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RENORMALIZED HALL CONDUCTIVITY FOR MAGNETIC ALLOYS*

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By means of the renormalized vertex procedure for the motion Green's function developed by the authors, the vertex function of magnetic alloys, based on the *s-d* exchange interaction, is solved exactly and the corresponding Hall conductivity tensors are obtained. It is found that the renormalized modification of the conductivity is very important in the cases of not too weak external magnetic field and slow relaxation time.

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I. INTRODUCTION

Since the discovery of Kondo effect the investigation of magnetic alloys containing dilute impurities have been one of the frontier subjects in the condensed physics and materials science[1, 2]. It is very important to study the transport quantities in order to explain the microscopic mechanism of dilute magnetic alloys. In general, the transport quantities in an applied electromagnetic field, such as Hall conductivity and susceptibility, are calculated by using Kubo-type formula. These results exhibit partly the response of transport quantities to an external field[3, 4]. Hänsch and Mahan developed the Kadanoff-Baym theory for many-particle systems[3] and established another transport equation[4, 5]. They used the transport equation to treat the electron-phonon interaction system. Their results are equivalent (or superior) to those given by the modified Kubo formula and Feynman graphic technique.

Recently, a set of self-consistent iterative recurrent methods about the vertex function has been developed by us for dilute magnetic alloys in an applied field, using the Mahan-Hänsch quantum transport equation to calculate the conductivity[6, 7]. We calculated corresponding conductivity and resistivity in the cases of vanishing and weak fields, respectively. In our previous work[7], we considered the *s-d* exchange-interaction between conduction electrons and localized moments in dilute magnetic alloys. The Hamiltonian reads

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$$H=\sum_{k,\sigma}\omega_{k\sigma}C_{k\sigma}^{+}C_{k\sigma}+\sum_{\sigma}\omega_{l\sigma}C_{l\sigma}^{+}C_{l\sigma}+H_{s\text{-}d} \tag{1}$$

and

$$H_{s\text{-}d}=\sum_{k,k'}(J_{k,k'}/N_0)[S_l^z(C_{k\uparrow}^{+}C_{k'\uparrow}-C_{k\downarrow}^{+}C_{k'\downarrow})+S_l^{+}C_{k\downarrow}^{+}C_{k'\uparrow}+S_l^{-}C_{k\uparrow}^{+}C_{k'\downarrow}], \tag{2}$$

where S_l^z is the z component of the spin operator associated with the impurity, $S_l^{+}=C_{l\uparrow}^{+}C_{l\downarrow}$ and $S_l^{-}=C_{l\downarrow}^{+}C_{l\uparrow}$ are raising and lowering operators, respectively, for the localized spin, and $J_{k,k'}$ denotes the strength of the exchange interaction which is assumed to be independent of k and k'. N_0 stands for the number of atoms of the system.

When an external magnetic field is not too weak and the relaxation time of the system is appropriate, the small parameter $h=\dfrac{He}{2c\Gamma}$ assumed in our previous work is of the order of one. Therefore, it is necessary to consider the contribution of the renormalized magnetic vertex function to the Hall conductivity. In this paper, it is our main goal to solve exactly the vertex function and evaluate the Hall conductivity.

II. HÄNSCH-MAHAN QUANTUM TRANSPORT EQUATION

Following Hänsch and Mahan[4], we write the transport equation for a steady-state system as

$$G^{>}\Sigma^{<}-\Sigma^{>}G^{<}=e\boldsymbol{E}\cdot\left[(\boldsymbol{V}_q+\nabla_q\mathrm{Re}\Sigma^r)\mathrm{Im}\Sigma^r+\nabla_q\mathrm{Im}\Sigma^r\right]A^2\frac{\partial n(\omega)}{\partial\omega}$$
$$+\mathrm{i}(e/c)\boldsymbol{V}_q\cdot(\boldsymbol{H}\times\nabla_qG^{<})-2n(\omega)(e/c)A^2\mathrm{Re}G^r\boldsymbol{V}_q\cdot(\boldsymbol{H}\times\nabla_q\mathrm{Re}\Sigma^r)\mathrm{Im}\Sigma^r, \tag{3}$$

