Controlling and synchronization of a hyperchaotic system based on passive control*

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In this paper, a new hyperchaotic system is proposed, and the basic properties of this system are analyzed by means of equilibrium point, Poincaré map, bifurcation diagram, and Lyapunov exponents. Based on the passivity theory, the controllers are designed to achieve the new hyperchaotic system globally, asymptotically stabilized at the equilibrium point, and also realize the synchronization between the two hyperchaotic systems under different initial values respectively. Finally, the numerical simulation results show that the proposed control and synchronization schemes are effective.

Keywords: hyperchaotic system, bifurcation diagram, Lyapunov exponent, passivity theory

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1. Introduction

Hyperchaotic systems not only have all the features and properties of chaotic systems, but also have more complicated nonlinear dynamic characteristics because they have more than one positive Lyapunov exponents and they are expanded in more than one direction. They have great potential applications in physics, engineering, mathematics, communication, and so on. Since hyperchaos was firstly reported by Rössler,[1] there have been considerable achievements in the study of hyperchaos. In the last decades, many hyperchaotic systems have been proposed and studied. Ott *et al.*[2] first presented a method of controlling chaos, which is currently known as the OGY method. Aside from the OGY method, there are many control techniques that have been developed for the controlling and synchronization hyperchaotic systems, such as linear state feedback, Lyapunov method, the feedback linearization, variable structure control, backstepping, delay feedback control, sliding mode control method, fuzzy control, and adaptive control.[3−11]

The passivity theory is considered as a powerful tool for analyzing the stability of nonlinear systems[12] The main interesting property is that passive systems are stable. Nowadays, many people are paying much attention to the passivity theory. In fact, passivity is a special case of the dissipative theory and passive control is an essential nonlinear control method. It starts from the energy of the system, and hence we should find out the energy function and design a controller to make the system passive. In this way, the global stability can be achieved. The aim of this work is to deal with how to design a controller. A linear feedback controller based on the passive technique has been designed[13,14], and also the synchronization between two hyperchaotic Lorenz systems under different initial conditions via a passive controller has been realized.[15] In both of these contributions, the passivity theory designed for only one output is used. Based on the properties of the passivity theory, we designed a novel passive controller with two outputs to realize the globally asymptotical stability of the new system and error system. Each method has its own character and is suitable to a certain area. The passive control is a smooth nonlinear state feedback method. It is simpler to achieve practically than the Lyapunov method, and the stabilization time is shorter than the feedback linearization and the adaptive control.

In this paper, a new four-dimensional (4D) hyperchaotic system is introduced, and its properties are analyzed, such as equilibrium point, 2D and 3D phase portraits, Poincaré map, bifurcation diagram, and Lyapunov exponents. Then, we introduce the passivity theory, and design the controllers based on

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the theory. The asymptotical stability of 4D hyperchaotic system is achieved globally, and two identical hyperchaotic systems are synchronized. Next, numerical simulations are given for the controlling and synchronization of the new hyperchaotic system based on the passivity controllers. Finally, conclusions are presented.

2. A new hyperchaotic system and its basic properties

In this section, a new hyperchaotic system is proposed, and its autonomy differential equations are described by

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = a(y-x), \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = bx + lxz + ew, \\ \dfrac{\mathrm{d}z}{\mathrm{d}t} = -hxy - ky^2 - cz - nw, \\ \dfrac{\mathrm{d}w}{\mathrm{d}t} = -dx, \end{cases} \tag{1}$$

where parameters $(a,\ b,\ c,\ d,\ e,\ h,\ l,\ k,\ n) = (10,\ 40,\ 2.5,\ 10,\ 1,\ 2,\ 1,\ 2,\ 1)$. In the following, we will analyze the dynamical properties of this new hyperchaotic system in detail.

