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WARD IDENTITIES IN PHASE SPACE AND THEIR APPLICATIONS*

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Starting from the phase space path integral, we have derived the Ward identities in canonical formalism for a system with regular and singular Lagrangian. This formulation differs from the traditional discussion based on path integral in configuration space. It is pointed out that the quantum canonical equations for systems with singular Lagrangians are different from the classical ones obtained from Dirac's conjecture. The preliminary applications of Ward identities in phase space to the Yang-Mills theory are given. Some relations among the proper vertices and propagators are deduced,the PCAC, AVV vertices and generalized PCAC expressions are also obtained. We have also pointed out that some authors in their early work had ignored the treatment of the constraints.

PACC: 1110;1130;1115

I. INTRODUCTION

Ward identities (or Ward-Takahashi identities) play an important role in modern field theories. The treatment of these identities are usually discussed in configuration space.[1,2] For the cases where the phase space generating functional cannot be simplified by carrying out explicit integration over momenta, or even if the integration over momenta can be carried out, the effective Lagrangian will sometimes show singularity[3]; especially, for the constrained Hamiltonian system with complex constraints[4], it is very difficult or even impossible to carry out the integration over momenta. In these cases the Ward identities cannot be derived via configuration space generating functional as traditional discussion. Therefore, based on the phase space generating functional, the investigation of its invariance under the transformation of canonical variables is necessary.

In this paper, starting from phase space generating functional of Green function, we have derived the Ward identities in phase space for a system with regular or singular Lagrangian. This formulation differs from traditional discussions based on configuration space functional integral. The canonical quantum equations for a constrained Hamiltonian system are deduced, which is different from the classical canonical equations arising from Dirac's conjecture. The preliminary applications of Ward identities in phase space to the Yang-Mills theories are given. Some relations among the proper vertices and propagators under the transformation of material field and gauge field (fixing the ghost fields) are derived using

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Ward identities in phase space. The expressions of PCAC and AVV vertices and generalized PCAC under the chirality transformation are obtained. It is pointed out that in Ref. [1] the analysis for the constraints was ignored by the authors.

II. THE REGULAR SYSTEM

Consider a system described by the field variables $\phi^\alpha(x)(\alpha = 1, 2, \cdots, N)$, where α denotes an index for different fields or different components of a field. The Lagrangian of the field is $\mathcal{L}(\phi^\alpha_r, \phi^\alpha_{r\mu}), \phi^\alpha_{r\mu} = \partial_\mu \phi^\alpha, \partial_\mu = \partial/\partial x^\mu, x = (t, \boldsymbol{x})$. For a system with a non-singular Lagrangian (called the regular system), the canonical Hamiltonian

$$H_c = \int \mathrm{d}^3x \mathcal{H}_c = \int \mathrm{d}^3x(\pi_\alpha \dot{\phi}^\alpha - \mathcal{L}) \tag{1}$$

is a functional of independent canonical variables $\phi^\alpha(x)$ and $\pi_\alpha(x) = \partial\mathcal{L}/\partial\dot{\phi}^\alpha(x)$. Introducing exterior sources J_α and J_1^α with respect to the ϕ^α and π_α respectively, the generating functional can be written as[5]

$$Z[J, J_1] = \int \mathcal{D}\phi^\alpha \mathcal{D}\pi_\alpha \exp\{\mathrm{i}[I^p + \int \mathrm{d}^4x(J_\alpha \phi^\alpha + J_1^\alpha \pi_\alpha)]\} , \tag{2}$$

where

$$I^p = \int \mathrm{d}^4\mathcal{L}^p = \int \mathrm{d}^4x(\pi_\alpha \dot{\phi}^\alpha - \mathcal{H}_c) \tag{3}$$

is canonical action of the field. Expanding the $Z[J, J_1]$ in the neighbourhood $J_\alpha = J_1^\alpha = 0$, one has

$$Z[J, J_1] = \sum_{n=0}^{\infty} \frac{\mathrm{i}^n}{n!} \int \mathrm{d}^4x_1 \cdots \mathrm{d}^4x_n G^p(x_1, \cdots, x_n) J_\alpha(x_1) \cdots J_\alpha(x_k) J_1^\alpha(x_{k+1}) \cdots J_1^\alpha(x_n) , \tag{4}$$

where

$$G^p(x_1, \cdots x_k, \cdots, x_n) = \frac{1}{\mathrm{i}^n} \frac{\delta^n Z[J, J_1]}{\delta J_\alpha(x_1) \cdots \delta J_\alpha(x_k) \delta J_1^\alpha(x_{k+1}) \cdots \delta J_1^\alpha(x_n)}\bigg|_{J_\alpha = J_1^\alpha = 0} . \tag{5}$$

Obviously,

$$\begin{aligned} G(x_1, x_2, \cdots, x_n) &= \frac{1}{\mathrm{i}^n} \frac{\delta^n Z[J, J_1]}{\delta J_\alpha(x_1) \delta J_\alpha(x_2) \cdots \delta J_\alpha(x_n)}\bigg|_{J_\alpha = J_1^\alpha = 0} \\ &= \langle 0|\mathrm{T}[\hat{\phi}^\alpha(x_1)\hat{\phi}^\alpha(x_2) \cdots \hat{\phi}^\alpha(x_n)]|0\rangle \end{aligned} \tag{6}$$

is an ordinary Green function.

In the generating functional (2), for the case the integration over momenta belongs to the category of the Gauss type or Feynman type, the expression (2) can be transformed to a path integral in configuration space[6], and the effective Lagrangian may be different from the original one. Ward identities in field theories are usually derived by using this configuration

space path integral. For the case the "mass" depends on coordinates (field variables), the description of the configuration space path integral was given by Lee and Yang.[7] For the case the "mass" depends on coordinates and momenta , the path integral in configuration space was discussed by Du *et al.*[3] The effective Lagrangians are singularity in the latter two cases. This singularity is expected to be canceled in the process of renormalization.[3] Generally, Eq. (2) cannot be simplified by carrying out explicit integration over momenta. Especially for the constrained Hamiltonian systems, to carry out integration over momenta is very difficult or impossible. Hence, the investigation of properties of the generating functional in phase space under transformation of canonical variables is necessary. The Ward identities in phase space can be deduced by using these transformation properties. Consequently, some new relations among the Green functions can be obtained.