where $G^{<}$ (or $G^{>}$) is the Kadanoff-Baym physical Green's function. $\Sigma^{<}$ (or $\Sigma^{>}$) stands for corresponding self-energies. $\Sigma^{>}$ represents the rate of conduction electrons scattered to leave from the configuration of energy ω and quasi-momentum $\boldsymbol{Q}$

$$\boldsymbol{Q}=\boldsymbol{q}+\frac{e}{2c}\boldsymbol{R}\times\boldsymbol{H}+e\boldsymbol{E}T_c\ , \tag{4}$$

where we take $\hbar=1$. $\Sigma^{<}$ is regarded as the rate of conduction electrons scattered to arrive at the corresponding energy-momentum configuration $(\omega, \boldsymbol{Q})$. The function $n(\omega)$ stands for an equilibrium Fermi distribution. $\boldsymbol{E}$ and $\boldsymbol{H}$ denote applied electric field and magnetic field, respectively. $\boldsymbol{V}_q$ is given by $\nabla_q\varepsilon_q$ and it denotes the group-velocity of electrons. Superscript r (or a) represents the retarded (or advanced) Green's function. In addition, the spectral function $A(\boldsymbol{R}, T_c; \boldsymbol{K}, \omega)$ is expressed as

$$A(\boldsymbol{R}, T_c; \boldsymbol{K}, \omega)=\frac{2\mathrm{Im}\Sigma^r(\boldsymbol{R}, T_c; \boldsymbol{K}, \omega)}{[\omega-\varepsilon_q-\mathrm{Re}\Sigma^r]^2+[\mathrm{Im}\Sigma^r(\boldsymbol{R}, T_c; \boldsymbol{K}, \omega)]^2}\ , \tag{5}$$

where $\boldsymbol{R}=(r_1+r_2)/2$ and $T_c=(t_1+t_2)/2$ denote the coordinates in the center of mass-time[5]. For simplicity, we assume that the system to be treated has a spherical Fermi surface, and we take also the standard approximation in which retarded self-energy has a negligible dependence upon the vector $\boldsymbol{q}$, but it depends strongly on the energy. Therefore, we can neglect all of the derivatives of self-energy with respect to the wave vector and rewrite the Hänsch-Mahan Eq. (3) as

$$G^{>}\Sigma^{<}-\Sigma^{>}G^{<}=\mathrm{i}(e/c)\,\boldsymbol{V}_{q}\cdot\,(\boldsymbol{H}\times\nabla_{q}G^{<})-e\boldsymbol{E}\cdot\,\boldsymbol{V}_{q}\Gamma(\omega)A^{2}\frac{\partial n(\omega)}{\partial\omega}\,, \tag{6}$$

where the shorthand $\Gamma(\omega)=-\mathrm{Im}\Sigma^{r}$ is used. The Green's functions $G^{<}(\boldsymbol{Q},\omega)$ and $G^{>}(\boldsymbol{Q},\omega)$ within the system of the Kadanoff-Baym-Mahan near the equilibrium will satisfy the following equations:

$$G^{>}=G^{<}-\mathrm{i}A\,, \tag{7}$$

and

$$G^{<}=\mathrm{i}n(\omega)A-\mathrm{i}\frac{\partial n(\omega)}{\partial\omega}\,\gamma(\boldsymbol{Q},\omega)\,, \tag{8}$$

where the first term of the right-hand side in Eq. (8) denotes the equilibrium quantities of the system. The function $\gamma(\boldsymbol{Q},\omega)$ in Eq. (8) represents an unknown vertex function. Once it is solved, we can write the electric current density $\boldsymbol{J}_{c}$ as

$$\boldsymbol{J}_{c}=e^{2}\int\frac{\mathrm{d}^{3}\boldsymbol{q}}{(2\pi)^{3}}\int\frac{\mathrm{d}\omega}{2\pi}\left[-\frac{\partial n(\omega)}{\partial\omega}\right]\boldsymbol{V}_{q}\gamma(\boldsymbol{Q},\omega)\,. \tag{9}$$

To evaluate the vertex function, it is necessary to determine the physical self-energy functions $\Sigma^{\gtrless}(\boldsymbol{Q},\omega)$. In general, the physical self-energies for different models have different forms. Therefore, they provide various vertex equations.

