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EFFECTS OF BOOTSTRAP CURRENT ON ENERGY CONFINEMENT OF TOKAMAK PLASMAS

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Based on the principle of profile consistency and η_i-mode induced anomalous transport model, the enery confinement in tokamak plasmas is studied with inclusion of the bootstrap current in which the poloidal magnetic field is consistently determined. The results show that bootstrap current has a significant effect on plasma confinement. If the bootstrap current is (15-30)% of the total current, global energy confinement time may increase by (20-45)%, and the maximum of related energy store is almost as much as twice the value without bootstrap current. Variation of the energy confinement time with the plasma parameters is of similar trend with Kaye-Goldston scaling. In addition, it can be seen that the radius of q=1 surface decreases with bootstrap current, which may be one of the possible mechanisms for some experimental results where sawtooth oscillation disappears because of auxiliary heating.

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I. INTRODUCTION

A major goal of tokamak fusion research is to understand and to improve energy confinement,therefore, the mechanism of anomalous transport and energy confinement of tokamak plasmas has long been an attractive subject for plasma physicists. Substantial progress in this field has been made during the last decade. It is generally believed that the confinement of ohmic plasma can be attributed to dissipative trapped eletron mode and η_i-mode for relative low and high density plasma,respectively. However, when auxiliary heating power such as neutral beam injection (NBI) and ion cyctotron resonance heating (ICRH) are applied, the picture of plasma confinement becomes much more complicated. Among the difficulties, the identification of the mechanism leading to plasma transition from low confinement mode (L-mode) to high confinement mode (H-mode) is an important one. Another question is the dependence of energy confinement time τ_E on the plasma current. Some authors have proposed mechanisms responsible for the two above mentioned questions. However , results of Christiansen *et al.*[1] implied that they may be two related problems: data analysis showed that the energy confinement on JET experiments is sensitive to current profile which, in the case of existence of additional heating power, is decoupled from temperature profile, and H-mode plasma may be related to the current profile broadening which is probably due to non-inductive bootstrap current.

Hence, one may ask how is the transport related to the current and its profile? In this article, based on the assumption of profile consistency and an η_i-mode induced anomalous transport model, we investigate the energy confinement of auxiliary heating plasma with emphasis placed on the effect of bootstrap current on confinement time τ_E. The motivation of the problem comes from recent experimental evidences [2,3] on JET and TFTR where bootstrap current has taken a significant portion of the total plasma current. In the next section, we derive the basic equations. In sec. Ⅲ, expressions for bootstrap current are presented. Numerical results are given in sec. Ⅳ and a brief summary and discussion will be found in sec. V.

Ⅱ. BASIC EQUATIONS

In the past few years, a lot of work has been engaged in ion-temperature-gradient-driven instability and the related anomalous thermal conductivity, because it is believed that η_i-mode may also be one of the most important candidates for explaining energy confinement of additional heating plasmas. One of them is the renormalized turbulence theory of Lee *et al.*[4] , which gives the following ion thermal conductivity:

$$\chi_i^{\mu} \approx 0.1\pi^2[\ln(1+\eta_i)]^2(1+\eta_i)2\rho_s^2C_s/\tau^2L_s \ , \tag{1}$$

where $\eta_i \equiv \mathrm{d}\ln T_i/\mathrm{d}\ln n$, $\tau \equiv T_e/T_i$, $L_s(\equiv qR/s)$ is the shear length. Notations without specification are conventional. In Eq.(1), η_i has been assumed to be larger than η_{ic}, the threshold for η_i-mode instablilities.