Consider the following infinitesimal transformation of canonical variables in phase space:

$$\begin{aligned}\phi^\alpha(x) \to \phi^{\alpha'}(x) &= \phi^\alpha(x) + \delta\phi^\alpha(x) = \phi^\alpha(x) + Q^\alpha(\phi, \pi, \varepsilon_1^\sigma, \varepsilon_2^\sigma) \ , \\ \pi_\alpha(x) \to \pi'_\alpha(x) &= \pi_\alpha(x) + \delta\pi_\alpha(x) = \pi_\alpha(x) + P_\alpha(\phi, \pi, \varepsilon_1^\sigma, \varepsilon_2^\sigma) \ ,\end{aligned} \tag{7}$$

where $\varepsilon_1^\sigma(x), \varepsilon_2^\sigma(x)(\sigma = 1, 2, \cdots, r)$ are arbitrary functions and they will vanish on the boundary of time-space domain. Let it be supposed that the Jacobian of the transformation (7) is $J[\phi, \pi, \varepsilon_1^\sigma, \varepsilon_2^\sigma]$ and the variation of canonical action under the transformation (7) is δI^p. The generating functional (2) is invariant under the transformation (7), and one finds the following Ward identities in phase space:

$$\begin{aligned}&\int \mathcal{D}\phi^\alpha \mathcal{D}\pi_\alpha \frac{\delta}{\delta\varepsilon_i^\sigma(x)} \left\{ J[\phi, \pi, \varepsilon_1^\sigma, \varepsilon_2^\sigma][1 + \mathrm{i}\delta I^p + \mathrm{i}\int \mathrm{d}^4x (J_\alpha \delta\phi^\alpha + J_1^\alpha \delta\pi_\alpha)] \right\}\Big|_{\varepsilon_1^\sigma = \varepsilon_2^\sigma} = 0 \\ &\qquad \times \exp\left\{ \mathrm{i}[I^p + \int \mathrm{d}^4x (J_\alpha \phi^\alpha + J_1^\alpha \pi_\alpha)] \right\} = 0 \qquad (i = 1, 2)\end{aligned} \tag{8}$$

or

$$\frac{\delta}{\delta\varepsilon_i^\sigma(x)} \left\{ J[\phi^\alpha, \pi_\alpha, \varepsilon_1^\sigma, \varepsilon_2^\sigma][1 + \mathrm{i}\delta I^p + \mathrm{i}\int \mathrm{d}^4x (J_\alpha \delta\phi^\alpha + J_1^\alpha \delta\pi_\alpha)] \right\}_{\substack{\phi^\alpha \to \frac{1}{\mathrm{i}}\delta/\delta J_\alpha \\ \pi_\alpha \to \frac{1}{\mathrm{i}}\delta/\delta J_1^\alpha}} Z[J, J_1] = 0 \ . \tag{9}$$

For example, let us consider an infinitesimal translation transformation

$$\begin{aligned}\phi^\alpha(x) \to \phi^{\alpha'}(x) &= \phi^\alpha(x) + \varepsilon_1^\alpha(x) \ , \\ \pi_\alpha(x) \to \pi'_\alpha(x) &= \pi_\alpha(x) + \varepsilon_{2\alpha}(x) \ ,\end{aligned} \tag{10}$$

the Jacobian of the transformation (10) equals unity. The variation of canonical action (3) is given by[4]

$$\delta I^p = \int \mathrm{d}^4x \left[\frac{\delta I^p}{\delta\phi^\alpha} \varepsilon_1^\alpha(x) + \frac{\delta I^p}{\delta\pi_\alpha} \varepsilon_{2\alpha}(x) + \frac{\mathrm{d}}{\mathrm{d}t}(\pi_\alpha(x)\varepsilon_1^\alpha(x)) \right] \ , \tag{11}$$

where

$$\frac{\delta I^{p}}{\delta\phi^{\alpha}}=-\dot{\pi}_{\alpha}-\frac{\delta H_{c}}{\delta\phi^{\alpha}}\ ,\quad \frac{\delta I^{p}}{\delta\pi_{\alpha}}=\dot{\phi}^{\alpha}-\frac{\delta H_{c}}{\delta\pi_{\alpha}}\ . \tag{12}$$

According to the boundary conditions of $\varepsilon_1^{\alpha}(x)$ and $\varepsilon_{2\alpha}(x)$ and the invariance of generating functional (2) under the transformation (10), one obtains the following Ward identities in phase space:

$$\int\mathcal{D}\phi^{\alpha}\mathcal{D}\pi_{\alpha}\left(\frac{\delta I^{p}}{\delta\phi^{\alpha}}+J_{\alpha}\right)\exp\left\{\mathrm{i}[I^{p}+\int\mathrm{d}^{4}x(J_{\alpha}\phi^{\alpha}+J_{1}^{\alpha}\pi_{\alpha})]\right\}=0\ , \tag{13a}$$

$$\int\mathcal{D}\phi^{\alpha}\mathcal{D}\pi_{\alpha}\left(\frac{\delta I^{p}}{\delta\pi_{\alpha}}+J_{1}^{\alpha}\right)\exp\left\{\mathrm{i}[I^{p}+\int\mathrm{d}^{4}x(J_{\alpha}\phi^{\alpha}+J_{1}^{\alpha}\pi_{\alpha})]\right\}=0\ . \tag{13b}$$

Let $J_{\alpha}=J_{1}^{\alpha}=0$, from Eqs. (13a) and (13b) one has

$$\langle 0|\mathrm{T}^{*}\left(\frac{\delta I^{p}}{\delta\phi^{\alpha}}\right)|0\rangle=0,\quad \langle 0|\mathrm{T}^{*}\left(\frac{\delta I^{p}}{\delta\pi_{\alpha}}\right)|0\rangle=0. \tag{14}$$

