Volume 3, Number 1 ACTA PHYSICA SINICA January, 1994
1004-423X/94/03010026-10 (Overseas Edition) © 1994 Science Press

THE PROCESS OF SPACE CURRENT NEUTRALIZATION OF INTENSE RELATIVISTIC ELECTRON BEAM UNDER AN EXTERNALLY APPLIED MAGNETIC GUIDE FIELD

LIU GUO-ZHI(刘国治)
Northwest Institute of Nuclear Technology, Xi'an 710024, China
LIU NAI-QUAN(刘乃泉) and XIE XI(谢 羲)
Department of Modern Applied Physics, Tsinghua University, Beijing 100084, China

(Received 7 September 1992)

The process of space current neutralization of intense relativistic electron beam under an externally applied magnetic guide field is discussed in this paper. Ionization by electron avalanching and by beam electrons impact and recombination is included in the calculation of plasma density buildup, with plasma heating by return current and two- stream instability taken into account. A code to evaluate the process of space current neutralization was set up. The calculations demonstrate that the optimum gas pressure increases as peak beam current increases, and it decreases as the risetime of beam pulse increases.

PACC: 4180D; 5260

I. INTRODUCTION

When intense relativistic electron beam enters the drift chamber filled with a neutral gas through the anode, which serves to isolate the diode from the neutral gas, plasma electrons are created by the ionizing collisions between beam electrons and neutral gas molecules. The plasma electrons are radially expelled from the beam region by the beam-generated electric field (before $f_e \approx 1$, the beam-generated electric field is radially dominated, on the order of 10^5 V/cm), leaving positive ions to neutralize the space charge[1]. The fractional beam space charge neutralization f_e is defined as

$$f_e = (n_i - n_e)/n_b \ , \tag{1}$$

n_i, n_e and n_b are the plasma ion density, the plasma electron density, and the beam electron density respectively. The complete beam space charge neutralization ($f_e \approx 1$) is reached after a critical time t_N, and the beam-generated electric field is mainly an axial inductive field, on the order of a few keV/cm. This inductive electric field has a major effect upon plasma production through electron avalanching. The exponentiation of secondary electron density during avalanching causes the gas to break down and the plasma electrons are oppositely accelerated by the inductive electric field, giving rise to the phenomenon of current neutralization. If the beam-generated plasma conductivity is large enough so that the magnetic diffusion time ($\tau_d \approx (4.0\pi\sigma a^2)/C$) is much larger than the beam pulse width, the magnetic field and net current will remain approximately constant, and are equal to values at

breakdown with the net current appromaximately expressed by

$$I_{\rm n} \approx I_{\rm b}(t_{\rm b}) \ . \tag{2}$$

Here a is the beam radius, σ is the beam-generated plasma conductivity, $t_{\rm b}$ is the gas breakdown time, the net current $I_{\rm n}$ equals $I_{\rm b}+I_{\rm p}$, and $I_{\rm p}$ is the plasma return current. The beam transport efficiency increases as the net current decreases. In order to achieve high current neutralization, two conditions must be fulfilled: 1, The conductivity at breakdown, σ, must be high ($t_{\rm d} \gg \tau$); 2, breakdown must occur early in the beam pulse ($t_{\rm b} \ll t_{\rm r}$). Because the beam-generated plasma conductivity in the neutral gas and the breakdown time ($t_{\rm b}$) decrease as the gas pressure increases, there must exists an optimum gas pressure minimizing the net current.

In this paper, a model and a corresponding code to evaluate the process of space current neutralization are set up, and the dependence of the net current, the beam-generated plasma's parameters and beam-generated electromagnetic field on the gas pressure and beam parameters is discussed, in order to get an optimum gas pressure for beam transport.

II. BEAM-GENERATED PLASMA PARAMETERS

The beam-generated plasma electron density $n_{\rm e}$ is calculated by taking into account beam impact ionization, plasma electron avalanching (plasma electron impact ionization) and recombination.