Of course, the η_i-turbulence will also lead to an anomalous electron thermal loss. Lee *et al.* pointed out that the associated electron thermal conductivity χ_e^{μ} will be less than χ_i^{μ} if collision is high such that the condition $\epsilon^{1/2}C_s/(L_s/\nu_e) < 1$ is satisfied. However, as noticed by Tang[5], in tokamaks with an additional heating power where plasma tends to be collisionless, χ_e^{μ} will tend to have the same magnitude and scaling as χ_i^{μ}. Hence,for simplicity, $\chi_e^{\mu} \approx \chi_i^{\mu}$ is assumed, and a simple steady-state energy balance equation of the following form is used :

$$P_H(r) = -\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}r(n_e\chi_e\mathrm{d}T_e/\mathrm{d}r + n_i\chi_i\mathrm{d}T_i/\mathrm{d}r). \tag{2}$$

To obtain Eq.(2), we have assumed that the sum of the radiation and charge-exchange losses is absent. Under the assumption that the density and temperature profiles of electrons take the same forms as those of ions, and let $P_H(r) = P_T h(r)$, we arrive at

$$\chi_e \approx \chi_i = -P_T\int_0^r \mathrm{d}r r h(r)/[(1+\tau)rn(r)\mathrm{d}T_i/\mathrm{d}r], \tag{3}$$

where the neutrality condition $n_i = n_e = n$ has been applied. Profile-consistency constrained temperature profile may take the form

$$T_{e,i}(r) = T_{e,io}(1-r^2/a^2)^{\alpha_T}, \tag{4}$$

with $\alpha_T = 2\alpha_q/3$ and $\alpha_q \approx q_a - 0.5$. As regards density profile, because η_i-turbutence has been supposed to be a dominant loss channel for plasma energy in the thermal insulated region between $q = 1$ and $q = 2$ surfaces, the condition $\eta_i > \eta_{ic}$ should be satisfied, and the local thermal conductivity $\chi^{\mu}(r)$ and the global thermal conductivity $\chi(r)$ should have the same tendency of variation as r varies, at least for $r_1 < r < r_2$, where r_1, r_2 denote the radius of $q = 1$ and $q = 2$ surfaces, respectively. Therefore, a suitable alternative for density profile is assumed as

$$n(r) = n_0 \exp(-\alpha_n r^2/a^2) \quad \text{for} \quad r < r_2 \tag{5a}$$

and

$$n(r) = An_0(1 - r^2/a^2)^{\alpha_n'} \quad \text{for} \quad r \geq r_2 \,. \tag{5b}$$

Expression (5) implies that particle loss mechanism is supposed to be different for the two separated regions. The function $n(r)$ and its first derivative at $r = r_2$ should be continuous, which gives

$$\alpha_n' = \alpha_n(1 - r_2^2/a^2)$$

and

$$A = \exp(-\alpha_n r_2^2/a^2)/(1 - r_2^2/a^2)^{\alpha_n'}.$$

The combination of Eqs. (4) and (5) yields

$$\eta_i = \begin{cases} \dfrac{\alpha_T}{\alpha_n} \dfrac{1}{1 - (r/a)^2} & \text{for} \quad r < r_2; \\ \alpha_T/\alpha_n' \quad \text{for} & r \geq r_2 \,. \end{cases} \tag{6}$$

Hence if $\eta_i > \eta_{ic}$ is satisfied on magnetic axis, it will be satisfied in the whole region as well. Therefore, it is reasonable to use Tang's approach [5] in which the entropy associated with χ_i^{μ} is set to equal the entropy associated with χ_i, namely

$$\int_{r_1}^{r_2} \mathrm{d}r r\chi_i(r)[\mathrm{d}\ln T_i/\mathrm{d}r]^2 = \int_{r_1}^{r_2} \mathrm{d}r r\chi_i^{\mu}(r)[\mathrm{d}\ln T_i/\mathrm{d}r]^2. \tag{7}$$

