0} = 1.7(x - 0.16)(\text{eV}) \,. \tag{7}$$

For Liu's sample[5], $x = 0.20$; for Chen's sample[6], $x = 0.22$. For the other materials, the width of energy gap at temperature of absolute zero and the temperature at which AHE begins to appear are shown in Table 1, from which we can see that the theoretical results are in agreement with experiment.

Based on our theoretical results, for Si sample the temperature at which AHE begins to appear should be higher than that (300 K) of Ge sample.

Table 1. Energy gaps for different materials and the highest temperature at which their AHE begin to appear.

Sample	HgCdTe[5]	HgCdTe[6]	InSb[11]	InAs[7]	Ge[1]
E_{g0}/eV	0.068	0.102	0.18	0.36	0.66
T/K	25	80	120	180	300

B. The effect of mobility on AHE

The effect of mobility on AHE is in two aspects.

1) For the samples such as InAs, HgCdTe and so on, their electron mobility is very high, but hole mobility is much lower than that of electron. Thus though the inversion layers formed by sample surface adsorption is very thin, its Hall electromotive force ε_n is strong enough to have AHE appear for p-type sample. It is this reason that AHE appears frequently for p-type HgCdTe and InAs samples.

2) The higher mobility is, the lower linear resistivity of p-n junction G_M will be. Suppose the linear resistivity of p-n junction for unit area is g_M. It is shown in Fig. 2 the temperature

dependence of G_M, the resistivity of p-type region ρ_p and the resistivity of n-type region ρ_n for Ge sample with inversion layers structure. We can see that g_M is much higher than ρ_p and ρ_n below intrinsic temperature. So ε_p contributes nothing to sample Hall voltage, and n-type Hall effect appears in this temperature range. For the samples such as InAs, HgCdTe and so on, their electron mobility is very high and carrier lifetime is very short, so g_M is lower than ρ_p and ρ_n below intrinsic temperature, and p-n junction does not play a key role. If we neglect the effect of p-n junction on sample Hall voltage and let $i = 1, j = 1, k = 2$; $i = 1, j = 2, k = 1$; $i = 2, j = 1, k = 1$, the results of Eq. (9) in Ref. [14] are just the three results of Petritz's two-layer model.[12] If the inversion layers is thin enough to match the width of p-n junction barrier, the contribution of Hall electromotive force in p-n junction to sample Hall voltage can not be neglected, which was particularly studied by Voroncof for this case.[8]

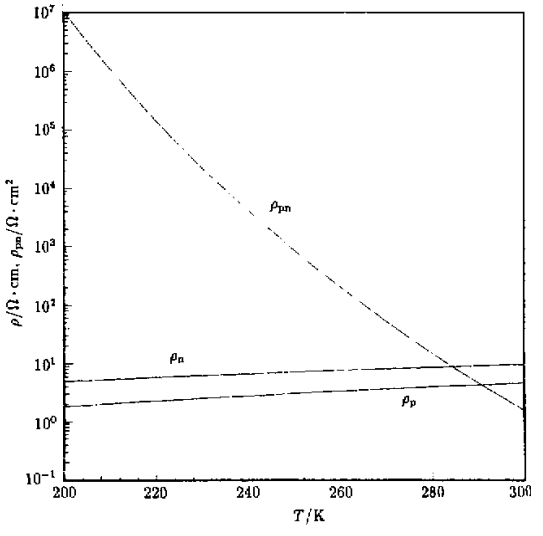

Fig. 2. Temperature dependence of g_M, the resistivity of p-type region ρ_p and the resistivity of n-type region ρ_n for Ge sample with inversion layers structure in which $N_A = 7.5 \times 10^{14}\,\mathrm{cm}^{-3}$, $N_D = 1.7 \times 10^{14}\,\mathrm{cm}^{-3}$.

C. The effect of sample structure on AHE for n-Ge sample

For n-Ge sample, the phenomena of AHE of second reversal type and single peak type were interpreted by inversion layers model[13] and interpenetration network model.[14] Now we study the phenomenon of AHE of single dip type for n-Ge sample.

