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HAWKING EFFECT OF DIRAC PARTICLES IN VAIDYA-SCHWARZSCHILD-DE SITTER SPACE-TIME*

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The solutions of the Dirac equation near the black hole event horizon and the universe horizon in the Vaidya-Schwarzschild-de Sitter space-time are given in this paper. Both the temperature of the Hawking radiation and the location of the evaporating black hole horizon and the universe horizon are determined exactly.

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I. INTRODUCTION

A simple, convenient and exact method determining both the location and the temperature of black hole event horizons is set up in Refs. [1] and [2]. The Hawking radiation of scalar particles in static, stationary, or non-static Riemann space-times was successfully researched there. In this paper, we generalize the method to the Dirac particles in the Vaidya-Schwarzschild-de Sitter space-time, and get certain satisfactory results.

II. NULL TETRAD AND SPIN COEFFICIENT

The Vaidya-Schwarzschild-de Sitter metric describing the evaporating black hole on the background of de Sitter space-time is given by Vick[3],

$$ds^2=[1-\frac{2m(v)}{r}-\frac{1}{3}\Lambda r^2]dv^2-2dvdr-r^2(d\theta^2+\sin^2\theta d\varphi^2). \tag{1}$$

The non-zero contravariant components of the metric tensor are

$$g^{01}=-1,\ g^{11}=-(1-\frac{2m}{r}-\frac{1}{3}\Lambda r^2),\ g^{22}=-1/r^2,\ g^{33}=-1/r^2\sin^2\theta. \tag{2}$$

We can set up the null tetrad as follows:

$$l_\mu=-\delta^0_\mu\ ,$$
$$n_\mu=-\frac{1}{2}(1-\frac{2m}{r}-\frac{1}{3}\Lambda r^2)\delta^0_\mu+\delta^1_\mu\ ,$$
$$m_\mu=-\frac{r}{\sqrt{2}}(\delta^2_\mu+i\sin\theta\delta^3_\mu)\ ,$$

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$$\bar{m}_\mu = -\frac{r}{\sqrt{2}}(\delta^2_\mu - \mathrm{i}\sin\theta\delta^3_\mu) \quad ,$$

and have

$$l^\mu = \delta^\mu_1,$$

$$n^\mu = -\delta^\mu_0 - \frac{1}{2}\left(1 - \frac{2m}{r} - \frac{1}{3}\Lambda r^2\right)\delta^\mu_1 \quad ,$$

$$m^\mu = \frac{1}{\sqrt{2}\,r}\left(\delta^\mu_2 + \frac{\mathrm{i}}{\sin\theta}\delta^\mu_3\right) \quad ,$$

$$\bar{m}^\mu = \frac{1}{\sqrt{2}\,r}\left(\delta^\mu_2 - \frac{\mathrm{i}}{\sin\theta}\delta^\mu_3\right) \quad . \tag{3}$$

It is easy to verify that Eq. (2) satisfies following relations and conditions,

$$l_\mu l^\mu = n_\mu n^\mu = m_\mu m^\mu = \bar{m}_\mu \bar{m}^\mu = 0 \quad ,$$

$$l_\mu n^\mu = -m_\mu \bar{m}^\mu = 1 \quad , \tag{4}$$

$$l_\mu m^\mu = l_\mu \bar{m}^\mu = n_\mu m^\mu = n_\mu \bar{m}^\mu = 0 \quad , \tag{5}$$

and

$$g_{\mu\nu} = l_\mu n_\nu + n_\mu l_\nu - m_\mu \bar{m}_\nu - \bar{m}_\mu m_\nu \quad . \tag{6}$$

According to Eq. (4.1a) in the paper of Newman and Penrose[4], we can calculate the spin coefficients and get

$$K = \lambda = \sigma = \nu = \tau = \pi = \varepsilon = 0,$$

$$\rho = -\frac{1}{r} \quad ,$$

$$\alpha = -\frac{1}{2\sqrt{2}\,r}\cot\theta = -\beta,$$

$$\mu = -\frac{1}{2r} + \frac{m}{r^2} + \frac{1}{6}\Lambda r,$$

$$\gamma = \frac{m}{2r^2} - \frac{1}{6}\Lambda r. \tag{7}$$

III. DIRAC EQUATION

The spinor base form of the Dirac equation is

$$\nabla_{a\dot{b}} P^a + \frac{1}{\sqrt{2}}\mathrm{i}\mu\bar{Q}_{\dot{b}} = 0 \quad ,$$

$$\nabla_{a\dot{b}} Q^a + \frac{1}{\sqrt{2}}\mathrm{i}\mu\bar{P}_{\dot{b}} = 0 \quad . \tag{8}$$