First, in order to calculate the equilibrium of the system, we suppose

$$\begin{cases} a(y-x) = 0, \\ bx + lxz + ew = 0, \\ -hxy - ky^2 - cz - nw = 0, \\ -dx = 0. \end{cases}$$

Hyperchaotic system (1) has only one real equilibrium of $O(0,0,0,0)$. The Jacobian matrix is equal to

$$\boldsymbol{J} = \begin{bmatrix} -a & a & 0 & 0 \\ b + lz & 0 & lx & e \\ -hy & -hx - 2ky & -c & -n \\ -d & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} -10 & 10 & 0 & 0 \\ 40 & 0 & 0 & 1 \\ 0 & 0 & -2.5 & -1 \\ -10 & 0 & 0 & 0 \end{bmatrix}.$$

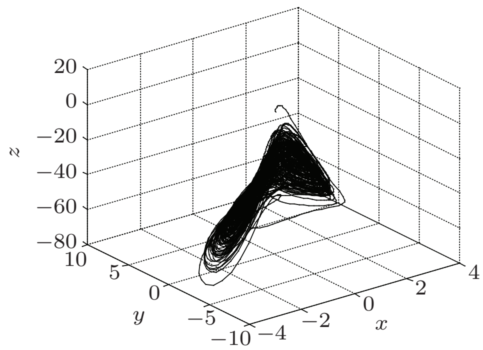

Fig. 1. Three-dimensional strange attractors of the system.

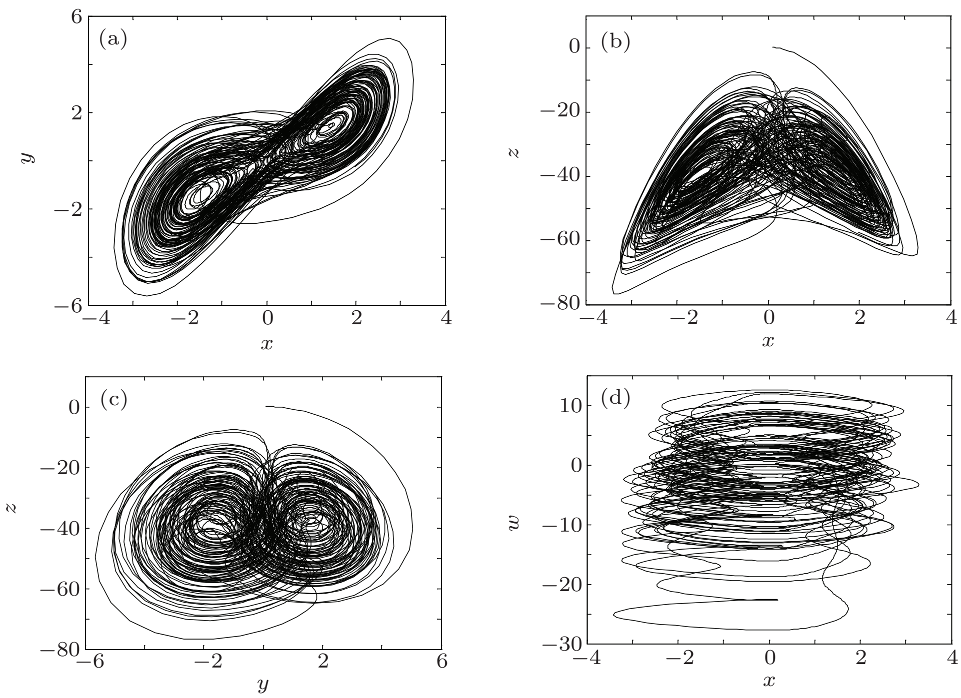

Fig. 2. Phase plane strange attractors: (a) x–y phase plane; (b) x–z phase plane; (c) y–z phase plane; (d) x–w phase plane.

Let $|\lambda I - J| = 0$, the eigenvalues corresponding to $O(0,0,0,0)$ are $\lambda_1 = -25.7096$, $\lambda_2 = -2.5000$, $\lambda_3 = 15.4580$, and $\lambda_4 = 0.2516$. Therefore, the equilibrium $O(0,0,0,0)$ is a saddle point of this new hyperchaotic system. Through numerical simulations, the hyperchaotic strange attractors of the system (1) are derived and shown in Figs. 1 and 2.[16,17]

The Poincaré mapping of this new hyperchaotic system is also analyzed. It is an intuitive method to determine whether a system is chaotic. It can be seen that the Poincaré mappings are a series of confused points in a specific area, as shown in Fig. 3. Thus, we are convinced that the system is a chaotic system.