In fact, Eq. (7) can be regarded as showing that $\chi_i(r)$ and $\chi_i^{\mu}(r)$ are equivalent in the sense of certain kind of average. From Eq. (7), we have T_{i0} for central ion temperature as

$$T_{i0}(\mathrm{keV}) \approx 0.674(P_T/n_0)^{0.4}(1+\tau)^{-0.4}\alpha_q^{-0.4}\tau^{0.2}A_i^{-0.2}C_i^{0.4}B_t^{0.8} \tag{8}$$

and

$$\chi_{io} \approx 1.77(P_T/n_0)^{0.6}R^{-1}(1+\tau)^{-0.6}\alpha_q^{-0.6}B_t^{-0.8}\tau^{-0.2}A_i^{0.2}C_i^{-0.4}, \tag{9}$$

with

$$C_i = \int_{r_1}^{r_2} \mathrm{d}r rF(r)[\mathrm{d}\ln T_i/\mathrm{d}r]^2 / \int_{r_1}^{r_2} \mathrm{d}r rG(r)[\mathrm{d}\ln T_i/\mathrm{d}r]^2, \tag{10}$$

$$\chi_i(r) = \chi_{i0}F(r), \quad \chi_i^{\mu}(r) = \chi_{i0}^{\mu}G(r), \tag{11}$$

$$\chi_{i0}^{\mu} = 3.19(A_i/r)^{1/2}T_{i0}^{3/2}/B_t^2R, \tag{12}$$

$$F(r) = \frac{\exp(\alpha_n r^2/a^2)}{(r/a)^2(1-r^2/a^2)^{\alpha_T-1}} \frac{\int_0^{r/a} \mathrm{d}x' x' h(x')}{\int_0^1 \mathrm{d}x' x' h(x')} \quad \text{for} \quad r < r_s, \tag{13a}$$

while

$$F(r) = \frac{1}{A(r/a)^2(1-r^2/a^2)^{\alpha_n'+\alpha_T^{-1}}} \frac{\int_0^{r/a} \mathrm{d}x' x' h(x')}{\int_0^1 \mathrm{d}x' x' h(x')} \quad \text{for} \quad r \geq r_s \tag{13b}$$

and

$$G(r) = [\ln(1+\eta_i)]^2(1+\eta_i)^2(1-r^2/a^2)^{\alpha_q}(s/q), \tag{14}$$

where η_i is given by Eq. (6). The definition for global energy confinement time results in

$$\tau_E(\mathrm{ms}) = 637a^2RB_t^{0.8}(n_0/P_T)^{0.6}(1+\tau)^{0.6}\tau^{0.2}A_i^{-0.2}\alpha_q^{-0.4}C_i^{0.4}W_p, \tag{15}$$

where

$$W_p = \int_0^a \mathrm{d}r r n(r) T_i(r)/(a^2 n_0 T_{i0}) . \tag{16}$$

III. BOOTSTRAP CURRENT

An expression for the diffusion-driven current (the initial term used for bootstrap current) was first given by Galeev in his pioneer work[6]. Following this initial work a full neo-classical transport theory was developed (Hinton *et al.*[7]). A simple version of the expressions mentioned above for bootstrap current in a tokamak with circular cross-section , which has been used by several authors(e. g. Becker[8]), reads

$$j_B(r) = \frac{1}{B_\theta(r)}(r/R)^{1/2}\mathrm{K}(r), \tag{17}$$

where $\mathrm{K}(r) = \mathrm{K}_{13}\{-\partial p_e/\partial r - (n_e/n_i)\partial p_i/\partial r + 1.17 n_e \cdot \partial T_i/\partial r\} + (2.5\mathrm{K}_{13} - \mathrm{K}_{23}) n_e \cdot \partial T_e/\partial r$, with $\mathrm{K}_{13} \approx 1.46(1+0.67/Z_{\mathrm{eff}})$ and $\mathrm{K}_{23} \approx 3.65(1+0.19/Z_{\mathrm{eff}})$. It is convenient to denote

$$I_{\rm B}(r) = 2\pi \int_0^r j_{\rm B}(r')r'{\rm d}r', \qquad I_{\rm OH}(r) = 2\pi \int_0^r j_{\rm OH}(r')r'{\rm d}r'$$

and

$$I(r) = 2\pi \int_0^r \{j_{\rm OH}(r') + j_{\rm B}(r')\}r'{\rm d}r',$$

where the ohmic current density $j_{\rm OH}(r) = j_0(1 - r^2/a^2)^{\alpha_q}$. Usually experiments are carried out with current feedback control,which implies that for a given plasma current $I_{\rm p}$ the ohmic current and the bootstrap current should satisfy