The symbol T^{*} stands for covariantized T product.[1] We functionally differentiate Eq. (13a) n times with respect to $J_{\alpha}(x)$ and let $t_1,t_2,\cdots,t_m\to\infty,t_{m+1},t_{m+2},\cdots,t_n\to-\infty$, then we obtains

$$\langle\mathrm{out},m|\frac{\delta I^{p}}{\delta\phi^{\alpha}}|n-m,\mathrm{in}\rangle=0\ . \tag{15}$$

Since m and n are arbitrary, from Eq. (15) one gets

$$\dot{\pi}_{\alpha}(x)=-\frac{\delta H_{c}}{\delta\phi^{\alpha}(x)}\ .$$

Similarly, starting from Eq. (13b) one can also obtains

$$\dot{\phi}^{\alpha}(x)=\frac{\delta H_{c}}{\delta\pi_{\alpha}(x)}\ . \tag{16}$$

The expression (16) is a quantum canonical equation of the system.

Next, let us consider the infinitesimal transformation in an extended phase space

$$\begin{aligned}x^{\mu'}&=x^{\mu}+\delta x^{\mu}=x^{\mu}+R_{\sigma}^{\mu}\varepsilon^{\sigma}(x)\ ,\\ \phi^{\alpha'}(x')&=\phi^{\alpha}(x)+\delta\phi^{\alpha}(x)=\phi^{\alpha}(x)+S_{\sigma}^{\alpha}\varepsilon^{\sigma}(x)\ ,\\ \pi'_{\alpha}(x')&=\pi_{\alpha}(x)+\delta\pi_{\alpha}(x)=\pi_{\alpha}(x)+T_{\alpha\sigma}\varepsilon^{\sigma}(x)\ ,\end{aligned} \tag{17}$$

where $\varepsilon^{\sigma}(x)(\sigma=1,2,\cdots,r)$ are infinitesimal arbitrary functions, the values of $\varepsilon^{\sigma}(x)$ and their derivatives on the boundary of time-space domain vanish, and

$$\begin{aligned}&R_{\sigma}^{\mu}=a^{\mu\nu(k)}\partial_{\nu(k)},\ S_{\sigma}^{\alpha}=b_{\sigma}^{\alpha\nu(l)}\partial_{\nu(l)},\ T_{\alpha\sigma}=C_{\alpha\sigma}^{\nu(m)}\partial_{\nu(m)}\ ,\\ &\nu(n)=\underbrace{\nu\mu\cdots\lambda\rho}_{n},\ \partial_{\nu(n)}=\underbrace{\partial_{\nu}\partial_{\mu}\cdots\partial_{\lambda}\partial_{\rho}}_{n}\ ,\end{aligned} \tag{18}$$

a, b and c are functions of x, ϕ^α and π_α. Under the transformation (17), the variation of canonical action is given by

$$\delta I^p = \int \mathrm{d}^4x \left\{ \frac{\delta I^p}{\delta \phi^\alpha} \bar{\delta}\phi^\alpha + \frac{\delta I^p}{\delta \pi_\alpha} \bar{\delta}\pi_\alpha + \partial_\mu [(\pi_\alpha \dot{\phi}^\alpha - \mathcal{H}_c)\delta x^\mu] + \frac{\mathrm{d}}{\mathrm{d}t}(\pi_\alpha \bar{\delta}\phi^\alpha) \right\} , \tag{19}$$

where

$$\bar{\delta}\phi^\alpha = \delta\phi^\alpha - \dot{\phi}^\alpha_{,\mu}\delta x^\mu , \quad \bar{\delta}\pi_\alpha = \delta\pi_\alpha - \pi_{\alpha,\mu}\delta x^\mu . \tag{20}$$

The Jacobian of transformation (17) is denoted by $J[\phi^\alpha_{,}\pi_\alpha, \varepsilon^\sigma]$. Because the boundary conditions of the functions $\varepsilon^\sigma(x)$ and the generating functional (2) is invariant under the transformation (17), which implies that $\delta Z/\delta\varepsilon^\sigma|_{\varepsilon^\sigma=0} = 0$, from expressions (2) and (19) one obtains the Ward identities in phase space

$$\begin{aligned} &\left[J^0_\sigma + \widetilde{S}^\alpha_\sigma \left(\frac{\delta I^p}{\delta \phi^\alpha} \right) - \widetilde{R}^\mu_\sigma \left(\phi^\alpha_{,\mu} \frac{\delta I^p}{\delta \dot{\phi}^\alpha} \right) + \widetilde{S}^\alpha_\sigma J_\alpha - \widetilde{R}^\mu_\sigma (\phi^\alpha_{,\mu} J_\alpha) + \widetilde{T}_{\alpha\sigma} \left(\frac{\delta I^p}{\delta \pi_\alpha} \right) \right. \\ &\left. - \widetilde{R}^\mu_\sigma \left(\pi_{\alpha,\mu} \frac{\delta I^p}{\delta \pi_\alpha} \right) + \widetilde{T}_{\alpha\sigma} J^\alpha_1 - \widetilde{R}^\mu (\pi_{\alpha,\mu} J^\alpha_1) \right]_{\substack{\phi^\alpha \to \frac{1}{\mathrm{i}} \frac{\delta}{\delta J_\alpha} \\ \pi_\alpha \to \frac{1}{\mathrm{i}} \frac{\delta}{\delta J^\alpha_1}}} Z[J, J_1] = 0 , \end{aligned} \tag{21}$$

where $J^0_\sigma = \delta J[\phi^\alpha_{,}\pi_\alpha, \varepsilon^\sigma]/\delta\varepsilon^\delta\big|_{\varepsilon^\sigma=0}$. In the above expressions we have used $J[\phi^\alpha_{,}\pi_\alpha, 0] = 1$. The $\widetilde{R}^\mu_\sigma$, $\widetilde{S}^\alpha_\sigma$ and $\widetilde{T}_{\alpha\sigma}$ are adjoint operators with respect to R^μ_σ, S^α_σ and $T_{\alpha\sigma}$ respectively.[9] We functionally differentiate Eq. (21) with respect to exterior sources, and set all exterior sources equal to zero, then we can deduce another Ward identities in phase space. If one replace π_α by $\partial\mathcal{L}/\partial\dot{\phi}^\alpha$ in Eq. (21), $\pi_\alpha \to \partial\mathcal{L}/\partial\dot{\phi}^\alpha$, the Ward identities (21) can be expressed in terms of variables in configuration space.