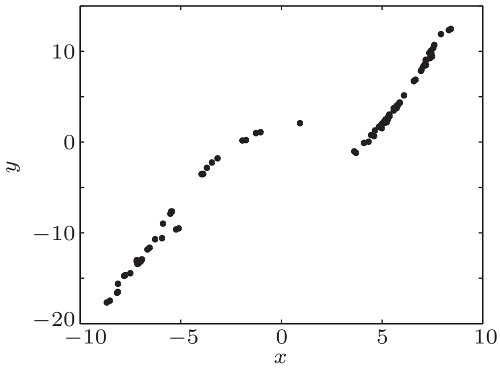

Fig. 3. Poincaré map of x–y plane.

The bifurcation diagram of x with increasing a is given in Fig. 4. We can see that the system shows abundant and complex dynamical behaviors with increasing parameter a. The divergence of system (1) is given by

$$\nabla V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} + \frac{\partial \dot{w}}{\partial w} = -a - c = -12.5.$$

The system (1) is dissipative with exponential convergence for the negative constant $\nabla V = -12.5$. Therefore, all the trajectories of the system will be restricted to a zero volume collection and fixed in an attractor.

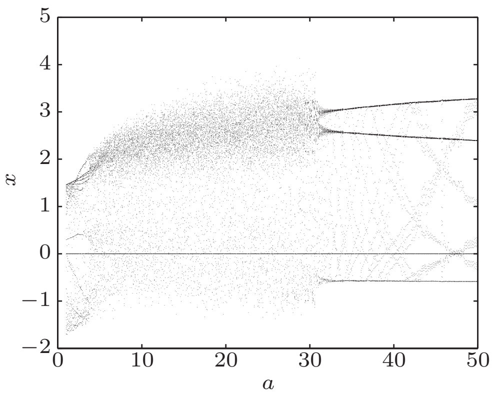

Fig. 4. Bifurcation diagram as a function of parameter a.

According to the chaos theory, the Lyapunov exponents measure the average divergence rate of the nearby trajectories in the continuous dynamical system. It is a quantitative method to determine whether a continuous dynamic system is chaotic. In general, a four-dimensional nonlinear system has two positive Lyapunov exponents, implying that it is hyperchaotic. The four Lyapunov exponents ($L_{\mathrm{E1}} = 0.6493$, $L_{\mathrm{E2}} = 0.2348$, $L_{\mathrm{E3}} = 0$, $L_{\mathrm{E4}} = -13.389$) of this nonlinear system are shown in Fig. 5.

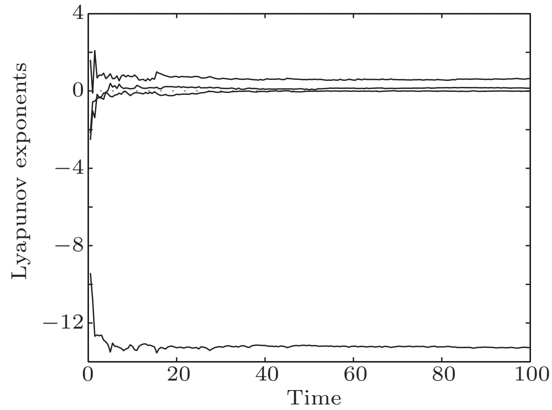

Fig. 5. Lyapunov exponents of the system.

Base on the Lyapunov exponents, we can calculate the Hausdroff (Lyapunov) dimension of the nonlinear autonomous system, i.e.,

$$\begin{aligned} D_{\mathrm{L}} &= j + \frac{1}{|L_{\mathrm{E}j+1}|} \sum_{i=1}^{j} L_{\mathrm{E}_i} \\ &= 3 + \frac{L_{\mathrm{E1}} + L_{\mathrm{E2}} + L_{\mathrm{E3}}}{|L_{\mathrm{E4}}|} = 3.066. \end{aligned}$$

Based on the above theoretical analysis of the system, the numerical simulations show that the system (1) is really a new hyperchaotic system, and has remarkable hyperchaotic dynamical properties and more complex topological structures.

