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THE TRANSMISSION PROPERTIES OF SOLITON ON THE CW BACKGROUND NEAR THE ZERO-DISPERSION WAVELENGTH*

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By using the perturbation theory and small-amplitude approximation with the self-induced Raman effect and the loss as well as gain of the fiber taken into account, the transmission properties of solitons in the normal dispersion region near the zero-group-dispersion wavelength are analytically investigated in this paper. The result shows that not only the small-amplitude dark soliton but also the small-amplitude bright soliton may exist in the normal dispersion region near the zero-group-dispersion point. Their stabilities depend on the propagating direction and the region in which they are situated in the parameter-space. The conversion between bright and dark solitons may be realized by controlling the relative intensities of gain and loss along the fiber. This may be applied to the logic or arithmetic devices in the integrated optical circuit.

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I. INTRODUCTION

In 1973, Hasegawa and Tappert[1] predicted that there exist bright or dark solitons in the negative or positive group-velocity dispersion. For the bright solitons the prediction was confirmed by the pioneer experiment of Mollenauer *et al.* in earlier time[2], so its theortical investigation and the application of it in communication have been well developed. However, the experimental investigations of dark solitons started only recently[3,4] because of the difficulty of generating appropriate dark pulses. Compared with the bright solitons, the interaction between dark solitons is weaker and the effect of background noise is smaller[5]. But the application of dark solitons in communication is not so optimistic as the bright solitons in present time due to its complex exprimental techiques. However, it is interesting in nonlinear dynamics of optical media, since some exprimental results of nonlinear dynamics for normal dispersive optical fiber may be treated with the help of dark-soliton phenomenology, including the small-amplitude bright soliton described later in this paper.

In this paper, the soltion phenomenon near the zero-group-dispersion(ZGD) point is analytically investigated by Raman spectra with loss and gain of the fiber taken into acount. Some experiments and theories have shown that the stimulated Raman effect produces a

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frequency shift for bright optical solitons[6-9], whereas the influence of self-induced Raman scattering on dark solitons is more destructive[4,10]. The results of the present work predict that, in the normal dispersion region near the ZGD point and on a CW pedestal (or CW background), there may exist small-amplitude bright solitons besides small-amplitude dark solitons. Their stabilities depend on the propagating direction and the region in which they are situated in the parameter-space.

II. SMALL-AMPLITUDE SOLITONS ON CW PEDESTAL

Near the ZGD point, because the second-order dispersion is getting smaller, the third-order dispersion is getting as important as the former. The transmission equation of soliton in the optical fiber with self-induced Raman effect, the loss and gain is[11]

$$i\frac{\partial u}{\partial \xi} - \sigma'\frac{\partial^2 u}{\partial \tau^2} - i\beta'\frac{\partial^3 u}{\partial t^3} + i(\gamma - g)u$$
$$+\left(1+\frac{i}{\omega_0 T}\frac{\partial}{\partial \tau}\right)\left[\alpha' |u|^2 u + (1-\alpha')u\int_0^{\infty} f_R(\tau-\tau')|u(\tau')|^2 d\tau'\right] = 0, \tag{1}$$

where $f_R(\tau-\tau')$ is the Raman response function, which is extremely short for pico-second (ps) solitons in fused silica, and according to Fig. 3 in Ref. [12] the response time of $f_R(t-t')$ is about 100 – 200 fs. Hence, an approximation for the response function can be successfully used to model the Raman contribution in the nonlinear schrödinger (NLS) equation by a local term $\frac{d\delta(t)}{dt}$, where $\delta(t)$ is the Dirac function. The part $\frac{i}{\omega_0 T}\frac{\partial}{\partial \tau}$ in the last term of Eq. (1) can be neglected for ps soliton pulses, then Eq. (1) can be rewriten as

$$i\frac{\partial u}{\partial x} - \sigma\frac{\partial^2 u}{\partial t^2} + 2|u|^2 u = \alpha u\frac{\partial |u|^2}{\partial t} - i\gamma_G u + i\beta\frac{\partial^3 u}{\partial t^3}, \tag{2}$$

where $\gamma_G \propto (\gamma - g)$. When $\beta = \gamma_G = \alpha = 0$, Eq. (2) has a dark soliton solution[13]