III. PHYSICAL SELF-ENERGY AND VERTEX EQUATION

Following our previous work[7, 8], the relation between the physical self-energy and Green's function of magnetic alloys can be specialized as

$$\Sigma^{>}(\boldsymbol{Q},\omega)=-\mathrm{i}n_{i}(J/N_{0})S_{i}^{z}+n_{i}\sum_{\boldsymbol{q}}\int\mathrm{d}\omega'\frac{(J/N_{0})^{2}}{\omega-\omega'}\times[S_{i}^{-}S_{i}^{+}+(S_{i}^{z})^{2}]G^{>}(\boldsymbol{Q}',\omega')\,, \tag{10}$$

and

$$\Sigma^{<}(\boldsymbol{Q},\omega)=\mathrm{i}n_{i}(J/N_{0})S_{i}^{z}+n_{i}\sum_{\boldsymbol{q}}\int\mathrm{d}\omega'\frac{(J/N_{0})^{2}}{\omega-\omega'}\times[S_{i}^{+}S_{i}^{-}+(S_{i}^{z})^{2}]G^{<}(\boldsymbol{Q}',\omega')\,, \tag{11}$$

where n_{i} is the concentration of impurities. Note that the self-energy expressions, Eqs. (10) and (11), are only a formal Born approximation because the Green's functions $G^{\gtrless}(\boldsymbol{Q},\omega)$ include higher order terms of (J/N_{0}). Substituting Eqs. (7), (8), (10) and (11) into Eq. (6), after making some arrangements, we obtain an identical vertex equation

$$\gamma(\boldsymbol{Q},\omega)=\frac{1}{2}A^{2}e\boldsymbol{E}\cdot\,\boldsymbol{V}_{q}-\boldsymbol{V}_{q}\cdot[\boldsymbol{h}\times\nabla_{q}\gamma(\boldsymbol{Q},\omega)]$$
$$-\frac{An_{i}}{2\Gamma(\omega)}[2n(\omega)S_{i}^{z}+S(S+1)+S_{i}^{z}]\sum_{\boldsymbol{q}}\int\mathrm{d}\omega'\frac{(J/N_{0})^{2}}{\omega-\omega'}\,\gamma(\boldsymbol{Q}',\omega'\omega'), \tag{12}$$

and

$$\boldsymbol{h}=\frac{\boldsymbol{H}e}{2\Gamma(\omega)c}\,, \tag{13}$$

which is a dimensionless reduced magnetic field (assuming that the magnetic field points in

z-direction). For not too weak magnetic field and slow relaxation time (or bigger $\Gamma(\omega)$), one has $h \leqslant 1$. Therefore, it can be treated as an expansion parameter. In addition, the relation between the group-velocity and the quasi-wave vector of electrons in weak fields must satisfy

$$V_q = \frac{Q}{m} , \tag{14}$$

where m is the effective mass of electron. Thus, the vertex function can be expanded in the order of the parameter h

$$\gamma(Q, \omega) = \sum_{n=0}^{\infty} h^n \gamma^{(n)}(Q, \omega). \tag{15}$$

Substituting Eq. (15) into Eq. (12) and collecting the contributions of the same order, we obtain

$$\gamma^{(0)}(Q, \omega) = \frac{1}{2} (e/m) A^2 E \cdot Q - \frac{An_i}{2\Gamma(\omega)} [2n(\omega) S_i^z + S(S+1) + S_i^z] \sum_q \int d\omega' \frac{(J/N_0)^2}{\omega - \omega'} \gamma^{(0)}(Q', \omega') , \tag{16}$$

$$h\gamma^{(1)}(Q, \omega) = -V_q \cdot [h \times \nabla_q \gamma^{(0)}(Q, \omega)] - \frac{An_i}{2\Gamma(\omega)} [2n(\omega) S_i^z + S(S+1) + S_i^z] \sum_q \int d\omega' \frac{(J/N_0)^2}{\omega - \omega'} h\gamma^{(1)}(Q', \omega') , \tag{17}$$

and

$$h\gamma^{(n)}(Q, \omega) = -V_q \cdot [h \times \nabla_q \gamma^{(n-1)}(Q, \omega)] - \frac{An_i}{2\Gamma(\omega)} [2n(\omega) S_i^z + S(S+1) + S_i^z] \sum_q \int d\omega' \frac{(J/N_0)^2}{\omega - \omega'} h\gamma^{(n-1)}(Q', \omega') . \tag{18}$$

The vertex function $\gamma^{(n)}(Q, \omega)$ in Eqs. (16)–(18) are coupled order by order in the order of the parameter h. The zeroth and the first order of the vertex function in Eq. (15) were evaluated in Refs. [7] and [8].