The rate equation for $n_{\rm e}$ is given by

$$\begin{aligned} \frac{\partial n_{\rm e}}{\partial t} &= \underset{①}{n_{\rm b}(t)n_{\rm g}(t)\sigma_{\rm b}V_{\rm b}} + \underset{②}{n_{\rm e}(t)\alpha_{\rm T}V_{\rm d}} \underset{③}{-\alpha_{\rm r1}n_{\rm e}(t)n_{\rm i}} \\ &\quad \underset{④}{-\alpha_{\rm r2}n_{\rm e}(t)n_{\rm i2}(t)} \underset{⑤}{-\alpha_{\rm a}n_{\rm e}(t)n_{\rm g}(t)} \ , \end{aligned} \tag{3}$$

where $n_{\rm g}(t)$ is the unionized gas density ($\rm cm^{-3}$); $n_{\rm b}(t)$ is the beam electron density ($\rm cm^{-3}$); $\sigma_{\rm b}$ is the beam-impact ionization cross section ($\rm cm^2$), as shown in Table 1[13]; $V_{\rm b}$ is the beam electron velocity (cm/s); $\alpha_{\rm T}$ is the first Townsend ionization coefficient, from experimental data[2-12]; $V_{\rm d}$ is the plasma electron drift velocity, from experimental data[4,8,9,12-24]; $\alpha_{\rm r1}$ is the recombination coefficient between plasma electron and ion A^+, and A is the unionized gas molecule (i.e. $\rm N_2$ for nitrogen), then the reaction equation is expressed as

$$\rm e^- + A^+ \overset{\alpha_{r1}}{\rightleftharpoons} A^* \ , \tag{4}$$

$\alpha_{\rm r2}$ is the recombination between plasma electron and ion A_2^+ (i.e. $\rm N_4^+$ for nitrogen), then the reaction equation is expressed as

$$\rm e^- + A_2^+ \overset{\alpha_{r2}}{\rightleftharpoons} 2A^+ \ . \tag{5}$$

Both α_{r1} and α_{r2} are given by experimental data[7,8,12,14,17,22,25–36], n_{i1} is the ion density for A^+, n_{i2} is the ion density for A_2^+, α_a is the attached coefficient between plasma electron and the unionized gas, then the reaction equation is expressed as

$$e^- + A \overset{\alpha_a}{=} 2A^- . \tag{6}$$

The first term in Eq.(3) is the rate of plasma electron generation by beam-impact ionization, playing an important role in the plasma electron generation before $f_e \approx 1$. The second term is the rate of plasma electron generation by impact ionization between gas molecules and plasma electrons due to the action of beam-generated electric field, playing an important role in the rate of plasma electron generation after $f_e \approx 1$. The third and fourth terms describe the rate of recombination between plasma electrons and ions, and can be ignored untill the plasma electron density is high enough. The fifth term is the attaching rate between plasma electrons and unionized gas molecules, and can be ignored for nitrogen and rare gas.

Table 1. The ionization cross-section σ_b.

Gas	N_2	H_2	Ne	Ar	He
$\sigma_b/10^{-12}\,cm^2$	2.16	0.2	0.4	0.9	0.2

The rate equation for plasma ion A^+ is given by

$$\frac{\partial n_{i1}}{\partial t} = \underset{①}{n_b(t)n_g(t)\sigma_b V_b} + \underset{②}{n_e(t)\alpha_r V} \underset{③}{-\alpha_{r1} n_e(t) n_{i1}(t)} \underset{⑥}{- K_d n_{i1}(t) n_g^2(t)} , \tag{7}$$

where K_d is the rate coefficient of reaction between plasma ion A^+ and unionized gas molecules[36–38], which is given by the reaction equation

$$A^+ + 2A \overset{K_d}{=} A_2^+ + A . \tag{8}$$

The rate equation for plasma ion A_2^+ is given by

$$\frac{\partial n_{i2}}{\partial t} = K_d n_{i1}(t) n_g^2(t) - \alpha_{r2} n_e(t) n_{i2}(t) . \tag{9}$$

The rate equation for negative ion A^- is given by

$$\frac{\partial n_i^-}{\partial t} = \alpha_a n_e(t) n_g . \tag{10}$$

In our calculation, the plasma parameters are evaluated by solving the Eqs.(3), (7), (9) and (10), with the coefficients in all the equations being obtained by using the experimental data.