3. Theory of passive control

The physical meaning of a passive system is that the energy of the nonlinear system increases less than the external injection. In other words, it is always accompanied by the loss of energy and cannot store more energy than that supplied externally. The passive system is a naturally stable system.

Consider the following affine nonlinear system:

$$\begin{cases} \dot{x} = f(x) + g(x)u, \\ y = h(x), \end{cases} \tag{2}$$

where $x \in \boldsymbol{R}^n$ is the state variable, $f(x)$ and $g(x)$ are the smooth vector fields, $u \in \boldsymbol{R}^m$ is the input, $y \in \boldsymbol{R}^m$ is the output, and $h(x)$ is a smooth mapping.

If the system (2) only has a storage function $H : \boldsymbol{R}^n \to \boldsymbol{R} \geq 0$, the system is dissipative with the supply rate $W[u(t), y(t)]$, that is,

$$H\,[x(T)] \leq H\,[x(0)] + \int_0^T W[u(\tau), y(\tau)]\,\mathrm{d}\tau,\quad T \geq 0. \tag{3}$$

When the system is dissipative with the supply rate $W(u, y) = u^{\mathrm{T}} y$, the system is of passivity, and apparent passivity is a special case of dissipative. More specifically, if the system (2) is of passivity, there exists a storage function $H(x)$ that is continuously differentiable positive semidefinite such that

$$u^{\mathrm{T}} y \geq \dot{H} = \frac{\partial H}{\partial x} f(x, u),\quad \forall (x, u) \in \boldsymbol{R}^n \times \boldsymbol{R}^m. \tag{4}$$

Actually, the inequality (4) can be expressed as

$$H[x(T)] - H[x(0)] \leq \int_0^{\mathrm{T}} u^{\mathrm{T}} y\,\mathrm{d}\tau. \tag{5}$$

Inequality (5) implies that the energy stored in the system is less than or equal to the supplied power, and the system's motion is always accompanied by the loss of energy. The $u^{\mathrm{T}} y$ is the supply rate of the external energy injected into the system which is accompanied by the input u.

When $t \geq 0$, there are real values β and $\rho \geq 0$, which satisfy

$$\int_0^t u^{\mathrm{T}}(\tau) y(\tau)\,\mathrm{d}\tau + \beta \geq \int_0^t \rho y^{\mathrm{T}}(\tau) y(\tau)\,\mathrm{d}\tau. \tag{6}$$

The system (2) is of passivity and can be formulated as follows:

$$\begin{cases} \dot{z} = q_0(z) + q_1(z, y) y, \\ \dot{y} = b(z, y) + a(z, y) u, \end{cases} \tag{7}$$

herein,

$$u = \alpha(y) + \beta(y) v. \tag{8}$$

If we can find a feedback controller with a similar form to Eq. (8), the nonlinear system (7) is equivalent to a passive system.[18]

4. Passive control of the hyperchaotic system

A new hyperchaotic system is presented in Section 2. In this section, the controlling of the new hyperchaotic system is studied based on the passive theory. The new hyperchaotic system (1) can be expressed as follows:

$$\begin{cases} \dot{z}_1 = a(y_1 - z_1), \\ \dot{z}_2 = -h y_1 z_1 - k y_1^2 - c z_2 - n y_2, \\ \dot{y}_1 = b z_1 + l z_1 z_2 + e y_2 + u_1, \\ \dot{y}_2 = -d z_1 + u_2, \end{cases} \tag{9}$$

where $z_1 = x$, $z_2 = z$, $y_1 = y$, and $y_2 = w$. Equation (9) can also be expressed in the normal form of Eq. (7) as follows:

$$\begin{cases} \boldsymbol{q}_0(z) = \begin{bmatrix} -a z_1 \\ -c z_2 \end{bmatrix}, \\ \boldsymbol{q}_1(z) = \begin{bmatrix} a & 0 \\ -h z_1 - k y_1 & -n \end{bmatrix}, \\ \boldsymbol{b}(z, y) = \begin{bmatrix} b z_1 + l z_1 z_2 + e y_2 \\ -d z_1 \end{bmatrix}, \\ \boldsymbol{a}(z, y) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \end{cases} \tag{10}$$