IV. EXACT SOLUTION OF THE VERTEX FUNCTION

In order to explain how to get an exact solution of the magnetic vertex function, let us review briefly our previous work. Obviously, $\gamma^{(0)}(Q, \omega)$ in Eq. (16) is corresponding to zero-field vertex function. Set

$$\gamma^{(0)}(Q, \omega) = (e/m) A(Q, \omega) \Lambda(Q, \omega) E \cdot Q , \tag{19}$$

where $\Lambda(Q, \omega)$ is another unknown function. Substituting Eq. (19) into Eq. (16), we have

$$\Lambda(Q, \omega) = \frac{1}{2} A(Q, \omega) + \frac{n_i}{2\Gamma(\omega)} [2n(\omega) S_i^z + S(S+1) + S_i^z] \sum_q \int d\omega' \frac{(J/N_0)^2}{\omega - \omega'} \times \frac{Q \cdot (Q+q)}{Q^2} A(Q', \omega') \Lambda(Q', \omega') . \tag{20}$$

To replace the summation over q in Eq. (20) by the corresponding integral, we set $\nu=\cos\theta$, in which θ represents the angle between $Q+q$ and Q, with $Q'^2=(Q+q)^2$, we then obtain

$$\int d^3q=\frac{2\pi}{Q}\int dq\ q\int_{Q-q}^{Q+q}dq'\ Q'=\frac{1}{V_q}\int dq\ q\int_{(Q-q)^2/m}^{(Q+q)^2/m}d\omega'.$$

Since the main contribution of the integration at low temperatures comes from electrons near the Fermi sphere, one has $Q\cong k_F\cong Q'$. Furthermore, the coupling constant is assumed as

$$J_{k,k'}=\begin{cases} J & |\omega-\omega_F|\leqslant D \\ 0 & \text{otherwise}\end{cases},$$

and one takes $V_q=V_F$. Similarly, we can easily prove

$$\frac{Q\cdot(Q+q)}{Q^2}=1+\frac{qv}{Q}=1-\frac{q^2}{2k_F^2}+O\left(\frac{\omega-\omega_F}{\omega_F},\ \frac{\omega'-\omega_F}{\omega_F}\right). \tag{21}$$

So, one has

$$\int d^3q\frac{Q\cdot(Q+q)}{Q^2}=\frac{1}{V_F}\int dq\ q\int_{\omega_F-D}^{\omega_F}d\omega'\left[1-\frac{q^2}{2k_F^2}\right]. \tag{22}$$

Furthermore, $\Lambda(Q,\omega)$ in Eq. (20) can be rewritten as

$$\Lambda(Q,\omega)=\frac{1}{2}A(Q,\omega)-\frac{n_i[2n(\omega)S_i^z+S(S+1)+S_i^z]}{2(2\pi)^3V_F\Gamma(\omega)}\int dq\ q\left[1-\frac{q^2}{2k_F^2}\right]$$
$$\times\int_{\omega_F-D}^{\omega_F}d\omega'\frac{(J/N_0)^2}{\omega-\omega'}A(Q',\omega')\Lambda(Q',\omega'), \tag{23}$$

which can be solved by using the self-consistent iterative method. Setting an integral kernel as

$$K(\omega,\omega')=-\frac{n_i(J/N_0)^2[2n(\omega)S_i^z+S(S+1)+S_i^z]}{(2\pi)^3V_F\Gamma(\omega)}\times\int dq\ q\left[1-\frac{q^2}{2k_F^2}\right]\frac{A(Q',\omega')}{\omega-\omega'}, \tag{24}$$

by iteration, $\Lambda(Q,\omega)$ in Eq. (23) takes the following form :

$$\Lambda(Q,\omega)=\frac{1}{2}A(Q,\omega)+\frac{1}{2}\sum_{n=1}^{\infty}\prod_{i=1}^{n}\int d\omega_iK(\omega_{i-1},\omega_i)A(\omega_n). \tag{25}$$