III. EQUATION FOR CURRENT NEUTRALIZATION

According to Maxwell equation, the dependence of electric field on the net current can be written as

$$\frac{\partial}{r\partial r}\left(r\frac{\partial E_z}{\partial r}\right)=\mu_0\frac{\partial J_{\rm nz}}{\partial t}\ , \tag{11}$$

$$\frac{\partial}{\partial r}\left(\frac{\partial}{r\partial r}(rE_\theta)\right)=\mu_0\frac{\partial J_{\rm n\theta}}{\partial t}\ , \tag{12}$$

where $J_{\rm nz}$ is the axial net current density, i.e.

$$J_{nz}=J_{\rm bz}+J_{\rm pz}\ , \tag{13}$$

$J_{n\theta}$ is the angle net current density, i.e.

$$J_{n\theta}=J_{\rm b\theta}+J_{\rm p\theta}\ , \tag{14}$$

$J_{\rm b}$ and $J_{\rm p}$ are beam current density and plasma current density respectively.

The plasma electron density is assumed to be high enough ($\geq 10^{13}\,{\rm cm}^{-3}$), so that the skin depth of electromagnetic field in the plasma (less than 0.16 cm) is much less than the beam radius (about 5 cm), and the net current flow is confined in the thin ring of thickness about the electromagnetic skin depth, justifying the assumption that the axial electric field is approximately uniform and the angle magnetic field is zero in the beam. Under this condition, the dependence of E_z and E_θ on $I_{\rm nz}$ and $I_{n\theta}$ can be written as

$$E_z({\rm V/cm})=-10^3\left(1+2\ln\frac{R}{a}\right)\frac{{\rm d}I_{\rm nz}}{{\rm d}t}\ , \tag{15}$$

$$E_\theta({\rm V/cm})=-2\pi\times10^3\frac{{\rm d}I_{\rm nz}}{{\rm d}t}\ , \tag{16}$$

where $I_{\rm nz}$ and $I_{n\theta}$ are the axial and angular beam current respectively (kA), with time in nanosecond and the beam radius in centimeter,

$$I_{\rm nz}=I_{\rm bz}+I_{\rm pz}\ , \tag{17}$$

$$I_{n\theta}=I_{\rm b\theta}+I_{\rm p\theta}\ , \tag{18}$$

$$I_{\rm bz}=\int_0^a J_{\rm bz}r{\rm d}r\ ,\quad I_{\rm pz}=\int_0^a J_{\rm pz}r{\rm d}r\ , \tag{19}$$

$$I_{\rm b\theta}=\int_0^a J_{\rm b\theta}r{\rm d}r\ ,\quad I_{\rm p\theta}=\int_0^a J_{\rm p\theta}r{\rm d}r\ . \tag{20}$$

According to the Ohm's law[39] we have

$$\frac{m_{\rm e}}{n_{\rm e}e^2}\frac{\partial J_{\rm nz}}{\partial t}=E_z-\frac{1}{\sigma_{\parallel}}J_{\rm pz}\ , \tag{21}$$

$$\frac{m_e}{n_e e^2}\frac{\partial J_{n\theta}}{\partial t} = E_\theta - \frac{\sigma_\perp}{\sigma_\perp^2 + \sigma_H^2} J_{p\theta} \ , \tag{22}$$

where $\sigma_\perp$ is the conductivity perpendicular to the magnetic field lines, given by

$$\sigma_\perp = \frac{\omega_p^2}{4\pi}\frac{\nu_e}{\nu_e^2 + \Omega_p^2} \ ; \tag{23}$$

σ_H is the Hall conductivity of plasma, given by

$$\sigma_H = \frac{\omega_p^2}{4\pi}\frac{\Omega}{\nu_e^2 + \Omega_p^2} \ , \tag{24}$$

and ν_e is the collision frequency between plasma electron and the gas molecules, i.e.

$$\nu_e = \frac{eE}{mV_d} \ . \tag{25}$$

The conductivity parallel to the magnetic field lines $\sigma_\parallel$ is given by

$$\frac{1}{\sigma_\parallel} = \frac{1}{\sigma_p} + \frac{1}{\sigma_{ee}} + \frac{1}{\sigma_{ia}} + \frac{1}{\sigma_w} \ , \tag{26}$$

where σ_p is the conductivity due to the collision between plasma electron and the gas molecules, i.e.