III. THE SINGULAR SYSTEM

Consider a system with singular Lagrangian (called the singular system) whose Hess matrix $H_{\alpha\beta} = \partial^2\mathcal{L}/\partial\dot{\phi}^\alpha\partial\dot{\phi}^\beta$ is degenerate. In this case, the system is described by the canonical variables which is subject to inherent phase space constraints, and is called a constrained Hamiltonian system. From the consistent (stationarity) conditions of constraints, one can define successively the secondary constraints according to the Dirac- Bergmann algorithm. Let $\Lambda_k (k = 1, 2, \cdots, K)$ be first-class constraints, $\theta_i (i = 1, 2, \cdots, I)$ be second-class constraints. The gauge conditions associated with first-class constraints are $\Omega_k (k = 1, 2, \cdots, K)$. The generating functional of Green function for this constrained Hamiltonian system is given

by[10]

$$Z[J,J_1]=\int \mathcal{D}\phi^\alpha\mathcal{D}\pi_\alpha\prod_{i,k,l}\delta(\theta_i)\delta(\Lambda_k)\delta(\Omega_l)\det|\{\Lambda_k,\Omega_l\}|[\det|\{\theta_i,\theta_j\}|]^{1/2}$$
$$\times\exp\{\mathrm{i}[I^P+\int \mathrm{d}^4x(J_\alpha\phi^\alpha+J_1^\alpha\pi_\alpha)]\}\ . \tag{22}$$

Using δ-function and integral properties of Grassmann variables $C_l(x)$ and $\overline{C}_k(x)$, one has

$$\det|\{A_k(x),B_l(y)\}|=\int \mathcal{D}C_l(y)\mathcal{D}\overline{C}_k(x)\exp[\mathrm{i}\int \mathrm{d}^4x\mathrm{d}^4y\overline{C}_k(x)\{A_k(x),B_l(y)\}C_l(y)]\ , \tag{23}$$

and the expression (22) can be written as

$$Z[J,J_1]=\int \mathcal{D}\phi^\alpha\mathcal{D}\pi_\alpha\mathcal{D}\Lambda_m\mathcal{D}C_l\mathcal{D}\overline{C}_k\exp\{\mathrm{i}[I^P_{\mathrm{eff}}+\int \mathrm{d}^4x(J_\alpha\phi^\alpha+J_1^\alpha\pi_\alpha)]\}\ , \tag{24}$$

where

$$I^P_{\mathrm{eff}}=\int \mathrm{d}^4x\mathcal{L}^P_{\mathrm{eff}}=\int \mathrm{d}^4x\{\mathcal{L}^P+\lambda_k\Lambda_k+\lambda_l\Omega_l+\lambda_i\theta_i$$
$$+\int \mathrm{d}^4y[\overline{C}_k(x)\{\Lambda(x),\Omega_l(y)\}C_l(y)+\frac{1}{2}\overline{C}_i(x)\{\theta_i(x),\theta_i(y)C_j(y)]\}\ . \tag{25}$$

The generating functional (24) is invariant under the transformation of canonical variables, thus one can still proceed in the same way as the case for regular system to derive corresponding Ward identities in phase space. In this case the only need is to replace I^P by I^P_{eff} in expression (21). Especially, the generating functional (24) is invariant under the translation transformation of the canonical variables, this leads to the quantum canonical equations

$$\dot{\pi}_\alpha(x)=-\frac{\delta H_{\mathrm{eff}}}{\delta\phi^\alpha(x)}\ ,\ \dot{\phi}^\alpha(x)=\frac{\delta H_{\mathrm{eff}}}{\delta\pi_\alpha(x)}\ , \tag{26}$$

where H_{eff} is the canonical Hamiltonian connected with effective Lagrangian $\mathcal{L}^P_{\mathrm{eff}}$.

In the classical theories of constrained Hamiltonian system Dirac conjectured that all first-class constraints (primary and secondary) are generators of the gauge transformation which generate equivalence transformation among physical states. If this conjecture holds true, then the dynamical equations of a constrained Hamiltonian system should be derived from the extended Hamiltonian H_e and one should take all first-class constraints into account in H_e (not only primary first-class constraints but also the secondary ones). From time to time there have been objections to Dirac's conjecture.[11,12] According to the above discussion we see that in the quantum theory of constrained Hamiltonian system the quantum canonical equation (26) is derived from effective Hamiltonian H_{eff} which involves not only first-class constraints but also the second-class ones (including secondary second-class constraints). This is significantly different from the classical theory of constrained Hamiltonian system. When we describe a constrained Hamiltonian system in quantum theory, a more fundamental

quantity is the generating functional, not classical equations of motion. Hence we can ignore whether Dirac's conjecture holds true or not in quantum theory of constrained system.

In the following we will give a preliminary application of Ward identities in canonical formalism to the gauge theories.