For the system (9), we need to design a smooth controller with a similar form to Eq. (8) to make the system passive. The controllers $[u_1, u_2]$ are needed to determine to stabilize the dynamical system (9) at the equilibrium point. We choose a storage function as follows:

$$V(z, y) = W(z) + \frac{1}{2} y^2, \tag{11}$$

where $W(z)$ is the Lyapunov function with $W(0) = 0$ and $W(z) = (z_1^2 + z_2^2)/2$. Then we have

$$\begin{aligned} \frac{\mathrm{d}V(z, y)}{\mathrm{d}t} &= \frac{\partial W(z)}{\partial z} \dot{z} + y \dot{y} = \frac{\partial W(z)}{\partial z} q_0(z) \\ &\quad + \frac{\partial W(z)}{\partial z} q_1(z, y) y + y b(z, y) \\ &\quad + y a(z, y) u. \end{aligned} \tag{12}$$

The system (7) complies with the external constraint $y = 0$, i.e., $\dot{z} = q_0(z)$. Then we have

$$\frac{\mathrm{d}}{\mathrm{d}t} W(z) = \frac{\partial W(z)}{\partial z} q_0(z) = [z_1, z_2][-a z_1, -c z_2]^{\mathrm{T}}$$

$$= -az_1^2 - cz_2^2 \le 0.$$

Since $W(z) \ge 0$ and $\dot{W}(z) \le 0$, $W(z)$ is the Lyapunov function of $q_0(z)$, and the $q_0(z)$ is globally asymptotically stable, which means the controlled system (9) is a minimum phase system based on the Lyapunov stability. Equation (12) becomes

$$\frac{\mathrm{d}}{\mathrm{d}t}V(z,y) \le \frac{\partial W(z)}{\partial z}q_1(z,y)y + yb(z,y) + ya(z,y)u. \tag{13}$$

We design the controller in the following form to make the system (7) passive.

$$u = a(z,y)^{-1}[-b^{\mathrm{T}}(z,y) - \frac{\partial}{\partial z}W(z)q_1(z,y) - \alpha y + v], \tag{14}$$

where α is a positive real value, v is an external signal which is connected to the reference input, and $W(z)$ is the Lyapunov function of $q_0(z)$.

Depending on Eq. (14), we select the following feedback controllers:

$$\begin{cases} u_1 = -(a+b)z_1 + (h-l)z_1z_2 - ey_2 \\ \qquad + 2ky_1z_2 - \alpha y_1 + v_1, \\ u_2 = -dz_1 + nz_2 - \alpha y_2 + v_2, \end{cases} \tag{15}$$

where $\alpha > 0$ is a positive constant, and v_1 and v_2 are external reference input signals. The hyperchaotic system (9) can be equivalent to a passive system and globally asymptotically stabilized at its zero equilibrium.

Substituting Eqs. (10) and (15) into Eq. (13) yields

$$\frac{\mathrm{d}}{\mathrm{d}t}V(z,y) \le -\alpha y^2 + vy. \tag{16}$$

Then, taking integration over both sides of Eq. (16), we obtain

$$V(z,y) - V(z_0,y_0) \le -\int_0^t \alpha y^2(\tau)\mathrm{d}\tau + \int_0^t v(\tau)y(\tau)\mathrm{d}\tau. \tag{17}$$

For $V(z,y) \ge 0$, let $V(z_0,y_0) = \beta$, then the inequality (17) can be rewritten as

$$\int_0^t v(\tau)y(\tau)\mathrm{d}\tau + \beta \ge \int_0^t \alpha y^2(\tau)\mathrm{d}\tau + V(z,y) \ge \int_0^t \alpha y^2(\tau)\mathrm{d}\tau. \tag{18}$$

It satisfies the passive definition (6). The hyperchaotic system (9) is rendered to be an output passive system under the feedback controllers. We can use the feedback controller (15) to stabilize system (9) at its zero equilibrium with external reference input signal $v = 0$.