$$\sigma_p = \frac{\omega_p^2}{4\pi\nu_e} \ ; \tag{27}$$

σ_{ee} is the two-stream interaction conductivity, given by

$$\frac{1}{\sigma_{ee}} \approx 1.6 \times 10^{19} j_b n_e^{-3/2} \exp\left(-\left(\frac{j_{min}}{j_b}\right)^3\right) \ ; \tag{28}$$

σ_{ia} is ion-acoustic conductivity, given by

$$\frac{1}{\sigma_{ia}} \approx 6 \times 10^6 n_e^{-1/2} \exp\left(-\frac{1}{0.7}\left(\frac{V_{cr}}{V_d}\right)^2\right) \ ; \tag{29}$$

σ_w is the wave interaction conductivity between plasma electrons and electromagnetic waves, given by

$$\frac{1}{\sigma_w} \approx 6.4 \times 10^4 T_e n_e^{-1/2} \exp\left(-\left(\frac{j_{min}}{j_b}\right)^2\right) \ , \tag{30}$$

$$V_{cr} = 9.8 \times 10^5 T_e^{1/2}\left\{1 + \left(\frac{m_i T_e}{m_e T_i}\right)^{1/2} \exp\left(1.5 - \frac{T_e}{T_i}\right)\right\} \ , \tag{31}$$

$$j_{min} = 7.5 \times 10^{-23} n_e^{3/2} \ln\Lambda / T_e^{3/2} (\mathrm{kA/cm^2}) \ , \tag{32}$$

where $\ln\Lambda$ is the coulomb logarithmic term.

The drift velocity, the temperature of plasma electrons and the temperature of plasma ions in the equation developed above are obtained from experimental data.

A code for evaluating the process of space current neutralization is set up by using the equations developed above. Given the kind of gas, magnetic flux density, gas pressure, and the waveform of current as well as the voltage of electron beam generated by the diode, the time dependences of net current, electric field, plasma current, plasma electron density, plasma density and plasma conductivity can be obtained by using this code.

IV. INFLUENCE OF GAS PRESSURE AND BEAM PARAMETERS ON THE BEAM TRANSPORT

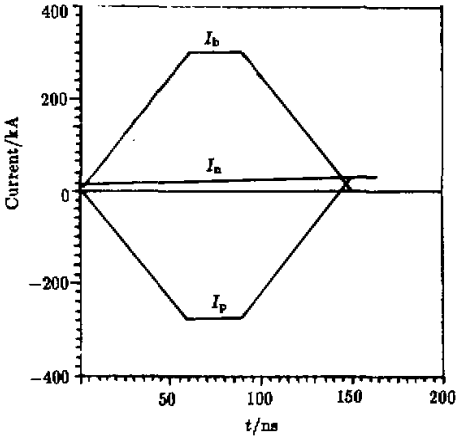

Fig. 1. Plot of net current and plasma current versus time for nitrogen pressure at 3.0 Torr (I_{peak} = 300 kA).

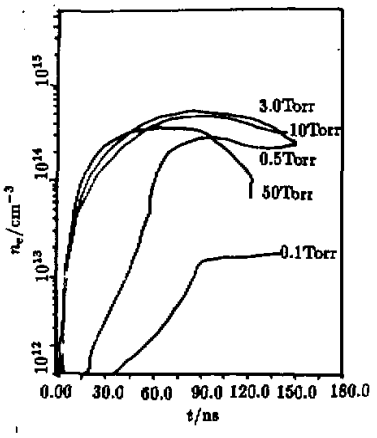

Fig. 2. Plot of plasma electron density versus time for several pressures in N_2 (I_{peak} = 500 kA).