IV. YANG-MILL THEORIES

The gauge invariant Lagrangian of $SU(3)$ is given by

$$\mathcal{L} = -\frac{1}{4}F_a^{\mu\nu}F_{\mu\nu}^a + \bar{\psi}[i\gamma^\mu(\partial_\mu - igB_\mu^a T^a) - m]\psi \ , \tag{27}$$

$$F_{\mu\nu}^a = \partial_\mu B_\mu^a - \partial_\nu B_\mu^a - g f_{bc}^a B_\mu^b B_\nu^c \ , \tag{28}$$

where T^a are generators of $SU(3)$, $T^a = \frac{1}{2}\lambda^a (a = 1, 2, \cdots, 8)$, λ^a are Gell-Mann matrix, and f_{bc}^a are structure constants of $SU(3)$. The canonical momenta of $\psi, \bar{\psi}$ and B_μ^a are

$$\pi = \partial\mathcal{L}/\partial\dot{\psi} = i\bar{\psi}\gamma_0, \ \ \bar{\pi} = \partial\mathcal{L}/\partial\dot{\bar{\psi}} = 0, \ \ \pi_a^\mu = \partial\mathcal{L}/\partial\dot{B}_\mu^a = -F_a^{0\mu} \ . \tag{29}$$

The canonical Hamiltonian of the system is given by

$$\begin{aligned} H_c &= \int d^3x \mathcal{H}_c = \int d^3x(\pi\dot{\psi} + \bar{\pi}\dot{\bar{\psi}} + \pi_a^\mu \dot{B}_\mu^a - \mathcal{L}) \\ &= \int d^3x \left[\frac{1}{2}\pi_i^a\pi_i^a - B_0^a(D_{bi}^a\pi_b^i - ig\pi T^a\psi) + \frac{1}{4}F_{ij}^a F_a^{ij} + \frac{1}{2}\pi\gamma^0\gamma^i(\partial^i\psi)\right. \\ &\quad \left. - \frac{1}{2}(\partial^i\pi)\gamma^0\gamma^i\psi - ig\pi\gamma^0\gamma^i\psi B_i^a T^a - im\pi\gamma^0\psi\right] \ , \end{aligned} \tag{30}$$

where

$$D_{b\mu}^a = \delta_b^a\partial_\mu + g f_{bc}^a B_\mu^c \ . \tag{31}$$

From Eq. (29) we see that the primary constraints are

$$\theta_1 = \pi - i\bar{\psi}\gamma_0 \approx 0, \ \theta_2 = \bar{\pi} \approx 0, \ \Lambda_{1a} = \pi_a^0 \approx 0 \ . \tag{32}$$

The total Hamiltonian is given by

$$H_t = \int d^3x(\mathcal{H}_c + \lambda_1\theta_1 + \lambda_2\theta_2 + \mu_1^a\Lambda_{1a}) \ . \tag{33}$$

From the stationarity conditions of constraints $\theta_i (i = 1, 2)$, $\{\theta_i, H_t\} \approx 0$, lead to the equations for determining the Lagrange multiplies $\lambda_i (i = 1, 2)$. The stationarity conditions of constraints Λ_{1a}, $\{\pi_a^0, H_t\} \approx 0$, yield the secondary constraints

$$\chi_a = D_{bi}^a\pi_b^i - ig\pi T^a\psi \approx 0 \ . \tag{34}$$

The stationarity of constraints χ_a does not induce any new constraints. One finds a linear combination of the constraints

$$\Lambda_{2a} = \chi_a + ig\pi T^a(\theta_1\psi - \overline{\psi}\theta_2) \approx 0 \ , \tag{35}$$

and it is easy to check that the constraints Λ_{1a} and Λ_{2a} are first-class, while the constraints θ_1 and θ_2 are second-class.

We adopt the quantization of path integral. For each first-class constraint a gauge condition should be chosen. Consider the Coulomb gauge

$$\Omega_2^a = \partial_i B_i^a \approx 0 \ . \tag{36}$$

For consistency of Ω_2^a, this has to be $\partial_0\Omega_2^a \approx 0$, and we may take other gauge conditions

$$\Omega_1^a = \partial_i \pi_i + M^{ab}B_b^0 \approx 0 \ , \tag{37a}$$

where

$$M_{ab} = \delta_{ab}\partial^i\partial_i - g f_{bc}^a B_i^c \partial^i \ . \tag{37b}$$

The factor $\det|\{\theta_i(x), \theta_j(y)\}|$ is independent of field variables, thus, we can omit this factor from the generating functional (22), but we have

$$\det|\{\Omega_a, \Lambda_b\}| = \det|M_{ab}\delta(\boldsymbol{x} - \boldsymbol{y})| \ . \tag{38}$$

Therefore, according to expressions (24) and (25), the generating functional of Green function can be written as

$$\begin{aligned} Z[J, J_1, \overline{\xi}, \xi_1, \xi, \overline{\xi}_1, \overline{\zeta}, \zeta, M, X, Y] = \int \mathcal{D}B_\mu^a \mathcal{D}\pi_a^\mu \mathcal{D}\psi \mathcal{D}\pi \mathcal{D}\overline{\psi} \mathcal{D}\overline{\pi} \mathcal{D}C \mathcal{D}\overline{C} \mathcal{D}\mu \mathcal{D}\omega \mathcal{D}\nu \\ \times \exp\left\{ \mathrm{i} \int \mathrm{d}^4x \left[\mathcal{L}_{\mathrm{eff}}^P + J_a^\mu B_\mu^a + J_{1\mu}^a \pi_a^\mu + \overline{\xi}\psi + \pi\xi_1 \right.\right. \\ \left.\left. + \overline{\psi}\xi + \overline{\xi}_1\overline{\pi} + \overline{\zeta}_a C_a + \overline{C}_a\zeta + M_k\mu_k + X_l\omega_l + Y_i\nu_i \right] \right\} , \end{aligned} \tag{39}$$

where

$$\mathcal{L}_{\mathrm{eff}}^P = \mathcal{L}^P + \overline{C}_a M^{ab} C_b + \mu_k \Lambda_k + \Omega_l \omega_l + \nu_i \theta_i \ , \tag{40}$$

μ_k, ω_l and ν_i are multiplier fields. For the sake of simplicity, the expression (39) can be written as

$$Z[J, J_1] = \int \mathcal{D}\phi^\alpha \mathcal{D}\pi_\alpha \exp\{\mathrm{i} \int \mathrm{d}^4x (\mathcal{L}_{\mathrm{eff}}^P + J_\alpha \phi^\alpha + J_1^\alpha \pi_\alpha)\} \ , \tag{41}$$

where ϕ^α denote various fields, $\phi^\alpha = (B_\mu^a, \psi, \overline{\psi}, C, \overline{C}, \mu, \omega, \nu)$, π_α are corresponding canonical momenta, $\pi_\alpha = (\pi_\alpha^\mu, \pi, \overline{\pi})$. The symbols J_α and J_1^α denote exterior sources with respect to ϕ^α and π_α respectively.