That is,

$$\partial_{a\dot{b}} P^a + \Gamma^a_{d a\dot{b}} P^d + \frac{1}{\sqrt{2}}\mathrm{i}\mu\bar{Q}_{\dot{b}} = 0 \quad ,$$

$$\partial_{a\dot{b}} Q^a + \Gamma^a_{d a\dot{b}} Q^d + \frac{1}{\sqrt{2}}\mathrm{i}\mu\bar{P}_{\dot{b}} = 0 \quad ; \tag{9}$$

so we get

$$(D + \varepsilon - \rho)F_1 + (\bar{\delta} + \pi - \alpha)F_2 - \frac{1}{\sqrt{2}}\mathrm{i}\mu G_1 = 0 \quad ,$$

$$(\Delta+\mu-\gamma)F_2+(\delta+\beta-\tau)F_1-\frac{1}{\sqrt{2}}\,i\mu G_2=0\ ,$$

$$(D+\varepsilon^*-\rho^*)G_2-(\delta+\pi^*-\alpha^*)G_1-\frac{1}{\sqrt{2}}\,i\mu F_2=0\ ,$$

$$(\Delta+\mu^*-\gamma^*)G_1-(\bar{\delta}+\beta^*-\tau^*)G_2-\frac{1}{\sqrt{2}}\,i\mu F_1=0\ . \tag{10}$$

Making use of Eq. (7), we have

$$(D-\rho)F_1+(\bar{\delta}-\alpha)F_2-\frac{1}{\sqrt{2}}\,i\mu G_1=0\ ,$$

$$(\Delta+\mu-\gamma)F_2+(\delta+\beta)F_1-\frac{1}{\sqrt{2}}\,i\mu G_2=0\ ,$$

$$(D-\rho^*)G_2-(\delta-\alpha^*)G_1-\frac{1}{\sqrt{2}}\,i\mu F_2=0\ ,$$

$$(\Delta+\mu^*-\gamma^*)G_1-(\bar{\delta}+\beta^*)G_2-\frac{1}{\sqrt{2}}\,i\mu F_1=0\ . \tag{11}$$

With Eq. (3) we obtain the explicit representatives of the differentiation operators as follows:

$$D=l^\mu\frac{\partial}{\partial x^\mu}=\frac{\partial}{\partial r}\ ,$$

$$\Delta=n^\mu\frac{\partial}{\partial x^\mu}=-\frac{\partial}{\partial v}-\frac{1}{2}\left(1-\frac{2m}{r}-\frac{1}{3}\Lambda r^2\right)\frac{\partial}{\partial r}\ ,$$

$$\delta=m^\mu\frac{\partial}{\partial x^\mu}=\frac{1}{\sqrt{2}\,r}\left(\frac{\partial}{\partial\theta}+\frac{i}{\sin\theta}\frac{\partial}{\partial\varphi}\right)\ ,$$

$$\bar{\delta}=\bar{m}^\mu\frac{\partial}{\partial x^\mu}=\frac{1}{\sqrt{2}\,r}\left(\frac{\partial}{\partial\theta}-\frac{i}{\sin\theta}\frac{\partial}{\partial\varphi}\right)\ . \tag{12}$$

Substituting Eqs. (7) and (12) into Eq. (11), we have

$$\left[\frac{\partial}{\partial r}+\frac{1}{r}\right]F_1+\left[\frac{1}{\sqrt{2}\,r}\left(\frac{\partial}{\partial\theta}-\frac{i}{\sin\theta}\frac{\partial}{\partial\varphi}\right)+\frac{1}{2\sqrt{2}\,r}\cot\theta\right]F_2-\frac{1}{\sqrt{2}}\,i\mu G_1=0\ ,$$