In order to understand the influences of gas pressure and beam parameters on the beam transport, we evaluated the process of space current neutralization for nitrogen. Figure 1 shows the calculated net current and plasma current in nitrogen, plotted as functions of time for a gas pressure of 3.0 Torr, with the waveform of electron beam current shown in the figure. From this figure, we know that a current flow in the plasma after the beam current equals zero is approximately the net current which flows in the plasma when the beam current flows in the drift chamber. The plasma conductivity under this condition is so high that the current in the plasma attenuates very slowly (the magnetic diffusion time is much larger than the beam plus width). The dependences of beam-generated plasma electron density and plasma conductivity on time are shown in Figs.2 and 3 respectively. From these figures we know that, when the neutral gas pressure is fairly low ($\leq$ 0.5 Torr), the gas breakdown occurs late, so that the net current is rather large, although the peak plasma conductivity is high, as shown in Fig.4. When the neutral gas pressure is high ($\geq$ 10 Torr),

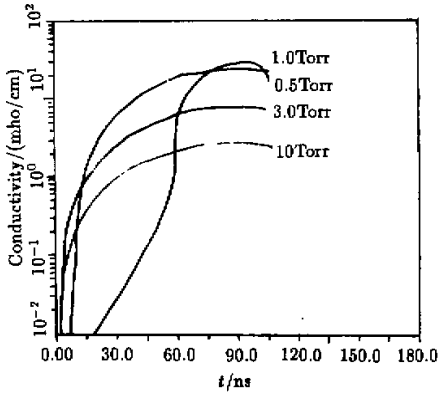

Fig. 3. Plot of plasma conductivity versus time for several pressures in N_2 (I_{peak} = 500 kA).

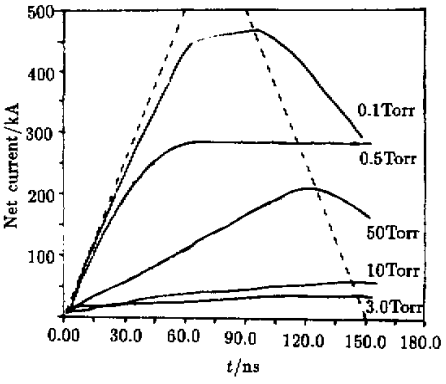

Fig. 4. Plot of net current versus time for several pressures in N_2 (I_{peak} = 500 kA).

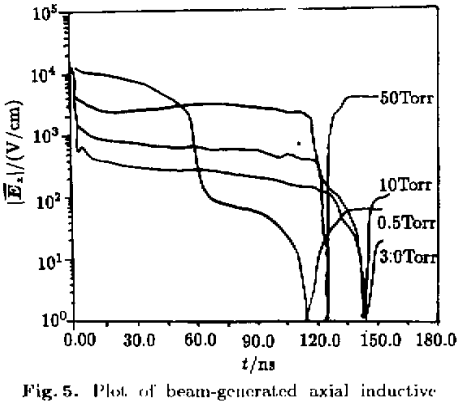

Fig. 5. Plot of beam-generated axial inductive electric field versus time for several pressures in N_2 (I_{peak} = 500 kA).

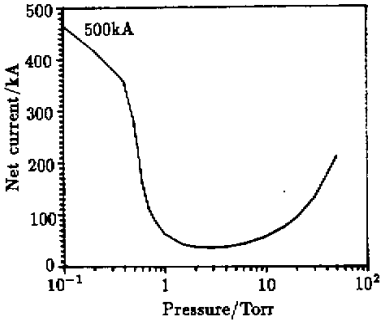

Fig. 6. Plot of peak net current versus nitrogen pressure.

the gas breakdown occurs early in the beam pulse, but the beam-generated plasma electron density and plasma conductivity are low, so that the net current increases with time, and the beam energy loss increases because of the beam-plasma interaction (ohmic heating). The beam-generated axial inductive electric field as a function of time for several gas pressures is shown in Fig.5. The dependence of the peak net current in the beam pulse on the neutral gas pressure is shown in Fig.6. The dependences of the time averages for the plasma conductivity and for the beam-generated axial inductive electric field on the neutral gas pressure are shown in Fig.7 and Fig.8 respectively. In calculations, the beam parameters are taken to be: peak beam current 500 kA, peak voltage 1.0 MV, risetime of beam pulse 60 ns, falltime 60 ns, and the flattime 30 ns. The optimum neutral nitrogen pressure for beam transport is