$$I_{\rm p} = I(a) = I_{\rm B}(a) + I_{\rm OH}(a). \tag{18}$$

This means that after the bootstrap current being taken into account, the ohmic current will be reduced. Equation (18) gives

$$j_0 = [1 - I_{\rm NB}(a)](\alpha_q + 1)I_{\rm p}/\pi a^2, \tag{19}$$

where

$$I_{\rm NB}(a) = I_{\rm B}(a)/I_{\rm p}$$

Note that

$$B_\theta(r) = (\mu_0/r)\int_0^r \{j_{\rm OH}(r') + j_{\rm B}(r')\}r'{\rm d}r' \quad \text{and} \quad {\rm d}I_{\rm B}(r)/{\rm d}r = 2\pi r j_{\rm B}(r),$$

then

$$B_\theta(r) = \frac{\mu_0}{2\pi r}[I_{\rm OH}(r) + I_{\rm B}(r)], \tag{20}$$

where

$$I_{\rm OH}(r) = \frac{\pi a^2}{\alpha_q + 1}[1 - (1 - r^2/a^2)^{\alpha_q + 1}].$$

Henece, from Eq. (17) we have

$$\frac{{\rm d}I_{\rm B}(r)}{{\rm d}r} = 1.6 \times 10^4 \frac{4\pi^2(a/R)^{1/2}r^2(r/a)^{1/2}{\rm K}(r)}{\mu_0\{I_{\rm OH}(r) + I_{\rm B}(r)\}}. \tag{21a}$$

The units of n,T in Eq. (21a) have been taken as $10^{20}\,m^{-3}$ and keV, respectively. Normalize $I_{\rm B}, I_{\rm OH}(r)$ with $I_{\rm p}$ and minor radius r with a, let $I_{\rm NB}(x) = I_{\rm B}(x)/I_{\rm p}, x = r/a$, then dimensionless equation for bootstrap current has the following form:

$$\frac{{\rm d}I_{\rm NB}}{{\rm d}x} = \frac{(1.6 \times 10^4)4\pi^2(a/R)^{1/2}a^3x^{5/2}{\rm K}(x)}{\mu_0 I_{\rm p}^2\{(1 - I_{\rm NB}(1))(1 - (1 - x^2)^{\alpha_q + 1}) + I_{\rm NB}(x)\}}, \tag{21b}$$

with the boundary condition $I_{NB}(x=0)=0$, and

$$K(x)=\frac{2n_0T_{i0}}{a}x\begin{cases}\exp(-\alpha_n x^2)(1-x^2)^{\alpha_T}(C_1/(1-x^2)+C_2) & x<r_2/a;\\ A(1-x^2)^{\alpha_n'+\alpha_T-1}(C_1+C_2') & x\geq r_2/a,\end{cases}\tag{22}$$

where$C_1=\alpha_T(\tau K_{23}-0.17K_{13}-1.5\tau K_{13}),C_2=\alpha_n K_{13}(1+\tau)$ and $C_2'=\alpha_n' K_{13}(1+\tau)$.Because of the bootstrap current, plasma current profile will be broadened, and variations in q-profile and s-profile will take place. From the definitions of safety factor and magnetic shear, and using the solution of Eq. (21b), we have

$$\frac{s}{q}=\frac{R}{B_t}\frac{\mu_0 j_0}{\alpha_q+1}\frac{1-(1-x^2)^{\alpha_q+1}}{x^2}-\mu_0 j_0(1-x^2)^{\alpha_q}-\frac{\mu_0 I_p}{2\pi a^2}\frac{dI_{NB}(x)}{xdx}+\frac{\mu_0 I_p}{\pi a^2}\frac{I_{NB}(x)}{x^2}.\tag{23}$$