$$u(x, t) = u_0\frac{(\lambda - iv)^2 + \exp Z}{1+\exp Z}\exp(i2u_0^2 x), \tag{3}$$

$$Z = 2vu_0(t - t_0 - 2\lambda u_0 x),\ \lambda = 1 - v^2. \tag{4}$$

Hence, unlike the bright soliton (two parameters) in anomalous dispersion region, the dark soliton is a single-parameter solution. When $\lambda = 0$, Eq. (3) results in a fundamental dark soliton

$$u(x, t) = u_0 \tanh(u_0 t)\exp(i2u_0^2 x); \tag{5}$$

when $v^2 \ll 1$, Eq. (3) gives a small-amplitude dark soliton

$$u(x, t) = [u_0 - \frac{1}{2}u_0 v^2 \operatorname{sech}^2(Z/2)]\exp[i2u_0^2 x + i\varphi(x, t)], \tag{6}$$

$$\varphi(x, t) = -2v/(1+e^Z), \tag{7}$$

$$Z = 2vu_0[t - c(1 - \frac{1}{2}v^2)x],\ c = \pm 2u_0. \tag{8}$$

Note that, the term sech^2 in Eq. (6) is the single-soliton solution of KdV equation. Probably for this reason, Kivshar[10] discovered the relationship between the small-amplitude soliton of NLS equation and KdV equation. In the case of $\beta\neq 0$, $\gamma_G\neq 0$, and $\alpha\neq 0$, we find a small-amplitude-soliton solution in the following form:

$$u(x,\ t)=[u_0+a(x,\ t)]\mathrm{e}^{2u_0^2x-i\varphi(x,\ t)}. \tag{9}$$

Substituting Eq. (9) into Eq. (2), we obtain

$$\frac{\partial a}{\partial x}-\sigma u_0\frac{\partial^2\varphi}{\partial t^2}-\sigma(2\frac{\partial\varphi}{\partial t}\frac{\partial a}{\partial t}+\alpha\frac{\partial^2\varphi}{\partial t^2})$$
$$=\gamma_G(u_0+a)+\beta[(\frac{\partial^3 a}{\partial t^3}-3u_0\frac{\partial\varphi}{\partial t}\frac{\partial^2\varphi}{\partial t^2})-(3a\frac{\partial\varphi}{\partial t}\frac{\partial^2\varphi}{\partial t^2}+3(\frac{\partial\varphi}{\partial t})\frac{2\partial a}{\partial t})], \tag{10}$$

$$u_0(\frac{\partial\varphi}{\partial x}-4u_0a)+[a\frac{\partial\varphi}{\partial x}-\sigma u_0(\frac{\partial\varphi}{\partial t})^2+\sigma\frac{\partial^2 a}{\partial t^2}-6u_0a^2]$$
$$=-2\alpha\frac{\partial a}{\partial t}(u_0+a)^2+[2a^3+\sigma a(\frac{\partial\varphi}{\partial t})^2]$$
$$+\beta\{(u_0+a)\frac{\partial^3\varphi}{\partial t^3}+[3\frac{\partial a}{\partial t}\frac{\partial^2\varphi}{\partial t^2}+3\frac{\partial\varphi}{\partial t}\frac{\partial^2 a}{\partial t^2}-(u_0+a)(\frac{\partial\varphi}{\partial t})^3]\}. \tag{11}$$

In the following, we will find the propagating solution in the form of perturbative series and make scales supposition as below:

$$\begin{aligned}&u_0=O(1),\ \beta=O(1),\ \sigma=O(1),\\&\gamma_G=O(\varepsilon^6),\ \alpha=O(\varepsilon^2),\\&a=\varepsilon^2a_0+\varepsilon^4a_1+\cdots,\\&\varphi=\varepsilon\varphi_0+\varepsilon^3\varphi_1+\cdots,\\&\tau=(t-cx)\varepsilon,\ z=\varepsilon^3x.\end{aligned} \tag{12}$$