Using the approximation of slow varying spectral functions[9], the n-fold coupled integral of the kernel can be simplified as

$$\prod_{i=1}^{n}\int d\omega_iK(\omega_{i-1},\omega_i)A(\omega_n)=\left[\int d\omega'K(\omega,\omega')\right]^nA(\omega_F). \tag{26}$$

Substituting Eq. (26) into Eq. (25), after some tedious treatments, we obtain

$$\Lambda(Q,\omega)=\frac{1}{2}A(Q,\omega)+\frac{n_i(J/N_0)^2[2n(\omega)S_i^z+S(S+1)+S_i^z]}{2(2\pi)^3V_F\Gamma(\omega)}$$
$$\times\ln[(\omega-\omega_F)/D]\int dq\ q\left[1-\frac{q^2}{2k_F^2}\right]\left[A(Q',\omega_F)\right]^2$$

$$\times\left[1-\frac{n_i(J/N_0)^2[2n(\omega)S_i^z+S(S+1)+S_i^z]}{2(2\pi)^3V_F\Gamma(\omega)}\right.$$

$$\left.\times\ln[(\omega-\omega_F)/D]\int dq\ q\left[1-\frac{q^2}{2k_F^2}\right]A(\boldsymbol{Q}',\ \omega_F)\right]^{-1}. \tag{27}$$

Inserting Eq. (19) into Eq. (17), one has

$$h\gamma^{(1)}(\boldsymbol{Q},\ \omega)=\left(\frac{e}{m^2}\right)A(\boldsymbol{Q},\ \omega)\Lambda(\boldsymbol{Q},\ \omega)\boldsymbol{E}\cdot[\boldsymbol{h}\times\boldsymbol{Q}]$$

$$-\frac{An_i}{2\Gamma(\omega)}[2n(\omega)S_i^z+S(S+1)+S_i^z]\sum_{\boldsymbol{q}}\int d\omega'\frac{(J/N_0)^2}{\omega-\omega'}h\gamma^{(1)}(\boldsymbol{Q}',\ \omega'). \tag{28}$$

To simplify Eq. (28), we set

$$h\gamma^{(1)}(\boldsymbol{Q},\ \omega)=\left(\frac{e}{m^2}\right)A(\boldsymbol{Q},\ \omega)\eta(\boldsymbol{Q},\ \omega)\boldsymbol{E}\cdot[\boldsymbol{h}\times\boldsymbol{Q}], \tag{29}$$

where $\eta(\boldsymbol{Q},\ \omega)$ is an unknown function. Inserting Eq. (29) into Eq. (28), we obtain

$$\eta(\boldsymbol{Q},\ \omega)=\Lambda(\boldsymbol{Q},\ \omega)-\frac{n_i[2n(\omega)S_i^z+S(S+1)+S_i^z]}{2(2\pi)^3V_F\Gamma(\omega)}\int dq\ q\left[1-\frac{q^2}{2k_F^2}\right]$$

$$\times\int_{\omega_F-D}^{\omega_F}d\omega'\frac{(J/N_0)^2}{\omega-\omega'}A(\boldsymbol{Q}',\ \omega')\eta(\boldsymbol{Q}',\ \omega'). \tag{30}$$

Next, inserting Eq. (27) into Eq. (30) and taking only the lower order in the iteration, the self-consistent iterative equation (30) can be changed into

$$\eta(\boldsymbol{Q},\ \omega)=\frac{1}{2}A(\boldsymbol{Q},\ \omega)-\frac{n_i[2n(\omega)S_i^z+S(S+1)+S_i^z]}{2(2\pi)^3V_F\Gamma(\omega)}\int dq\ q\left[1-\frac{q^2}{2k_F^2}\right]$$

$$\times\int_{\omega_F-D}^{\omega_F}d\omega'\frac{(J/N_0)^2}{\omega-\omega'}A(\boldsymbol{Q}',\ \omega')\eta(\boldsymbol{Q}',\ \omega'). \tag{31}$$