The equations for $q=1$and $q=2$ surfaces with the bootstrap current taken into account are as follows:

$$q_a x_1^2-I_{NB}(x_1)=(1-I_{NB}(1))(1-(1-x_1^2)^{\alpha_q+1}),\tag{24}$$

$$q_a x_2^2/2-I_{NB}(x_2)=(1-I_{NB}(1))(1-(1-x_2^2)^{\alpha_q+1}).\tag{25}$$

Now, Eqs. (8) and (21) are coupled by Eqs. (10),(14),(23),(24) and (25), which means T_{io} and $I_{NB}(x)$ should be determined simultaneously. Therefore, two-fold iteration should be employed because of $I_{NB}(1)$ on the right hand side of Eq. (21), where $I_{NB}(1)$ denotes the ratio of bootstrap current to total plasma current I_p.

IV. NUMERICAL RESULTS

To solve Eq. (21b) for $I_{NB}(x)$, r_2 in Eq.(5) should be determined beforehand. But r_2 itself will also be affected by current profile. To avoid this difficulty, we first assume r_2 be the value without bootstrap current taken into account. The posterior-estimate indicates that this approach is self-consistent, radius of $q=2$ surface is hardly influenced by bootstrap current. To obtain global energy confinement time τ_E for typical situations, we choose $Z_{eff}=2, A_i=1, \tau=1$ and $h(r)$, the heating power deposition profile, to be of the form of $(1-r^2/a^2)^{\alpha_p}$.

Figures 1–8 indicate dependences of global confinement time τ_E on different plasma parameters. Dashed line and solid line correspond to the cases with and without effects of bootstrap current, respectively. Figure 1 shows that τ_E increases with the peakedness of density profile, which agrees with the fact that pellet injection is favourable for energy confinement. If $\alpha_n\to 0$, the bootstrap current will be very small, and τ_E with and without bootstrap current tends to be the same. Figure 2 demonstrates the variation of τ_E with

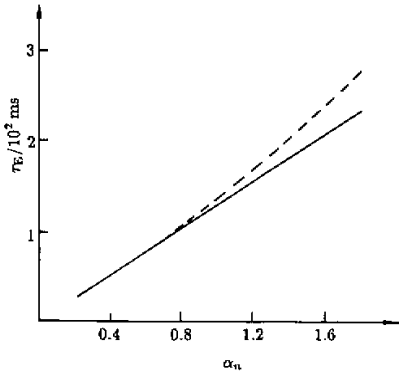

Fig. 1. τ_E vs α_n, the peakedness of density profile, for $n_0(10^{20}\,m^{-3}) = 1.0$, $P_T = 10\,\mathrm{MW}$ $B_t = 5.2\,\mathrm{T}$, R=2.48 m, a=0.85 m, $I_p = 2.2\,\mathrm{MA}$, $\alpha_p = 2.0$ and $q_a = 3.44$.

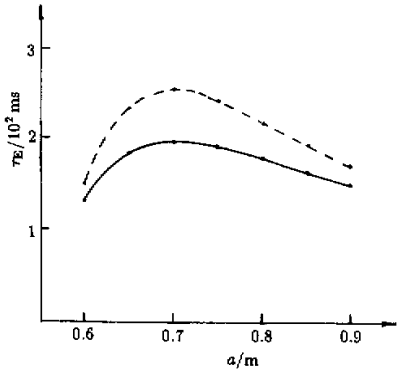

Fig. 2. τ_E vs a, the tokamak minor radius, for $n_0(10^{20}\,m^{-3}) = 1.5$, $P_T = 10\,\mathrm{MW}$, $B_t = 5.2\,\mathrm{T}$, R=2.48 m, $I_p = 1.7\,\mathrm{MA}$, $\alpha_n = 1.5$ and $\alpha_p = 2.0$.