5. Passive synchronization of the hyperchaotic system

In this section, we will introduce synchronization between two identical hyperchaotic systems using passive control. First, we choose the new hyperchaotic system which is described by Eq. (1) as the drive system. The drive system is described as follows:

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1), \\ \dot{x}_2 = bx_1 + lx_1x_3 + ex_4, \\ \dot{x}_3 = -hx_1x_2 - kx_2^2 - cx_3 - nx_4, \\ \dot{x}_4 = -dx_1. \end{cases} \tag{19}$$

The response system is given by

$$\begin{cases} \dot{y}_1 = a(y_2 - y_1), \\ \dot{y}_2 = by_1 + ly_1y_3 + ey_4 + u_1, \\ \dot{y}_3 = -hy_1y_2 - ky_2^2 - cy_3 - ny_4, \\ \dot{y}_4 = -dy_1 + u_2, \end{cases} \tag{20}$$

where u_1 and u_2 are controllers which are designed for achieving synchronization between the drive system and the response system. We set $e_1 = y_1 - x_1$, $e_2 = y_2 - x_2$, $e_3 = y_3 - x_3$, and $e_4 = y_4 - x_4$, then the error system between the drive system and the response system can be expressed as

$$\begin{cases} \dot{e}_1 = a(e_2 - e_1), \\ \dot{e}_2 = be_1 + lx_1e_3 + lx_3e_1 + le_1e_3 + ee_4 + u_1, \\ \dot{e}_3 = -hx_1e_2 - hx_2e_1 - he_1e_2 - 2kx_2e_2 \\ \qquad - ke_2^2 - ce_3 - ne_4, \\ \dot{e}_4 = -de_1 + u_2. \end{cases} \tag{21}$$

The new error system (21) can be expressed in the normal form as

$$\begin{cases} \dot{z}_1 = a(y_1 - z_1), \\ \dot{z}_2 = -hx_1y_1 - hx_2z_1 - hy_1z_2 - 2kx_2y_1 \\ \qquad - ky_1^2 - cz_3 - ny_2, \\ \dot{y}_1 = bz_1 + lx_1z_2 + lx_3z_1 + lz_1z_3 + ey_2 + u_1, \\ \dot{y}_2 = -dz_1 + u_2, \end{cases} \tag{22}$$

and

$$\begin{cases} \boldsymbol{q}_0(z) = \begin{bmatrix} -az_1 \\ -cz_2 - hx_2z_1 \end{bmatrix}, \\ \boldsymbol{q}_1(z) = \begin{bmatrix} a & 0 \\ -hz_1 - kx_1 - 2kx_2 - ky_1 & -n \end{bmatrix}, \\ \boldsymbol{b}(z,y) = \begin{bmatrix} \boldsymbol{b}z_1 + lz_1z_2 + lx_1z_2 + lx_2z_1 + ey_2 \\ -dz_1 \end{bmatrix}, \\ \boldsymbol{a}(z,y) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \end{cases} \tag{23}$$

where $z_1 = e_1$, $z_2 = e_3$, $y_1 = e_2$, and $y_2 = e_4$. The aim is to determine the controllers $[u_1, u_2]$ to achieve the synchronization between the drive system (19) and the response system (20). We choose a storage function as follows:

$$V(z,y) = W(z) + \frac{1}{2}y^2, \tag{24}$$

where $W(z)$ is the Lyapunov function with $W(0) = 0$ and $W(z) = (z_1^2 + z_2^2)/2$.