We do not investigate the problem whether the generating functional (39) can be simplified by carrying out explicit integration over momenta. We shall discuss straightforward the transformation propertities of the generating functional under the transformation of canonical variables. Consider an infinitesimal local transformation of canonical variables

$$\begin{aligned} B_\mu^{a'}(x) &= B_\mu^a(x) + D_{b\mu}^a\varepsilon^b(x), \qquad \pi_a^{\mu'}(x) = \pi_a^\mu(x) + gf_{ab}^c\pi_c^\mu(x)\varepsilon^b(x) , \\ \psi'(x) &= \psi(x) + igT^b\psi(x)\varepsilon^b(x), \quad \pi'(x) = \pi(x) - ig\pi(x)T^b\varepsilon^b(x) , \\ \overline{\psi}'(x) &= \overline{\psi}(x) - igT^b\overline{\psi}(x)\varepsilon^b(x), \quad \overline{\pi}'(x) = \overline{\pi}(x) + igT^b\overline{\pi}(x)\varepsilon^b(x) . \end{aligned} \tag{42}$$

The Jacobian of this transformation is denoted by $J[\phi,\pi,\varepsilon]$, the quantities $\delta J[\phi,\ \pi,\ \varepsilon]/\delta\varepsilon|_{\varepsilon=0}$ are independent of field variables.[1] The term $\mathcal{L}^p$ in Eq. (40) is invariant under the transformation (42), but we have

$$\delta(\overline{C}_a M^{ab} C_b) = -gf_{ec}^a\overline{C}^a D_{bi}^c\varepsilon^b(x)\partial^i C_e = E_b^i(\overline{C},C)\partial_i\varepsilon^b(x) + E_b^0(\overline{C},\partial_i C,B_i)\varepsilon^b(x) , \tag{43a}$$

$$\delta\theta_1 = 0 , \tag{43b}$$

$$\delta\theta_2 = igT^b\overline{\pi}\varepsilon^b(x) = \Theta_{2b}(\overline{\pi})\varepsilon^b(x) , \tag{43c}$$

$$\delta\Lambda_{1a} = gf_{bc}^a\pi_c^0\varepsilon^b(x) = \Lambda_{1ab}(\pi^0)\varepsilon^b(x) , \tag{43d}$$

$$\delta\Lambda_{2a} = \Lambda_{2ab}^i(\pi^i)\partial_i\varepsilon^b(x) + \Lambda_{2ab}(B_i,\pi^i,\psi,\overline{\psi},\overline{\pi})\varepsilon^b(x) , \tag{43e}$$

$$\begin{aligned} \delta\Omega_1^a =& \Omega_{1b}^{a0}\nabla^2\partial_0\varepsilon^b(x) + \Omega_{1b}^a(B_0)\nabla^2\varepsilon^b(x) + \Omega_{1b}^{ai0}(B_i)\partial_i\partial_0\varepsilon^b(x) \\ &+ \Omega_{1b}^{ai}(\pi^i,\partial_i B_0,B_\mu)\partial_i\varepsilon^b(x) + \Omega_{1b}^{a1}(\partial\pi^i,\partial_i B_0,B_\mu,\nabla^2 B_0)\varepsilon^b(x) , \end{aligned} \tag{43f}$$

$$\delta\Omega_2^a = \Omega_{2b}^a\nabla^2\varepsilon^b(x) + \Omega_{2b}^{ai}(B_i)\partial_i\varepsilon^b(x) + \Omega_{2b}^{a0}(\partial_i B_i)\varepsilon^b(x) . \tag{43g}$$

Let

$$E_b = E_b^0 - \partial_i E_b^i , \tag{44}$$

$$\begin{aligned} F_b =& \nu_2\Theta_{2b} + \mu_1^a\Lambda_{1ab} + \mu_2^a\Lambda_{2ab} - \partial_i(\mu_2^a\Lambda_{2ab}^i) - \partial_0\nabla^2(\omega_1^a\Omega_b^{a0}) + \nabla^2(\omega_1^a\Omega_{1b}^a) \\ &+ \partial_0\partial_i(\omega_1^a\Omega_1^{ai0}) - \partial_i(\omega_1^a\Omega_{1b}^{ai}) + \omega_1^a\Omega_{1b}^{a1} + \nabla^2(\omega_2^a\Omega_{2b}^a) - \partial_i(\omega_2^a\Omega_{2b}^{ai}) + \omega_2^a\Omega_{2b}^{a0} . \end{aligned} \tag{45}$$

The invariance of generating functional (41) under the transformation (42) implies $\delta\ Z/\delta\varepsilon^b(x)\big|_{\varepsilon^b} = 0$, this leads to

$$\begin{aligned} &(\delta J[\phi,\pi,\varepsilon]/\delta\varepsilon^b)_{\varepsilon^b=0}Z[J,J_1] + \int \mathcal{D}\phi^\alpha\mathcal{D}\pi_a\Big[E_b + F_b - \partial_\mu J_b^\mu + gf_{bc}^a B_\mu^c J_a^\mu + gf_{ab}^c\pi_c^\mu J_{1\mu}^a + igT^b\bar{\xi}\psi \\ &- igT^b\pi\xi_1 - igT^b\overline{\psi}\xi + igT^b\bar{\xi}_1\overline{\pi}\Big]\exp\left\{i\int d^4x\left[\mathcal{L}_{\text{eff}}^p + J_\alpha\phi^\alpha + J_1^\alpha\pi_\alpha\right]\right\} = 0 \end{aligned} \tag{46}$$

or

$$\left[(\delta J[\phi,\pi,\varepsilon]/\delta\varepsilon^b)_{\varepsilon^b=0}+E_b+F_b-\partial_\mu J_b^\mu+gf_{bc}^aJ_a^\mu\frac{\delta}{\delta J_c^\mu}+gf_{ab}^cJ_{1\mu}^a\frac{\delta}{\delta J_{1\mu}^c}+\mathrm{i}gT^b\bar{\xi}\frac{\delta}{\delta\bar{\xi}}\right.$$
$$\left.-\mathrm{i}gT^b\xi_1\frac{\delta}{\delta\xi_1}-\mathrm{i}gT^b\xi\frac{\delta}{\delta\xi}+\mathrm{i}gT^b\bar{\xi}_1\frac{\delta}{\delta\bar{\xi}_1}\right]Z[J,J_1]=0\ , \tag{47}$$

where E_b depends on ghost fields C_a and $\overline{C}_a$, but F_b depends on multiplier fields.