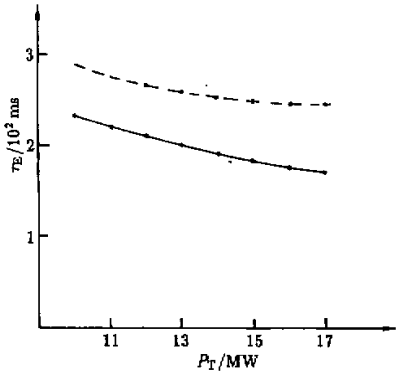

Fig. 3. τ_E vs P_T, the total heating power, for $n_0(10^{20}\,\mathrm{m}^{-3})$=1.5, $B_t = 4.6\,\mathrm{T}$, R=2.48 m, a=0.85 m, $I_p = 2.0\,\mathrm{MA}$, $\alpha_n = 1.5$ and $\alpha_p = 2.0$.

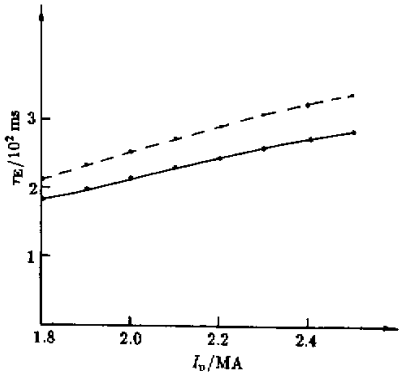

Fig. 4. τ_E vs I_p, the plasma current, for $n_0(10^{20}\,m^{-3}) = 1.5$, $P_T = 10\,\mathrm{MW}$, $B_t = 5.2\,\mathrm{T}$, R=2.48 m, a=0.85 m, $\alpha_n = 1.5$ and $\alpha_p = 2.0$.

minor radius a. To understand this result we should point out that there are disagreements among experimental results. In some cases, τ_E increases with minor radius, but in others, it behaves contrarily (especially for large tokamaks). Figure 3 shows that energy confinement will deteriorate with total heating power, P_T, increasing. However, when P_T is large enough, so that bootstrap current can play a significant role in total plasma current, the improvement of energy confinement due to bootstrap current will offset the deterioration due to the increase of power, and energy confinement will no longer deteriorate. Generally speaking, the larger the value of bootstrap current is, the larger the difference in τ_E with and without

bootstrap current will be. Therefore, we find from Figs. 1–8 that in the present day tokamaks (TFTR-like) there is a substantial part of energy confinement time that is due to bootstrap current. Figure 9 shows that while keeping q_a unchanged, the radius of $q = 1$ surface will decrease with bootstrap current. The decrease of r_1 due to bootstrap current may be a candidate for explanation of experimental results where sawtooth osillations disappeared because of auxiliary heating power.

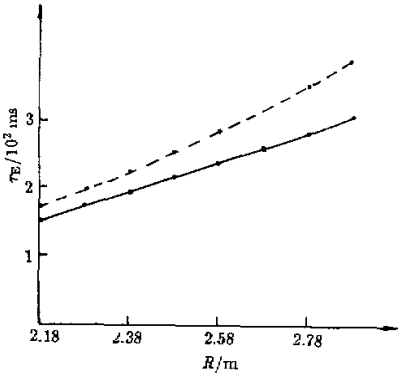

Fig. 5. τ_E vs R, the major radius. Other parameters are $n_0(10^{20}m^{-3}) = 1.5, P_T = 10$ MW, $B_t = 5.2$ T, a=0.85 m, $I_p = 2.0$ MA, α_n=1.5 and α_p=2.0.

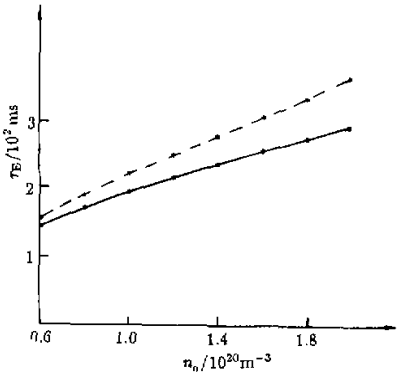

Fig. 6. τ_E vs n_0, the density on axis, for P_T = 10 MW, B_t = 5.2 T, R=2.48 m, a=0.85 m, I_p = 2.2 MA, α_n = 1.5 and α_p = 2.0.