about 2.5 Torr, that is, the net current and the time average for the beam-generated axial inductive electric field is minimum when the neutral nitrogen pressure is about 2.5 Torr, and the nitrogen pressure window for beam transport is about 1.5~ 4.0 Torr. The dependence of the peak net current on the neutral gas pressure for several peak beam currents is shown in Fig.9, from which we know that the optimum gas pressure increases and the gas pressure window decreases as the peak beam current increases. The influence of the risetime of beam pulse on the peak net current is shown in Fig.10. The smaller the risetime of beam pulse, the higher the optimum gas pressure, showing that the influence of the shape of beam pulse on the beam transport is rather large. In this paper, we only give the results for nitrogen, and the code can be used for other gases.

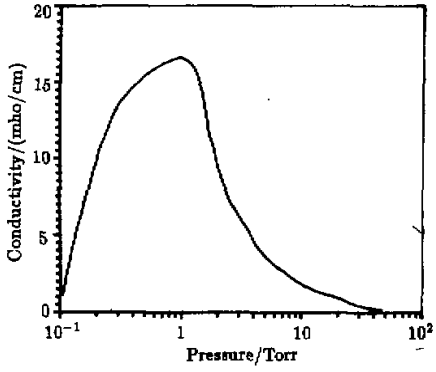

Fig. 7. Plot of time average of conductivity versus nitrogen pressure (I_{peak} = 500 kA).

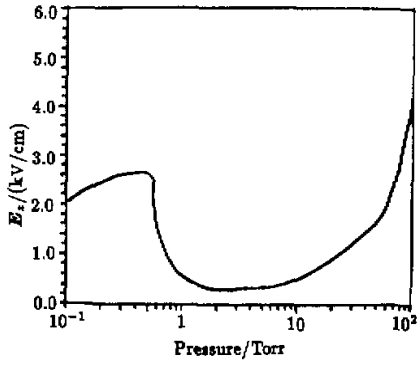

Fig. 8. Plot of time average of beam-generated axial inductive electric field versus nitrogen pressure (I_{peak} = 500 kA).

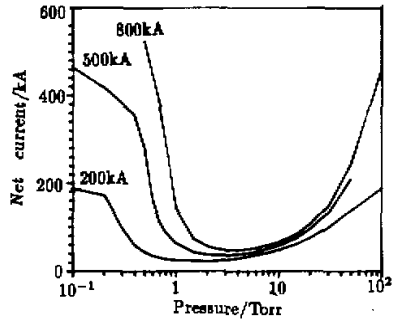

Fig. 9. Plot of net current versus nitrogen pressure for several peak beam currents (beam radius is constant).

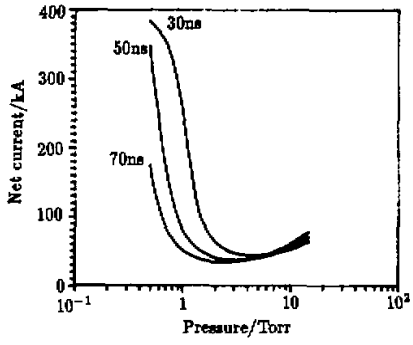

Fig. 10. Influence of risetime of electron beam pulse on the net current for nitrogen.

V. CONCLUSIONS

In this paper, we have first obtained the equations of beam-generated plasma parameters, plasma current and beam-generated electromagnetic field on the condition that there exists an applied magnetic field. We know, by calculation, that the optimum gas pressure increases, as the peak beam current increases; and the smaller the risetime of the electron beam pulse, the higher the optimum gas pressure. The optimum nitrogen pressure is about 2.5 Torr, when the peak beam current is 500 kA, the peak voltage is 1.0 MV, and the risetime of beam pulse is 60 ns, showing that the net current and the time average of electric field are minimum when the gas pressure is about 2.5 Torr. The window of nitrogen pressure for beam transport is 1.5~ 4.0 Torr.