For the n-Ge sample with inversion layers structure, p-n junction play a key role. In low temperature range, the linear resistivity of p-n junction G_M is so high that the contribution of the Hall electromotive force for p-type region to sample Hall voltage V_H is zero. If

G_M becomes low enough to match the resistivity of p-type region or n-type region with temperature increasing, the p-type region begin to affect sample Hall voltage. As the Hall electromotive force of p-type region ε_p is in the opposite direction of the Hall electromotive force of n-type region ε_n, two results will be shown with temperature further increasing as follows. One is that the p-type region is so large that the contribution of ε_p to sample Hall voltage V_H is more than that of n-type regions. In this case, sample Hall coefficient R changes its sign from negative to positive, this is AHE of the second reversal type. The other one is that p-type region is not large enough to change the sign of sample Hall coefficient R. In this case the absolute value of R begins to decrease, but if temperature increase up to the intrinsic temperature of p-type region, ε_p changes its direction, and the absolute value of R begins to increase. This is AHE of the single dip type. From Fig. 3 we can see that with the increase of p-type region, the Hall effect changes from normal n-type Hall effect to AHE of single dip type, then to AHE or second reversal type, and for AHE of single dip type, the lager p-type region is, the lower dip will be.

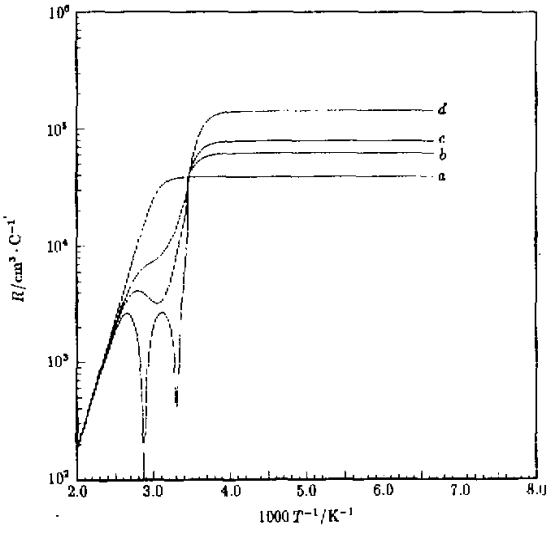

Fig. 3. For n-Ge sample with inversion layers structure, with the increase of p-type region, the Hall effect changes from normal n-type Hall effect to AHE of the single dip type, then to AHE of the second reversal type, the proportion of p-type region in the sample (%): curve *a*: 0.5; curve *b*: 39; curve *c*: 53; curve *d*: 74.

III. EXPERIMENTAL

In order to verify the effect of sample resistivity and high temperature heat treatment on sample Hall effect, we adopted the n-Ge samples with the resistivity between the resistivity

of Konorva's sample and the resistivity of Xing's sample in our experiment. By the method of thermal defect compensation, we got the heterogenous Ge samples.

A. Sample preparation

Adopting two n-Ge samples with the same resistivity 18Ω·cm and the same sizes (produced by Beijing Semiconductor Plant), we made 670°C heat treatment for one sample and call it sample A, made 730°C heat treatment for the other sample and call it sample B. The heat treatment curves are shown in Fig. 4. Nitrogen is used to protect samples from being oxidized for all heat treatments.

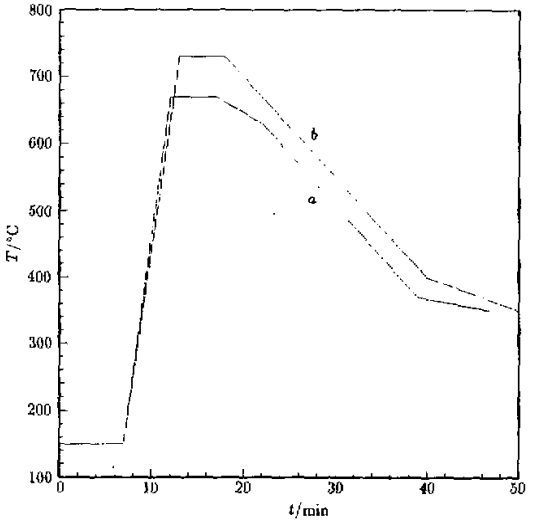

Fig. 4. Heat treatment curves for sample A and B.