Substituting Eq. (12) into Eqs. (10) and (11), we obtain

$$c\frac{\partial a_0}{\partial\tau}+\sigma u_0\frac{\partial^2\varphi_0}{\partial\tau^2}=0, \tag{13}$$

$$(c\frac{\partial a_1}{\partial\tau}+\tau u_0\frac{\partial^2\varphi_1}{\partial\tau^2})+\sigma(2\frac{\partial\varphi_0}{\partial\tau}\frac{\partial a_0}{\partial\tau}+a_0\frac{\partial^2\varphi_0}{\partial\tau^2})-\frac{\partial a_0}{\partial z}$$
$$=\gamma_Gu_0-\beta[\frac{\partial^3 a_0}{\partial t^3}-3u_0\frac{\partial\varphi_0}{\partial\tau}\cdot\frac{\partial^2\varphi_0}{\partial\tau^2}], \tag{14}$$

$$u_0(c\frac{\partial\varphi_0}{\partial\tau}+4u_0a_0)=0, \tag{15}$$

$$u_0(c\frac{\partial\varphi_1}{\partial\tau}+4u_0a_1)+ca_0\frac{\partial\varphi_0}{\partial\tau}+\sigma u_0(\frac{\partial\varphi_0}{\partial\tau})^2-\sigma\frac{\partial^2 a_0}{\partial\tau^2}+6u_0a_0^2-u_0\frac{\partial\varphi_0}{\partial z}$$
$$=2\alpha u_0^2\frac{\partial a_0}{\partial\tau}-\beta u_0\frac{\partial^3\varphi_0}{\partial\tau^3}, \tag{16}$$

where we have set $\varepsilon=1$. The solvable condition for Eqs. (13) and (15) is

$$c^2=4\sigma u_0^2, \tag{17}$$

which results in

$$\frac{\partial\varphi_0}{\partial\tau}=-\frac{4u_0}{c}a_0=-\frac{c}{u_0\sigma}a_0. \tag{18}$$

Equation (17) means that solution (9) is only valid for normal dispersion region ($\sigma>0$). Substituting Eq. (18) into Eqs. (14) and (16), and combining them together yield

$$A_1\frac{\partial a_0}{\partial z}+A_2a_0\frac{\partial a_0}{\partial\tau}-\frac{\partial^3a_0}{\partial\tau^3}=-\varepsilon p(a_0), \tag{19}$$

$$A_1=2c/(2\beta c+\sigma^2), \tag{20}$$

$$A_2=12u_0(2\sigma^2+c\beta)/[\sigma(2\beta c+\sigma^2)], \tag{21}$$

$$\varepsilon p(a_0)=[\gamma_G cu_0-2\alpha u_0^2\sigma\frac{\partial^2a_0}{\partial\tau^2}]/(2\beta c+\sigma^2). \tag{22}$$

By transformation of

$$w=A_2a_0/6,\quad y=-z/A_1, \tag{23}$$

the left side of Eq. (19) can be reduced to the form of standard KdV type

$$\frac{\partial w}{\partial y}-6w\frac{\partial w}{\partial\tau}+\frac{\partial^3w}{\partial\tau^3}=\varepsilon\bar{P}(w), \tag{24}$$

with

$$\varepsilon\bar{P}(w)=B_1-B_2\frac{\partial^2w}{\partial\tau^2}, \tag{25}$$

$$B_1=A_2\gamma_G cu_0/6(2\beta c+\sigma^2), \tag{26}$$

$$B_2=2\alpha u_0^2\sigma/(2\beta c+\sigma^2). \tag{27}$$

When $\varepsilon\bar{P}(\omega)=0$, Eq. (24) is just the standard KdV equation. In the case of $\varepsilon\bar{P}(\omega)\neq0$, the parameters in the solution of KdV equation can be regarded as the slowly-varying functions of y, i.e.

$$w=-2\mu^2(y)\,\mathrm{sech}^2Z,\quad Z=\mu(y)(\tau-\xi(y)). \tag{28}$$

Then the evolution solutions of $\mu(y)$ and $\xi(y)$ are[14]

$$\frac{\mathrm{d}\mu}{\mathrm{d}y}=-\frac{1}{4\mu}\int_{-\infty}^{\infty}\varepsilon\bar{P}(w)\,\mathrm{sech}^2Z\mathrm{d}Z, \tag{29}$$

$$\frac{\mathrm{d}\xi}{\mathrm{d}y}=4\mu^2-\frac{1}{4\mu^3}\int_{-\infty}^{+\infty}\bar{P}(w)[Z+\frac{1}{2}\sinh Z]\,\mathrm{sech}^2\mathrm{d}Z. \tag{30}$$