$$\left[-\frac{\partial}{\partial v}-\frac{1}{2}\left(1-\frac{2m}{r}-\frac{1}{3}\Lambda r^2\right)\frac{\partial}{\partial r}-\frac{1}{2r}+\frac{m}{2r^2}+\frac{1}{3}\Lambda r\right]F_2$$

$$+\left[\frac{1}{\sqrt{2}\,r}\left(\frac{\partial}{\partial\theta}+\frac{i}{\sin\theta}\frac{\partial}{\partial\varphi}\right)+\frac{1}{2\sqrt{2}\,r}\cot\theta\right]F_1-\frac{1}{\sqrt{2}}\,i\mu G_2=0\ ,$$

$$\left[\frac{\partial}{\partial r}+\frac{1}{r}\right]G_2-\left[\frac{1}{\sqrt{2}\,r}\left(\frac{\partial}{\partial\theta}+\frac{i}{\sin\theta}\frac{\partial}{\partial\varphi}\right)+\frac{1}{2\sqrt{2}\,r}\cot\theta\right]G_1-\frac{1}{\sqrt{2}}\,i\mu F_2=0\ ,$$

$$\left[-\frac{\partial}{\partial v}-\frac{1}{2}\left(1-\frac{2m}{r}-\frac{1}{3}\Lambda r^2\right)\frac{\partial}{\partial r}-\frac{1}{2r}+\frac{m}{2r^2}+\frac{1}{3}\Lambda r\right]G_1$$

$$-\left[\frac{1}{\sqrt{2}\,r}\left(\frac{\partial}{\partial\theta}-\frac{i}{\sin\theta}\frac{\partial}{\partial\varphi}\right)+\frac{1}{2\sqrt{2}\,r}\cot\theta\right]G_2-\frac{1}{\sqrt{2}}\,i\mu F_1=0\ . \tag{13}$$

Given

$$F_1=e^{in\varphi}f_1(v,\ r,\ \theta)\ ,$$
$$F_2=e^{in\varphi}f_2(v,\ r,\ \theta)\ ,$$
$$G_1=e^{in\varphi}g_1(v,\ r,\ \theta)\ ,$$
$$G_2=e^{in\varphi}g_2(v,\ r,\ \theta)\ ,$$

Eq. (13) can be reduced to

$$(r\frac{\partial}{\partial r}+1)f_1+\frac{1}{\sqrt{2}}\left[\frac{\partial}{\partial\theta}+\frac{n}{\sin\theta}+\frac{1}{2}\cot\theta\right]f_2=\frac{1}{\sqrt{2}}\ i\mu rg_1\ ,$$
$$\left[-r\frac{\partial}{\partial\theta}-\frac{1}{2}(r-2m-\frac{1}{3}\Lambda r^3)\frac{\partial}{\partial r}-\frac{1}{2}(1-\frac{m}{r}-\frac{2}{3}\Lambda r^2)\right]f_2$$
$$+\frac{1}{\sqrt{2}}(\frac{\partial}{\partial\theta}-\frac{n}{\sin\theta}+\frac{1}{2}\cot\theta)f_1=\frac{1}{\sqrt{2}}\ i\mu rg_2\ ,$$
$$(r\frac{\partial}{\partial r}+1)g_2-\frac{1}{\sqrt{2}}(\frac{\partial}{\partial\theta}-\frac{n}{\sin\theta}+\frac{1}{2}\cot\theta)g_1=\frac{1}{\sqrt{2}}\ i\mu rf_2\ ,$$
$$\left[-r\frac{\partial}{\partial v}-\frac{1}{2}(r-2m-\frac{1}{3}\Lambda r^3)\frac{\partial}{\partial r}-\frac{1}{2}(1-\frac{m}{r}-\frac{2}{3}\Lambda r^2)\right]g_1$$
$$-\frac{1}{\sqrt{2}}(\frac{\partial}{\partial\theta}+\frac{n}{\sin\theta}+\frac{1}{2}\cot\theta)g_2=\frac{1}{\sqrt{2}}\ i\mu rf_1\ . \tag{14}$$