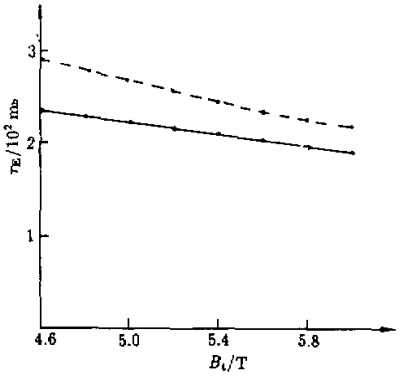

Fig. 7. τ_E vs B_t, the toroidal field, for $n_0(10^{20}m^{-3})$=1.5, P_T=10 MW, R = 2.48 m, a = 0.85 m, I_p = 2.0 MA, α_n = 1.5 and α_p = 2.0.

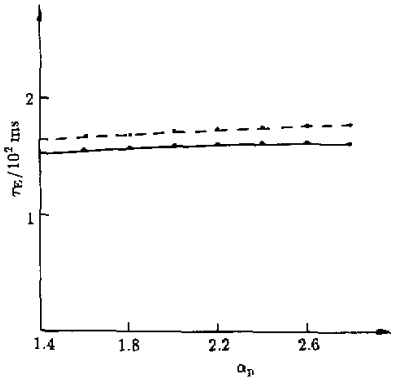

Fig. 8. τ_E vs α_p, the parameter of heating power deposition profile, for $n_0(10^{20}m^{-3}) = 1.0$, $P_T = 10$ MW, $B_t = 5.2$ T, $R = 2.48$ m, $a = 0.85$ m, $I_p = 2.2$ MA and $\alpha_n = 1.2$.

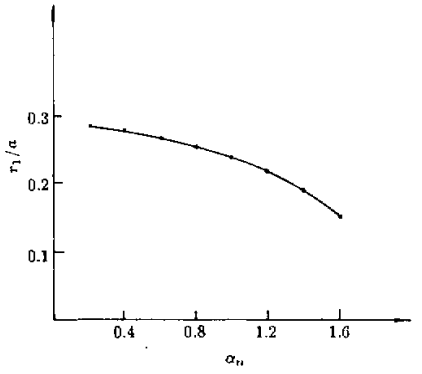

Fig. 9. The normalized radius of $q = 1$ surface as a function of α_n (or bootstrap current),for parameters occuring in Fig. 1.

V. SUMMARY AND DISCUSSION

New results for dependences of global confinement time on different plasma parameters are presented. It can be seen that for typical situations, effect of bootstrap current could be significant. If the bootstrap current is $15\% - 30\%$ of the total plasma current, global energy confinement time may increase by $20\% - 45\%$, and the maximum of related energy store will be nearly twice as much as that in the case where bootstrap current is absent. It can be concluded from calculation that bootstrap current is an increasing function of α_n, heating power P_T, plasma density n_0, plasma current I_p, major radius R, and a decreasing function of B_t. The dependence on minor radius is somewhat different (see Fig. 2). In addition, it can be found easily that τ_E for the chosen model increases with I_p (just as many previous authors have attempted to explain), R and n_0, but decreases with B_t, P_T. These results are in a reasonable agreement with the so-called Kaye-Goldston scaling law[9]

$$\tau_E \propto I_p^{1.24} R^{1.65} \alpha^{-0.49} n^{0.26} B_t^{-0.09} P_T^{-0.58}.$$

It should be pointed out that there exists another mechanism,namely, a transport due to nonlinear unstable microtearing mode[10] , which may be responsible for the observed energy confinement,because it has similar parameter dependence as those of η_i-turbulence. In this sense, there still exist some uncertainties about which mechanism (η_i-mode, microtearing mode, or combination of them) bears the real responsibility in energy confinement of auxiliary heating plasmas. But it is concluded that there is a significant part of global confinement time attributed to bootstrap current in large tokamak devices.

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