Substituting Eq. (25) into Eqs. (29) and (30) yields

$$\xi(y)=4\int_0^y\mu^2(y')\mathrm{d}y'+\xi_0, \tag{31}$$

$$\frac{\mathrm{d}\mu}{\mathrm{d}y}=\frac{8}{15}B_2\mu^3-\frac{1}{2}B_1\frac{1}{\mu}. \tag{32}$$

Returning to the original scales and noting that $z=x$ when $\varepsilon=1$, Eq. (32) becomes

$$\mu\frac{\mathrm{d}\mu}{\mathrm{d}x}=-b_1[\mu^4-b_2/b_1], \tag{33}$$

$$b_1=\frac{8}{15}\frac{\alpha\tau u_0^2}{c},\quad b_2=\frac{\gamma_G u_0^2(2\sigma^2+c\beta)}{2\sigma(2\beta c+\sigma^2)}. \tag{34}$$

In the case of $b_2/b_1>0$,

$$\mu^2=(b_2/b_1)^{1/2}\tanh[2\sqrt{b_2b_1}\,(x-x_0)], \tag{35}$$

$$x_0=\frac{1}{4\sqrt{b_2b_1}}\ln\left[\frac{b_2/b_1-\mu_0^2}{b_2/b_1+\mu_0^2}\right]<0; \tag{36}$$

when $b_2/b_1<0$,

$$\mu^2=-\mathrm{sgn}(b_1)\cdot|b_2/b_1|^{1/2}\tan[2\sqrt{|b_1b_2|}\ (x-x_0)], \tag{37}$$

$$x_0=\frac{1}{\mathrm{sgn}(b_1)\cdot 2\sqrt{|b_2/b_1|}}\arctan\left[\frac{\mu_0^2}{|b_2/b_1|^{1/2}}\right], \tag{38}$$

where μ_0 is the amplitude of μ at $x=0$, $\mathrm{sgn}(b_1)$ is the sign of b_1. Returning to the original scales, Eq. (28) becomes

$$a_0(x,\ t)=-\frac{A_0}{u_0}\ \mu^2(x)\,\mathrm{sech}^2[\mu(x)(t-cx+\zeta(x))], \tag{39}$$

$$A_0=\sigma(2\beta c+\sigma^2)/(2\sigma^2+c\beta). \tag{40}$$

This is the small-amplitude soliton solution on the CW pedestal.

III. TRANSMISSION PROPERTIES OF SMALL-AMPLITUDE BRIGHT AND DARK SOLITONS

According to Eqs. (35) and (37), in the case of $b_2/b_1>0$, the solitons can stably propagate and its final stable amplitude is

$$\mu_\infty^2=[|15\gamma_G c(2\sigma^2+\beta c)/(16\alpha\sigma^2(2\beta c+\sigma^2))|]^{1/2}. \tag{41}$$

However, when $b_2/b_1<0$, the soliton amplitude will increase, and finally the small amplitude approximation (i.e. Eq. (12)) would no longer be in force. Note that when μ^2 is getting very large, which is corresponding to $v\gg\lambda$ or $\lambda\to 0$, the small-amplitude dark soliton (hereafter, shortly dark soliton) becomes fundamental dark soliton (i.e. Eq (5)), while the small-amplitude bright soliton (hereafter, shortly bright soliton) may evolve into a large-amplitude bright soliton or elapse from the CW pedestal. The distance for the solitons to get to its stable states or the distance for solitons to change its small-amplitude states is

$$\Delta x=\left|\frac{15c(2\beta c+\sigma^2)}{16\alpha\gamma_G u_0^4(2\sigma+\beta c)}\right|^{1/2}. \tag{42}$$