Let

$$\mathscr{D}_0=r\frac{\partial}{\partial r}+1\ ,$$
$$\mathscr{D}_1=-r\frac{\partial}{\partial v}-\frac{1}{2}(r-2m-\frac{1}{3}\Lambda r^3)\frac{\partial}{\partial r}-\frac{1}{2}(1-\frac{m}{r}-\frac{2}{3}\Lambda r^2)\ ,$$
$$\mathscr{D}_+=\frac{1}{\sqrt{2}}(\frac{\partial}{\partial\theta}+\frac{n}{\sin\theta}+\frac{1}{2}\cot\theta)\ ,$$
$$\mathscr{D}_-=\frac{1}{\sqrt{2}}(\frac{\partial}{\partial\theta}-\frac{n}{\sin\theta}+\frac{1}{2}\cot\theta)\ ,$$

Eq. (14) can be written as

$$\mathscr{D}_0f_1+\mathscr{D}_+f_2=\frac{1}{\sqrt{2}}\ i\mu rg_1\ ,$$
$$\mathscr{D}_-f_1+\mathscr{D}_1f_2=\frac{1}{\sqrt{2}}\ i\mu rg_2\ ,$$
$$\mathscr{D}_0g_2-\mathscr{D}_-g_1=\frac{1}{\sqrt{2}}\ i\mu rf_2\ ,$$
$$\mathscr{D}_1g_1-\mathscr{D}_+g_2=\frac{1}{\sqrt{2}}\ i\mu rf_1\ . \tag{15}$$

After the separation of variables as[5]

$$f_1(v,\ r,\ \theta)=R_-(r,\ v)S_-(\theta)=R(r,\ v)S(\theta)\ ,$$
$$f_2(v,\ r,\ \theta)=R_+(r,\ v)S_+(\theta)\ ,$$
$$g_1(v,\ r,\ \theta)=R_+(r,\ v)S_-(\theta)\ ,$$

$$g_2(v, r, \theta)=R_-(r, v)S_+(\theta) \ ,$$

Eq. (15) can be reduced to

$$\begin{aligned}
&\mathscr{D}_0RS+\mathscr{D}_+R_+S_+=\frac{1}{\sqrt{2}}\ \mathrm{i}\mu rR_+S \ ,\\
&\mathscr{D}_-RS+\mathscr{D}_1R_-S_+=\frac{1}{\sqrt{2}}\ \mathrm{i}\mu rRS_+ \ ,\\
&\mathscr{D}_0RS_+-\mathscr{D}_-R_+S_-=\frac{1}{\sqrt{2}}\ \mathrm{i}\mu rR_+S_+ \ ,\\
&\mathscr{D}_1R_+S-\mathscr{D}_+RS_+=\frac{1}{\sqrt{2}}\ \mathrm{i}\mu rRS \ ;
\end{aligned} \tag{16}$$

or

$$\begin{aligned}
&\frac{1}{R_+}\ (\mathscr{D}_0R-\frac{\mathrm{i}}{\sqrt{2}}\ \mu rR_+)=-\frac{\mathscr{D}_+S_+}{S} \ ,\\
&\frac{1}{R}\ (\mathscr{D}_1R_+-\frac{\mathrm{i}}{\sqrt{2}}\ \mu rR)=-\frac{\mathscr{D}_-S}{S_+} \ ,\\
&\frac{1}{R_+}\ (\mathscr{D}_0R-\frac{\mathrm{i}}{\sqrt{2}}\ \mu rR_+)=\frac{\mathscr{D}_-S}{S_+} \ ,\\
&\frac{1}{R}\ (\mathscr{D}_1R_+-\frac{\mathrm{i}}{\sqrt{2}}\ \mu rR)=\frac{\mathscr{D}_+S_+}{S} \ .
\end{aligned} \tag{17}$$

Given $\dfrac{-\mathscr{D}_+S_+}{S}=\lambda$, we have

$$\mathscr{D}_0R=(\frac{1}{\sqrt{2}}\ \mathrm{i}\mu r+\lambda)R_+ \ , \tag{18}$$

$$\mathscr{D}_1R_+=(\frac{1}{\sqrt{2}}\ \mathrm{i}\mu r-\lambda)R \ . \tag{19}$$

By means of Eqs. (18) and (19), we get the de-coupled Dirac equation represented with general derivative in the Vaidya-Schwarzschild-de Sitter space-time ,

$$\mathscr{D}_0\frac{\mathscr{D}_1R_+}{\frac{1}{\sqrt{2}}\ \mathrm{i}\mu r+\lambda}=(\frac{1}{\sqrt{2}}\ \mathrm{i}\mu r+\lambda)R_+ \ . \tag{20}$$