Comparing Eq. (31) with Eq. (23), we get $\eta(\boldsymbol{Q},\ \omega)=\Lambda(\boldsymbol{Q},\ \omega)$. In the same way, we obtain the vertex function of arbitrary order:

$$\gamma^{(n)}(\boldsymbol{Q},\ \omega)=\begin{cases}[-1]^{n/2}A(\boldsymbol{Q},\ \omega)\Lambda(\boldsymbol{Q},\ \omega)(e/m^2)\boldsymbol{E}\cdot\boldsymbol{Q}, & n=\text{even};\\ [-1]^{(n-1)/2}A(\boldsymbol{Q},\ \omega)\Lambda(\boldsymbol{Q},\ \omega)(e/m^2)\boldsymbol{E}\cdot(\boldsymbol{h}\times\boldsymbol{Q}), & n=\text{odd}.\end{cases} \tag{32}$$

Substituting Eq. (32) into Eq. (15), we further obtain the exact solution of the vertex function

$$\gamma(\boldsymbol{Q},\ \omega)=\left(\frac{e}{m(1+h^2)}\right)A(\boldsymbol{Q},\ \omega)\Lambda(\boldsymbol{Q},\ \omega)\boldsymbol{E}\cdot\boldsymbol{Q}$$

$$+\left(\frac{e}{m^2(1+h^2)}\right)A(\boldsymbol{Q},\ \omega)\eta(\boldsymbol{Q},\ \omega)\boldsymbol{E}\cdot[\boldsymbol{h}\times\boldsymbol{Q}]. \tag{33}$$

By comparing (33) with (34) in Ref. [8], it is easy to find that the renormalized vertex function is $1/(1+h^2)$ times less than that before renormalization.

V. HALL CONDUCTIVITY AND KONDO TEMPERATURE

Substituting the first term on the right-hand side of Eq. (33) into the current density in Eq. (9), we have

$$\sigma_{ij}^{(1)}=\frac{e^2}{m^2}\int\frac{d^3q}{(2\pi)^3}\int\frac{d\omega}{2\pi}\left[-\frac{\partial n(\omega)}{\partial\omega}\right]Q_iQ_jA(Q,\omega)\Lambda(Q,\omega)\left[\frac{1}{1+h^2}\right], \tag{34}$$

where $i, j=x, y, z$. With the help of Eq. (34), we can arrive at[7, 8]

$$\sigma^{(1)}=\frac{e^2n_0}{m}\int d\omega\left[-\frac{\partial n(\omega)}{\partial\omega}\right]\left[\frac{1}{\Gamma(\omega)(1+h^2)}\right]\times\left[1-M+\frac{M}{1-N\ln[(\omega-\omega_F)/D]}\right], \tag{35}$$

with

$$M=\int dq\ q\left[1-\frac{q^2}{2k_F^2}\right]\left[A(Q',\ \omega_F)\right]^2\times\left[2\Gamma(\omega)\int dq\ q\left[1-\frac{q^2}{2k_F^2}\right]A(Q',\ \omega_F)\right]^{-1} \tag{36}$$

and

$$N=\frac{n_i(J/N_0)^2[2n(\omega)S_i^z+S(S+1)+S_i^z]}{2(2\pi)^3V_F\Gamma(\omega)}\int dq\ q\left[1-\frac{q^2}{2k_F^2}\right]A(Q',\ \omega_F)\ . \tag{37}$$