The sign of A_0 (in Eq. (39)) determines the type of solitons, that is, $A_0>0$ or $a_0<0$ is corresponding to the dark solitons and $A_0<0$ or $a_0>0$ corresponding to the bright solitons. The regions for the dark and bright soliton to exist, in the case of $\gamma_G>0$ (i.e. $\gamma>g$ or $g=0$), are given in Fig. 1. In the left-half parameter space, i.e. in the case of $\beta<0$, there still exist the small-amplitude solitons which can propagate along two directions. In the regions I and III, the solitons are stable. In region VII, the amplitude of dark solitons is continually increasing. While, in region II, the bright soliton is going to disappear. However, for the realistic optical fiber, the only case is $\beta>0$. From Fig. 1 and Table 1, in the right-half parameter space, there exist left-directed and right-directed propagating solitons. For the right-directed soliton ($c>0$), its amplitude is getting large

and becomes stable at $\mu_{\alpha c}^2$. Its propagating velocity and centre-frequency shift are

$$V=\frac{\mathrm{d}}{\mathrm{d}x}\,[cx+\xi(x)]=c[1-\frac{2(2\beta c+\sigma^2)}{c^2}\,\mu^2(x)], \tag{43}$$

$$\Delta\omega=\frac{\mathrm{d}\varphi}{\mathrm{d}t}=\frac{\mathrm{d}}{\mathrm{d}t}\,[-\frac{4u_0}{c}\int_0^{\tau}a_0(x,\ \tau)\mathrm{d}\tau]=\frac{4A_0}{c}\,\mu^2(x), \tag{44}$$

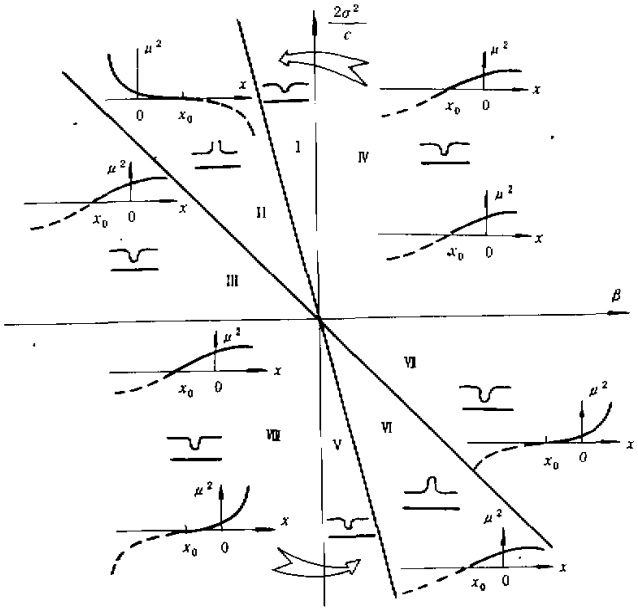

Fig. 1. Wave shapes and evolution curves of soliton amplitude in each region of parameter space.

Table 1. Signs and change tendency of relavant quantities in each region of parameter space.

Regions	I	II	III	IV	V	VI	VII	VIII
A_0	+	−	+	+	+	−	+	+
b_2/b_1	+	−	+	+	−	+	−	−
b_1	+	+	+	+	−	−	−	−
ΔV-μ^2 relation	−	+	+	−	−	+	+	−
ΔV-χ relation	−	−	+	−	−	+	+	−
$\Delta\omega_0$-μ^2 relation	+	−	+	+	−	+	−	−
$\Delta\omega_0$-χ relation	+	+	+	+	−	+	−	−

respectively. From Eq. (44), $\Delta\omega$ will finally tend to a stable value. In the right-half space, i.e. $\beta>0$, in the left-directed propagating case ($c<0$), the dark solitons in regions V and VII are unstable with the amplitude continually increasing, and finally they will evolve into fundamental dark solitons. The centre frequency is getting close to the ZGD point and the velocity is decreasing with respect to x in region V and increasing in region

VII. The bright soliton in region VI is getting large and becomes a stable soliton ($\mu^2 \to \mu_\alpha^2$), and its velocity as well as its centre frequency is getting stable, too, though the centre frequency is getting far away from the ZGD point at the beginning. If $\mu_\alpha^2 > c^2/2(2\beta c+\sigma^2)$, there must exist one point x_1 so that, when $x>x_1$, the dark soliton in region IV will propagate along the right direction until time t_1, and then turn its direction to the left. Similarly, in region V, the dark soliton will turn its direction to the right after a distance x_2 determined by $\mu^2(x_2)=c^2/2(2\beta c+\sigma^2)$. In this paper, the relevant behaviors of solitons in regions IV, V, and VII show a consistency with that in Ref. [10]. That is, the left-directed-propagating soliton will evolve into a fundamental dark soliton, while the right-directed-propagating soliton is getting to the stable state with an amplitude μ_α^2. From Eq. (41), when $\gamma_G=0$ and $\beta=0$, it is just the conclusion of Refs. [10] and [15]. Hence, the self-induced Raman effect is destructive for the small-amplitude dark solitons. But the loss and gain of an optical fiber is constructive.