REFERENCES

[1] Liu Guo-zhi, Thesis for Ph. D. (Tsinghua University, March, 1992) (in Chinese).
[2] S. C. Hagdon and O. M. Williams, *J. Phys. D*, **9**(1976), 523.
[3] L. M. Chanin and G. D. Rork, *Phys. Rev.*, **132**(1963), 2547.
[4] A. W. Ali, *Laser and Particle Beams*, **16**(1988), 105.
[5] S. C. Brown, Basic Data of Plasma Physics (The Technology Press of The MIT, 1959).
[6] D. J. Rose, *Phys. Rev.*, **104**(1956), 273.
[7] C. S. Lakshminarasimha and J. Lucas, *J. Phys. D*, **10**(1977), 313.
[8] Hans Schlumbohm, *Z. Physik*, **184**(1965), 492.
[9] A. Von Engel, *Ionized Gases (Second Edition) (Oxford University Press, 1965)*.
[10] G. R. Govinda Raju, *Int. J. Electron*, **45**(1976), 65.
[11] G. R. Govinda Raju and M. S. Dincer, *IEEE Trans. on Plasma Science*, **18**(1990), 819.
[12] N. Koutdeon, J. Lucas and Le Virr, *J. Phys. D*, **5**(1972), 956.
[13] H. Arai and H. Hotta, *J. Chem. Phys.*, **75**(1981), 2252.
[14] Earl W. McDaniel, Collision Phenomena in Ionized Gases (New York, Wiley, 1964).
[15] J. Dutton, *J. Phys. Chem. Ref. Data*, **4**(1975), 577.
[16] Hans Schlumbohm, *Z. Physik*, **182**(1964), 306.
[17] H. T. Saelee, *J. Phys. D*, **10**(1977), 343.
[18] T. Itoh, *J. Appl. Phys.*, **31**(1960), 744.
[19] P. Felsenthal and Others, *Phys. Rev.*, **A139**(1965), 1796.
[20] Y. Ohmori, *J. Phys. D*, **21**(1988), 742.
[21] A. B. Wedding, H. A. Blevin and J. Fletcher, *J. Phys. D*, **18**(1985), 2361.
[22] L. A. Newman and T. A. Detemple, *J. Appl. Phys.*, **47**(1976), 1912.
[23] W. Rozmerski and K. Leja, *J. Phys. D*, **17**(1984), 279.
[24] K. A. Bunting and A. E. D. Heglen, *Int. J. Electron*, **31**(1971), 9.
[25] A. A. Garamon, *J. Phys. D*, **10**(1977), 991.
[26] N. Koutdeon, Le Virr and J. Lucas, *J. Phys. D*, **6**(1973), 1237.
[27] W. H. Kasner and Manfred. A. Biodi, *Phys. Rev.*, **A139**(1965), 317.
[28] P. Mul and J. Wu. McGman, *J. Phys. B*, **14**(1979), 1591.
[29] D. Auerbach, *J. Phys. B*, **12**(19,7), 3797.
[30] H. S. W. massey, E. H. S. Burhor and H. b. Gilbody, Electronic and Ionic Impact Phenomena, Volume II (Second Edition) (Oxford University Press, 1969).
[31] H. S. W. massey and H. B. gilbody, Electronic and Ionic Impact Phenomena, Volume IV (Second Edition) (Oxford Univerisity Press, 1974).
[32] *Raund N. Franklin, Plasma Phenomena in Gas Discharges (Oxford University Press, 1976)*.
[33] H. S. W. massey, Electronic and Ionic Impact Phenomena, Volume II (Second Edition) (Oxford University

Press, 1969).

[34] Essam Nasser, Fundamentals of Gaseous Ionization and Plasma Electronic (Oxford University Press, 1971).

[35] J. Lucas, D. A. Price and J. L. Moruzzi, *J. Phys. D*,**6**(1973), 1503.

[36] John R. Cary, *Phys. Fluids*, **23**(1980), 1005.

[37] Koniehi Owo, *Jpn. J. Appl. Phys.*, **20**(1981), 1515.

[38] D. Prono, B. Ecker and J. Benford, *Phys. Rev. Lett.*, **35**(1975), 438.

[39] J. Cox, *Phys. Fluids*, **13**(1970), 182.

[40] D. A. Hammer, K. A. Gerber and A. W. Ali, *IEEE Trans. on Plasma Science*, **7**(1979), 83.

[41] S. K. Iyyengar and V. K. Rohatgi, *Phys. Fluids*, **B1**(1989), 1860.