Given

$$R_+=r^{-1}\rho(r, v) \ ,$$

and substituting $\mathscr{D}_1$ and $\mathscr{D}_0$ into Eq. (20), we have

$$\begin{aligned}
&(1-\frac{2m}{r}-\frac{1}{3}\Lambda r^2)\frac{\partial^2\rho}{\partial r^2}+2\frac{\partial^2\rho}{\partial v\partial r}+\left[(\frac{1}{r}+\frac{m}{r^2}-\frac{4}{3}\Lambda r)-(1-\frac{2m}{r}-\frac{1}{3}\Lambda r^2)\right.\\
&\left.\times\frac{\mu^2r+\mathrm{i}\sqrt{2}\ \mu r}{\mu^2r^2+2\lambda^2}\right]\frac{\partial\rho}{\partial r}+\frac{2}{r}\left[1-\frac{\mu^2r^2-\mathrm{i}\sqrt{2}\ \mu\lambda}{\mu^2r^2+2\lambda^2}\right]\frac{\partial\rho}{\partial v}+\left[-\frac{m}{r}-\frac{2}{3}\Lambda r^2\right.\\
&\left.-(\mu^2r^2+2\lambda^2)-\frac{\mu^2r-\mathrm{i}\sqrt{2}\ \mu\lambda}{\mu^2r^2+2\lambda^2}\right](m-\frac{1}{3}\Lambda r^3)\frac{\rho}{r^2}=0 \ .
\end{aligned} \tag{21}$$

Making use of the tortoise transformation[1, 2].

$$r_* = r + \frac{1}{2\kappa(v_0)} \ln[r - r_H(v)] \ ,$$
$$v_* = v - v_0 \ , \tag{22}$$

we have

$$\frac{\partial}{\partial r} = [1 + \frac{1}{2\kappa(r-r_H)}] \frac{\partial}{\partial r_*} \ ,$$
$$\frac{\partial^2}{\partial r^2} = [1 + \frac{1}{2\kappa(r-r_H)}]^2 \frac{\partial^2}{\partial r_*^2} - \frac{1}{2\kappa(r-r_H)^2} \frac{\partial}{\partial r_*} \ ,$$
$$\frac{\partial}{\partial v} = \frac{\partial}{\partial v_*} - \frac{\dot{r}_H}{2\kappa(r-r_H)} \frac{\partial}{\partial r_*} \ , \tag{23}$$
$$\frac{\partial^2}{\partial v \partial r} = \frac{\dot{r}_H}{2\kappa(r-r_H)^2} \frac{\partial}{\partial r_*} + [1 + \frac{1}{2\kappa(r-r_H)}] \frac{\partial^2}{\partial v_* \partial r_*}$$
$$- [1 + \frac{1}{2\kappa(r-r_H)}] \frac{\dot{r}_H}{2\kappa(r-r_H)} \frac{\partial^2}{\partial r_*^2} \ .$$

Substituting Eq. (23) into Eq. (21), and multiplying it by the factor $2\kappa(r-r_H)/[2\kappa(r-r_H)+1]$, we obtain