IV. EFFECT OF LOSS AND GAIN AS WELL AS CONVERSION BETWEEN BRIGHT AND DARK SOLITONS

When neglecting the gain dispersion and gain saturation, the quantity g in Eq. (1) is a $u(x, t)$-independent constant[16]. In such a case, the solutions (39) and (40) are also valid for the soliton evolution in a fiber amplifier. According to Eqs. (34) and (40)–(44), in the case of $g>\gamma$ (i.e. $\gamma_G<0$), the signs of the quantities A_0, b_1, ΔV, $\Delta\omega$ except b_2/b_1 in corresponding regions are the same as those in Table 1. That the sign of A_0 is not changed means that the wave shapes in every parameter region are not changed. While the sign change of b_2/b_1 means the type of evolution curve of the soliton amplitude μ^2 with respect to x is changed, i.e. the types $\tanh(\cdot)$ and $\tan(\cdot)$ are exchanged. Suppose one controls the gain distribution along a fiber so that $\gamma_G=\gamma-g<0$ in some sections and $\gamma_G>0$ in others, and selects the fiber parameters so as to make them positive in regions V and VI. In such a case, when a soliton propagates along a fiber from the negative-γ_G section ($\gamma_G<0$) to the positive-γ_G section ($\gamma_G>0$), the unstable dark soliton will evolve into a stable one, and the stable bright and dark solitons in regions VI and IV will evolve into the amplitude-continually-increasing solitons, i.e., the small amplitude one will become a fundamental one in region IV, and may become a large-amplitude bright soliton in region VI, where the small-amplitude approximation is no longer in force; and conversely, they will also hold. In addition, if one adjusts the relative intensity of σ and β so as to make the parameter point pass through the boundary of regions V and VI, the quantity $(2\sigma^2+\beta c)$ will decrease to zero and then change its sign. The conversion between dark and bright solitons may be realized by controlling the parameters σ and β along the fiber to make the above changes in the parameter space take place. However, this conversion is not possible when the parameter point pass through the boundary of regions VII and VI because $\mu^2 \to \infty$ at the boundary according to Eqs. (34) and (37), where the small-amplitude approximation is not valid. In view of this, we suppose that, if the parameters are sectionally controlled along a fiber and the two

neighbouring sections are located in regions V and VI in parameter space respectively, with a negative value of γ_G in region V and a positive value of γ_G in region VI, the conversion between dark and bright solitons may be realized automatically when the soliton is propagating along the fiber. If the information of such a conversion can be detected, it may be applied to the logic or arithmetic devices in an integrated optical circuit.

V. SUMMARY AND CONCLUSION

In the case of taking the gain and loss as well as the self-induced Raman effect into account, there may exist the small-amplitude dark and bright solitons in the normal dispersion region near ZGD point. These solitons have following properties: (1) In the motion to the right, there only exist the amplitude-increasing dark solitons with a stable final state. (2) In the motion to the left, there exist the stable bright solitons in region VI and unstable small-smplitude dark solitons with the final state of fundamental ones. (3) In some cases, the stable dark soliton in region IV will turn its direction to the left after traveling some distance. Similarly, the unstable dark soliton in region V may turn its direction to the right after traveling certain distance. (4) When $\gamma_G<0$, the above three behaviors, except the amplitude evolution, are not changed; the only difference is the exchange between stable and unstable final states in the amplitude evolution. (5) By controlling the parameters along the fiber to make the point in parameter space pass through the boundary of regions V and VI, and at the same time, correspondingly controlling the gain and loss index, the conversion between bright and dark solitons may be realized, which may be applied to the logic or arithmetic devices in an integrated optical circuit. Since the fiber for such uses is working in the normal dispersion region and too low a loss is not necessary, the scheme is easily realized.

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