B. Experimental results

Under the condition of constant current, the temperature dependence of Hall coefficient R is measured for sample A and B. The results are shown in Fig. 5. The AHE of second reversal type appears for sample A. The temperature at which AHE begin to appear is 278 K. This temperature is exactly between 200 K for Konorova's sample and 300 K for Xing's sample, which further verify the theoretical result that the lower sample resistivity is, the higher temperature at which AHE begins to appear. Normal p-type Hall effect appears for sample B. Combining our results with Konorova's results, it seems that the higher temperature of heat treatment is, the larger p-type regions formed by thermal defect compensation is.

Figure 6 shows temperature dependence of resistivity for different samples. We can see that much difference appears for sample resistivity after heat treatment, in which a "flight" appears along $\lg R$-T curve for sample A in temperature range of appearance of AHE. This phenomenon was studied in Ref. [13].

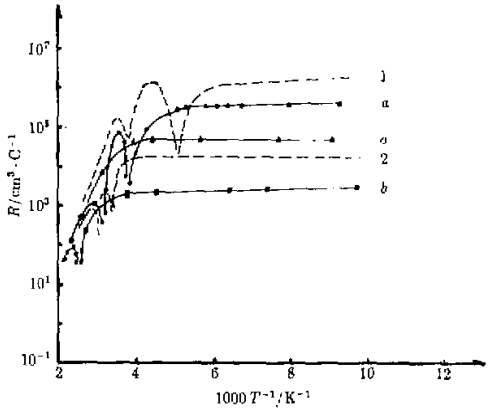

Fig. 5. The experimental curves for temperature dependence of Hall coefficient R. Curve o: the sample before heat treatment; curve a: sample A; curve b: sample B; curve 1: Konorova's sample; curve 2: xing's sample.

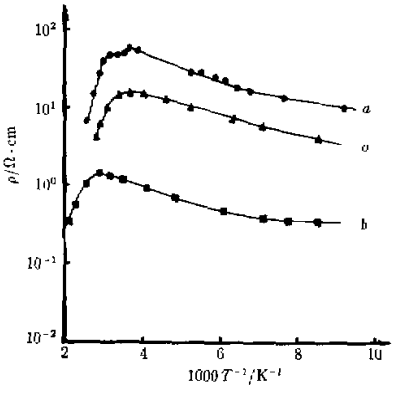

Fig. 6. The experimental curves for temperature dependence of resistivity. Curve o: the sample before heat treatment; curve a: sample A; curve b: sample B.

IV. CONCLUSIONS

1. Though the models in Refs. [13, 14] were proposed for AHE of n-Ge sample, they can be used to study AHE for other materials.

2. The width of sample energy gap and sample resistivity are two main parameters to dominate the temperature at which AHE begins to appear. The wider the sample energy gap is, the higher the temperature at which AHE begins to appear; the lower the sample resistivity (the lower donor concentration or accepter concentration) is, the higher the temperature at which AHE begins to appear.

3. For Ge sample, sample structure is the main factor to dominate the type of AHE. For n-Ge sample with inversion layers structure, with the increase of p-type region, the Hall effect changes from normal n-type Hall effect to AHE of single dip type, then to AHE of second reversal type. For n-Ge sample with interpenetrating network structure, with the increase of p-type region, the Hall effect changes from normal n-type Hall effect to AHE of single peak type, then to normal p-type Hall effect, and the larger p-type region is, the higher peak is for AHE of single peak type and the lower dip is for AHE of single dip type.

4. For the samples with high electron mobility and short carrier lifetime such as HgCdTe and InAs, p-n junction does not play a key role, the type of AHE is dominated by the sizes of p-type region, the width of p-n junction barrier, temperature dependence of mobility and so on, which can be thoroughly discussed combining the results of Refs. [13, 14] with specific parameters of sample.

5. Our experiment results not only verify the theoretical result: the lower the sample resistivity is, the higher the temperature at which AHE begins to appear, but also show that high temperature heat treatment can invert the type of semiconductor sample partly or wholly once more.

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