$$\left\{ \frac{(r-2m-\frac{1}{3}\Lambda r^3)[1+2\kappa(r-r_H)] - 2r\dot{r}_H}{2\kappa(r-r_H)r} \right\} \frac{\partial^2 \rho}{\partial r_*^2} + 2\frac{\partial^2 \rho}{\partial v_* \partial r_*} + \frac{2\kappa(r-r_H)}{[1+2\kappa(r-r_H)]}$$
$$\times \left\{ (1 - \frac{2m}{r} - \frac{1}{3}\Lambda r^2) \left[\frac{-1}{2\kappa(r-r_H)^2} \right] + \frac{2\dot{r}_H}{2\kappa(r-r_H)^2} + \left[\frac{1}{r} + \frac{m}{r^2} - \frac{4}{3}\Lambda r - \frac{r-2m-\frac{1}{3}\Lambda r^3}{r} \right.\right.$$
$$\left. \times \frac{\mu^2 r - i\sqrt{2}\,\mu\lambda}{\mu^2 r^2 + 2\lambda^2} \right] \frac{[1+2\kappa(r-r_H)]}{2\kappa(r-r_H)} - \left[\frac{2}{r} - \frac{2(\mu^2 r - i\sqrt{2}\,\mu\lambda)}{\mu^2 r^2 + 2\lambda^2} \right] \frac{\dot{r}_H}{2\kappa(r-r_H)} \left. \right\} \frac{\partial \rho}{\partial r_*}$$
$$+ \frac{2\kappa(r-r_H)}{2\kappa(r-r_H)+1} \left[\frac{2}{r} - \frac{\mu^2 r^2 - i\sqrt{2}\,\mu\lambda}{\mu^2 r^2 + 2\lambda^2} \right] \frac{\partial \rho}{\partial v_*} - \frac{2\kappa(r-r_H)}{2\kappa(r-r_H)+1} \left[\frac{m}{r^3} + \frac{\Lambda}{3} \right.$$
$$\left. + \frac{\mu^2 r^2 - i\sqrt{2}\,\mu\lambda}{\mu^2 r^2 + 2\lambda^2} \left(\frac{m}{r^2} - \frac{1}{3}\Lambda r \right) + \left(\frac{2\lambda^2}{r^2} + \mu^2 \right) \right] \rho = 0 \ . \tag{24}$$

We want the coefficient of $\frac{\partial^2 \rho}{\partial r_*^2}$ to approach 1 when $r \to r_H$, that is

$$\lim_{\substack{r \to r_H \\ v \to v_0}} \frac{(r-2m-\frac{1}{3}\Lambda r^3)[1+2\kappa(r-r_H)] - 2r\dot{r}_H}{2\kappa(r-r_H)r} = 1 \ . \tag{25}$$

It means that the numerator of the coefficient has to vanish when r goes to r_H, thereby we get the equation determining the event horizon,

$$(r_H - 2m - \frac{1}{3}\Lambda r_H^3) - 2r_H \dot{r}_H = 0 \ , \tag{26}$$

When r_H represents the black hole event horizon, from Eq. (26), we get

$$r_H = r_+ = 2\sqrt{\frac{1-2\dot{r}_+}{\Lambda}} \cos(\alpha_1 + \frac{\pi}{3}) \ , \tag{27}$$

where

$$\alpha_1=\frac{1}{3}\arccos\left[\frac{3m\Lambda^{1/2}}{(1-2\dot{r}_+)^{3/2}}\right].$$

When r_H is the universe horizon, we get

$$r_H=r_c=2\sqrt{\frac{1-2\dot{r}_c}{\Lambda}}\cos(\alpha_2-\frac{\pi}{3}), \tag{28}$$

where

$$\alpha_2=\frac{1}{3}\arccos\left[\frac{3m\Lambda^{1/2}}{(1-2\dot{r}_c)^{3/2}}\right].$$

Applying the L'Hospital rule to Eq. (25), we obtain the temperature function κ,

$$\kappa=\frac{1-2\dot{r}_H-\Lambda r_H^2}{2[2m(v_0)+\frac{1}{3}\Lambda r_H^3]}. \tag{29}$$

For the black hole event horizon r, we have

$$\kappa_+(v_0)=\frac{1-2\dot{r}_+-\Lambda r_+^2}{2[2m(v_0)+\frac{1}{3}\Lambda r_+^3]}, \tag{30}$$

and for the universe horizon r, we get

$$\kappa_c(v_0)=\frac{1-2\dot{r}_c-\Lambda r_c^2}{2[2m(v_0)+\frac{1}{3}\Lambda r_c^3]}. \tag{31}$$

Near event horizons, Eq. (24) is reduced to

$$\frac{\partial^2\rho}{\partial r_*^2}+2\frac{\partial^2\rho}{\partial v_*\partial r_*}+a\frac{\partial\rho}{\partial r_*}=0, \tag{32}$$

where

$$a=\frac{m}{r_H^2}-\frac{1}{3}\Lambda r_H.$$

Separating variables as $\rho(v_*, r_*)=e^{-i\omega v_*}\mathscr{R}(r_*)$ and substituting it into Eq. (32), we get

$$\mathscr{R}''+(a-2i\omega)\mathscr{R}'=0, \tag{33}$$

its solution is

$$\mathscr{R}(r_*)=c_1e^{(2i\omega-a)r_*}+c_2, \tag{34}$$

so we have

$$\rho=e^{-i\omega v_*}[c_1e^{(2i\omega-a)r_*}+c_2], \tag{35}$$

where both c_1 and c_2 are integral constants. It is easy to check

$$\rho_{in}=e^{-i\omega v_*},$$
$$\rho_{out}=e^{-i\omega v_*}e^{2i\omega r_*}e^{-ar_*}. \tag{36}$$