Considering that $\Gamma(\omega)$ is a slow varying function near the Fermi sphere, and assuming $\Gamma(\omega)\cong\Gamma(\omega_F)$ in the reduced magnetic field h, we can take the renormalized factor $1/(1+h^2)$ in Eq. (35) out of the integral symbol. In addition, by defining another dimensionless factor $\zeta=(\omega-\omega_F)/T$, the formula (35) can be rewritten as

$$\sigma^{(1)}=\frac{\sigma^{(0)}}{1+h^2}\ , \tag{38}$$

and

$$\sigma^{(0)}=C+\frac{X}{1-Z\ln(T/D)}\ , \tag{39}$$

with

$$C=\frac{e^2n_0}{m}\int d\xi\left[-\frac{\partial n(\xi)}{\partial\xi}\right]\left[\frac{1-M}{\Gamma(\omega)}\right]. \tag{40}$$

$$X=\frac{e^2n_0}{m}\int d\xi\left[-\frac{\partial n(\xi)}{\partial\xi}\right]\frac{M}{(1-N\ln\xi)\Gamma(\xi)}\ , \tag{41}$$

$$Z=\frac{e^2n_0}{Xm}\int d\xi\left[-\frac{\partial n(\xi)}{\partial\xi}\right]\frac{MN}{(1-N\ln\xi)^2\Gamma(\xi)}\ . \tag{42}$$

With similar steps, substituting the second term on the right-hand side of Eq. (33) into Eq. (9) and using the symmetry of the integral on the Fermi sphere surface, we have

$$\sigma_{xy}^{(2)}=\sigma_{xz}^{(2)}=\sigma_{yz}^{(2)}=\frac{1}{3}\left[\frac{h}{1+h^2}\right]\sigma^{(0)}, \tag{43}$$

and

$$\sigma_{xx}^{(2)}=\sigma_{yy}^{(2)}=\sigma_{zz}^{(2)}=0 . \tag{44}$$

Note that the conductivity, Eq. (39), is of diagonal with off-diagonal elements equal zero, whereas it is just the opposite in Eqs. (43) and (44). Considering two parts of the contributions of vertex function to the Hall conductivity tensors, we obtain

$$\sigma_{xx}=\frac{1}{3}\left[\frac{1}{1+h^2}\right]\sigma^{(0)}, \tag{45}$$

and

$$\sigma_{xy}=\frac{1}{3}\left[\frac{h}{1+h^2}\right]\sigma^{(0)} . \tag{46}$$

With the help of Eqs. (45) and (46), the conductivity tensors with different limits can be obtained as follows:

(I) $h=0$, $\sigma_{xx}=\frac{1}{3}\sigma^{(0)}$, $\sigma_{xy}=0$;

(II) $h\ll 1$, $\sigma_{xx}=\frac{1}{3}\sigma^{(0)}$, $\sigma_{xy}=\frac{h}{3}\sigma^{(0)}$;

(III) $h=1$, $\sigma_{xx}=\frac{1}{6}\sigma^{(0)}=\sigma_{xy}$.

Here (I) and (II) correspond to the results in Refs. [7] and [8], respectively. It is shown in (III) that the renormalized modification of the conductivity is very important for the slower relexation system. The so-called Kondo temperature can also be derived from Eq. (39)

$$T_K=D\exp\left(\frac{1}{Z}\right) . \tag{47}$$

At the absolute zero temperature, the conductivity tensors take finite values. This shows that the general divergence associated with the zero temperature is removed in our calculation.

In conclusion, we use the self-consistent iterative method to calculate the renormalized vertex function and obtain the exact Hall conductivity tensors. We find that the conductivity is $(1+h^2)^{-1}$ times less than that before the renormalized modification[8].

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REFERENCES

[1] J. Kondo, *Prog. Theor. Phys.* (*Kyoto*), 32(1964). 37.

[2] N. Andrei, *Rev. Mod. Phys.*, **2**(1983), 331.
[3] L. P. Kadanoff and G. Baym, Quantum Statistical Mechanics (Benjamin, New York, 1962).
[4] W. Hänsch and G. D. Mahan, *Phys. Rev.*, **B28**(1983), 1902.
[5] W. Hänsch and G. D. Mahan, *Phys. Rev.*, **B28**(1983), 1886.
[6] Yi Lin and Yao Kai-lun, *Physica*, **B165 & 166**(1990), 259.
[7] Yao Kai-lun and Yi Lin, *Physica*, **B165 & 166**(1990), 261.
[8] Yi Lin and Yao Kai-lun, *Phys. Rev.*, **B46**(1992), 1202; Yao Kai-lun and Yi Lin, *Phys. Rev.*, **B44**(1991), 190.
[9] J. Ziman, Electrons and Phonons (Clarendon, Oxford, 1960).