As usual, let $Z[J,J_1]=\exp\{\mathrm{i}W[J,J_1]\}$ and use the definition of generating functional of proper vertices $\Gamma[\phi,\pi]$ which is given by performing a functional Legendre transformation[13], we have

$$\Gamma[\phi^\alpha,\pi_\alpha]=W[J,J_1]-\int\mathrm{d}^4x(J_\alpha\phi^\alpha+J_1^\alpha\pi_\alpha) \tag{48}$$

and

$$\begin{aligned}&\delta W/\delta J_\alpha(x)=\phi^\alpha(x),\qquad &&\delta\Gamma/\delta\phi^\alpha(x)=-J_\alpha(x)\ ,\\&\delta W/\delta J_1^\alpha(x)=\pi_\alpha(x),\qquad &&\delta\Gamma/\delta\pi_\alpha(x)=-J_1^\alpha(x)\ .\end{aligned} \tag{49}$$

Thus, the expression (47) can be written as

$$(\delta J[\phi,\ \pi,\ \varepsilon]/\delta\varepsilon^b)_{\varepsilon^b=0}+E_b+F_b+\partial_\mu\frac{\delta\Gamma}{\delta B_\mu^b}-gf_{bc}^aB_\mu^a\frac{\delta\Gamma}{\delta B_\mu^c}-gf_{bc}^a\pi_a^\mu\frac{\delta\Gamma}{\delta\pi_c^\mu}-\mathrm{i}gT^b\psi\frac{\delta\Gamma}{\delta\psi}$$
$$+\mathrm{i}gT^b\pi\frac{\delta\Gamma}{\delta\pi}+\mathrm{i}gT^b\bar{\psi}\frac{\delta\Gamma}{\delta\bar{\psi}}-\mathrm{i}gT^b\bar{\pi}\frac{\delta\Gamma}{\delta\bar{\pi}}=0\ . \tag{50}$$

Differentiating Eq. (50) with respect to $\psi(x_2)$ and $\bar{\psi}(x_3)$, and setting all fields (including multiplier fields) equal to zero, $\phi^\alpha=\pi_\alpha=0$, we obtain

$$\partial_{x_1}^\mu\left.\frac{\delta^3\Gamma}{\delta\bar{\psi}(x_3)\delta\psi(x_2)\delta B_b^\mu(x_1)}\right|_{\phi^\alpha=\pi_\alpha=0}=gT^b\delta(x_1-x_2)\left.\frac{\delta^2\Gamma}{\delta\bar{\psi}(x_3)\delta\psi(x_1)}\right|_{\phi^\alpha=\pi_\alpha=0}$$
$$-gT^b\delta(x_1-x_3)\left.\frac{\delta^2\Gamma}{\delta\bar{\psi}(x_3)\delta\psi(x_2)}\right|_{\phi^\alpha=\pi_\alpha=0}\ . \tag{51}$$

Differentiating Eq.(50) with respect to $B_\nu^a(x_2)$ and $B_\Lambda^c(x_3)$ and setting $\phi^\alpha=\pi_\alpha=0$ yields

$$\partial_{x_1}^\mu\left.\frac{\delta^3\Gamma}{\delta B_c^\lambda(x_3)\delta B_a^\nu(x_2)\delta B_b^\mu(x_1)}\right|_{\phi^\alpha=\pi_\alpha=0}=gf_{b\sigma}^a\delta(x_1-x_2)\left.\frac{\delta^2\Gamma}{\delta B_c^\lambda(x_3)\delta B_\sigma^\nu(x_1)}\right|_{\phi^\alpha=\pi_\alpha=0}$$
$$+gf_{b\sigma}^c\delta(x_1-x_3)\left.\frac{\delta^2\Gamma}{\delta B_a^\nu(x_2)\delta B_\sigma^\lambda(x_1)}\right|_{\phi^\alpha=\pi_\alpha=0}\ . \tag{52}$$

The expressions (51) and (52) indicate that there are some relationships among the propagators and proper vertices. On performing the Fourier transformation one translates Eqs. (51) and (52) into momentum space, and then can obtain corresponding relationships for the propagators and proper vertices in momentum space.

Differentiating Eq. (50) with respect to $\pi_a^\nu(x_2)$ and $\pi_c^\lambda(x_3)$ and setting $\phi_\alpha = \pi_\alpha = 0$ yields

$$\partial_{x_1}^\mu \frac{\delta^3\Gamma}{\delta\pi_c^\lambda(x_3)\delta\pi_a^\nu(x_2)\delta B_b^\mu(x_1)}\bigg|_{\phi^\alpha=\pi_\alpha=0} = g f_{b\sigma}^a \delta(x_1-x_2) \frac{\delta^2\Gamma}{\delta\pi_c^\lambda(x_3)\delta\pi_\sigma^\nu(x_1)}\bigg|_{\phi^\alpha=\pi_\alpha=0} + g f_{b\sigma}^c \delta(x_1-x_3) \frac{\delta^2\Gamma}{\delta\pi_a^\nu(x_2)\delta\pi_\sigma^\lambda(x_1)}\bigg|_{\phi^\alpha=\pi_\alpha=0} . \tag{53}$$

The propagator for π in phase space has been introduced in Ref. [5], and the expression (53) indicates that the propagator for π should satisfy this relation. Similarly, differentiating Eq. (50) many times with respect to field variables, one can obtain various Ward identities for proper vertices in phase space.