Near event horizons, $r_*\sim\frac{1}{2\kappa}\ln(r-r_H)$, so the outgoing wave can be written as

$$\rho_{out}=e^{-i\omega v_*}(r-r_H)^{i\omega/\kappa}(r-r_H)^{-a/2\kappa}. \tag{37}$$

We analytically continue it to the region inside the horizon through the complex lower semi-plane as[6]

$$(r-r_H)\rightarrow |r-r_H|e^{-i\pi}=(r_H-r)e^{-i\pi} \quad , \tag{38}$$

then we have

$$\rho_{\text{out}}\Rightarrow\widetilde{\rho}_{\text{out}}=e^{i\omega v_*}e^{2i\omega r_*}e^{-ar_*}e^{i\pi a/2\kappa}e^{\pi\omega/\kappa} \quad . \tag{39}$$

The scattering probability of the outgoing wave at the horizon is

$$|\frac{\rho_{\text{out}}}{\widetilde{\rho}_{\text{out}}}|=e^{-2\pi\omega/\kappa} \quad . \tag{40}$$

Following the method suggested by Sannan[7], we get

$$N_\omega=\frac{1}{e^{\omega/\kappa k_B T}+1} \quad , \tag{41}$$

where k_B is Boltzmann's constant.

IV. DISCUSSION

Making use of the method to determe the temperature and the location of event horizon of black hole, suggested in Refs. [1] and [2], we have dealt with the Hawking radiation of the Dirac particles in the Vaidya-Schwarzschild-de Sitter space-time in this paper. The representatives of location and temperature of event horizons, Eqs. (27)−(31), are the same as those obtained by means of the Klein-Gordon equation.

We can re-write Eqs. (30) and (31) as

$$\kappa_+(v_0)=\frac{1-2\dot{r}_+-\Lambda r_+^2}{2(2m+\frac{1}{3}\Lambda r_+^3)}=\frac{\Lambda}{6}\frac{(r_{++}-r_+)(r_+-r_-)}{r_+(1-2\dot{r}_+)} \quad , \tag{42}$$

$$\kappa_c(v_0)=\frac{1-2\dot{r}_c-\Lambda r_c^2}{2(2m+\frac{1}{3}\Lambda r_c^3)}=\frac{\Lambda}{6}\frac{(r_c-r'_+)(r_c-r'_-)}{r_c(1-2\dot{r}_c)} \quad , \tag{43}$$

where

$$r_{++}=2\sqrt{\frac{1-2\dot{r}_+}{\Lambda}}\cos(\alpha_1-\frac{\pi}{3}),\ r_-=-2\sqrt{\frac{1-2\dot{r}_+}{\Lambda}}\cos\alpha_1 \ ,$$

$$r'_+=2\sqrt{\frac{1-2\dot{r}_c}{\Lambda}}\cos(\alpha_2+\frac{\pi}{3}),\ r'_-=-2\sqrt{\frac{1-2\dot{r}_c}{\Lambda}}\cos\alpha_2 \ . \tag{44}$$

When the two event horizons approach each other (i.e. $r_+\rightarrow r_c$), from Eqs. (27) and (28), we have

$$\cos(\alpha_2-\frac{\pi}{3})=\cos(\alpha_1+\frac{\pi}{3}) \quad . \tag{45}$$

On the other hand, it is easy to see from the representatives of α_1 and α_2 that $\alpha_1=\alpha_2$ when the two horizons touch each other.

At that time, $r_{++}=r_+$, and $r'_+=r_c$, so we have

$$\kappa_+(v_0)=0,$$
$$\kappa_c(v_0)=0. \tag{46}$$

It means that the temperatures of the two horizons go to zero. This result violates the third law of thermodynamics, therefore, we conclude that r_+ cannot touch r_c.

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