In the transformation (42), the change of ghost fields $C_a(x)$ and $\overline{C}_a(x)$ are not considered. If one further considers both the transformation of gauge fields and ghost fields, the more general Ward identities for gauge-ghost fields proper vertices can be obtained.

For later use we list the $SU(3)$ currents (vector, axial-vector, scalar, and pseudoscalar) constructed from Ferim field $\psi(x)$,

$$\begin{aligned} &V_\mu^a(x) = \overline{\psi}(x)\gamma_\mu T^a\psi(x),\ A_\mu^a(x) = \overline{\psi}(x)\gamma_\mu\gamma_5 T^a\psi(x) , \\ &S^a(x) = \overline{\psi}(x)T^a\psi(x), \qquad P^a(x) = \mathrm{i}\overline{\psi}(x)\gamma_5 T^a\psi(x) . \end{aligned} \tag{54}$$

Introducing the external sources $v_a^\mu(x)$, $a_a^\mu(x)$, $s_a(x)$ and $p_a(x)$ for these currents respectively, the extended generating functional can be constructed as

$$Z[J, J_1, v, a, s, p] = \int \mathcal{D}\phi^\alpha \mathcal{D}\pi_\alpha \exp\left\{\mathrm{i}\int \mathrm{d}^4x \left[\mathcal{L}_{\mathrm{eff}}^{p} + J_\alpha\phi^\alpha + J_1^\alpha\pi_\alpha + v_a^\mu V_\mu^a + a_a^\mu A_\mu^a + s_a S^a + p_a P^a\right]\right\} . \tag{55}$$

The variation of canonical action is given by

$$\delta I^{p} = \int \mathrm{d}^4x \varepsilon^a(x)(\partial^\mu A_\mu^a - 2mP^a - g f_{bc}^a A_\mu^b B^{c\mu}) \tag{56}$$

under the chirality transformation

$$\begin{aligned} &\psi'(x) = (1+\varepsilon^a(x)\gamma_5 T^a)\psi(x) ,\ \pi'(x) = \pi(x)(1-\mathrm{i}\varepsilon^a(x)\gamma_5 T^a) , \\ &\overline{\psi}'(x) = \overline{\psi}(x)(1+\mathrm{i}\varepsilon^a(x)\gamma_5 T^a) ,\ \overline{\pi}'(x) = (1-\mathrm{i}\varepsilon^a(x)\gamma_5 T^a)\overline{\pi}(x) . \end{aligned} \tag{57}$$

The Jacobian of the transformation (57) equals unity. Because $\delta\theta_2 = -\mathrm{i}\varepsilon^a(x)\gamma_5 T^a\theta_2$, the invariance of generating functional (55) under the transformation (57) implies that

$$\begin{aligned} &\int \mathcal{D}\phi^\alpha\mathcal{D}\pi_\alpha \left[\partial^\mu A_\mu^a - 2mP^a - g f_{bc}^a A_\mu^b B^{c\mu} - \mathrm{i}\nu_2\gamma_5 T^a\theta_2 + \mathrm{i}\overline{\xi}\gamma_5 T^a\psi - \mathrm{i}\pi\gamma_5 T^a\xi_1 \right. \\ &+ \mathrm{i}\overline{\psi}\gamma_5 T^a\xi - \mathrm{i}\overline{\xi}_1\gamma_5 T^a\pi + v_\mu^b f_{bc}^a A^{c\mu} + a_\mu^b f_{bc}^a V^{c\mu} - s^b d_{bc}^a P^c + p^b d_{bc}^a S^c\Big] \\ &\times \exp\left\{\mathrm{i}\int \mathrm{d}^4x \left[\mathcal{L}_{\mathrm{eff}}^{p} + J_\alpha\phi^\alpha + J_1^\alpha\pi_\alpha + v_a^\mu V_\mu^a + a_a^\mu A_\mu^a + s_a S^a + p_a P^a\right]\right\} = 0 . \end{aligned} \tag{58}$$

Let us set all external sources (and $\theta_2 \approx 0$) equal to zero, we obtains

$$\langle 0| \left[\partial^\mu \widehat{A}^a_\mu(x) - 2m\widehat{P}^a(x) - g f^a_{bc}\widehat{A}^b_\mu(x)\widehat{B}^{c\mu}(x)\right] |0\rangle = 0 \ . \tag{59}$$

This is just an expression for partial conservation of axial-vector currents (PCAC). Differentiating Eq. (58) with respect to $v^b_\nu(y)$ and $v^c_\lambda(z)$ and setting all external sources equal to zero yields

$$\begin{aligned} &\langle 0|\mathrm{T}^* \left[(\partial^\mu \widehat{A}^a_\mu(x) - 2m\widehat{P}^a(x) - g f^a_{de}\widehat{A}^d_\mu(x)\widehat{B}^{e\mu}(x))\widehat{V}^b_\nu(y)\widehat{V}^c_\lambda(z)\right] |0\rangle \\ &= \mathrm{i}\delta(x-y)\left[f^a_{be}\langle 0|\mathrm{T}[\widehat{A}^e_\nu(x)\widehat{V}^c_\lambda(y)]|0\rangle\right] + \mathrm{i}\delta(x-z)\left[f^a_{ce}\langle 0|\mathrm{T}[\widehat{A}^e_\lambda(x)\widehat{V}^b_\nu(y)]|0\rangle\right] \ . \end{aligned} \tag{60}$$

This is the axial-vector Ward identity for the AVV vertex. For the case $f^a_{bc} = 0$ in Eqs. (59) and (60), the result has been given in Ref.[1]. The Lagrangian used in Ref.[1] is not gauge invariant but a singular one, and there are some inherent constraints in phase space. The treatment of constraints was ignored for derving Ward identity in Ref.[1] by the authors. This is a supplementary discussion given from another point of view.

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