The system (7) is consistent with the external constraint $y = 0$, i.e., $\dot{z} = q_0(z)$. We can obtain $|x_2| < 6$ from the phase plant in Fig. 2, and $h^2x_2^2/(4a^2) - c < 0$ with the parameters $(a,c,h) = (10, 2.5, 2)$. Then we have

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}W(z) &= \frac{\partial W(z)}{\partial z}q_0(z) \\ &= -az_1^2 - cz_2^2 - hx_2z_1z_2 \\ &= -a\left(z_1 + \frac{hx_2}{2a}z_2\right)^2 + \left(\frac{h^2x_2^2}{4a^2} - c\right)z_2^2 \\ &< 0. \end{aligned}$$

Since $W(z) \geq 0$ and $\dot{W}(z) \leq 0$, $W(z)$ is the Lyapunov function of $q_0(z)$, which means the controlled system (21) is a minimum phase system based on the Lyapunov stability.

Depending on Eq. (14), we select the following feedback controller:

$$\begin{cases} u_1 = -(a+b)z_1 + (h-l)z_1z_2 + (h-l)x_1z_2 \\ \qquad - lz_1x_3 - ey_2 + 2ky_2z_2 \\ \qquad + kz_2y_1 - \alpha y_1 + v_1, \\ u_2 = -dz_1 + nz_2 - \alpha y_2 + v_2. \end{cases} \tag{25}$$

In this way, the error system (21) can be equivalent to a passive system and globally asymptotically stabilized.

6. Simulation results

Simulation results are presented in this section to demonstrate the effectiveness of the proposed control and synchronization scheme. The simulation results are carried out using the MATLAB software and the fourth order Runge-Kutta method. A time step size is chose to be 0.001. Let $\alpha = 1$ and $v = 0$. To control the new system, the initial values of the system are set to be $(x, y, z, w) = (0.1, 0.1, 0.1, 0.1)$. The results are shown in Figs. 6 and 7, from which we can see that the controlled system stabilizes at the equilibrium point $O(0, 0, 0, 0)$.

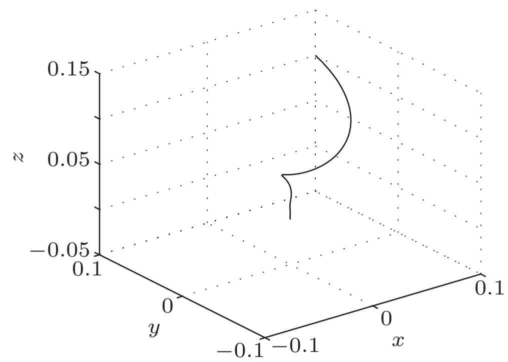

Fig. 6. Three-dimensional (x, y, z) plot of the controlled system via passive control.

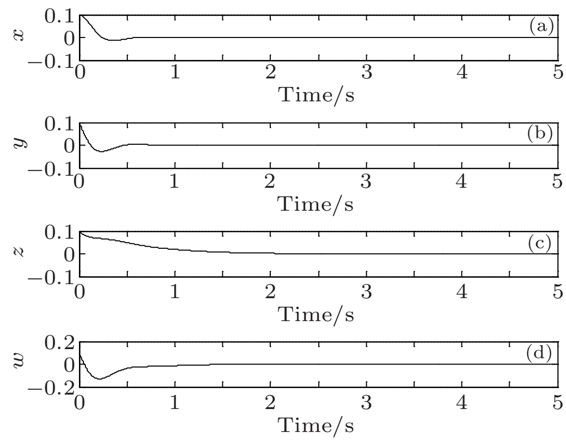

Fig. 7. Time evolution of stabilization via passive control. Time evoltuion of (a) x, (b) y, (c) z, and (d) w.

For the synchronization of the drive system and the response system, the initial values of the systems are set to be $(x_1, x_2, x_3, x_4) = (1, 2, 3, 4)$, and $(y_1, y_2, y_3, y_4) = (0.1, 0.2, 0.3, 0.4)$. At 5 s, the controller is added to the response system. Figure 8 shows that the state trajectories of the response system asymptotically approach the drive system. From Fig. 9 we can see that the errors are indeed close to zero. The results imply that the two identical new hyperchaotic systems synchronize with each other.

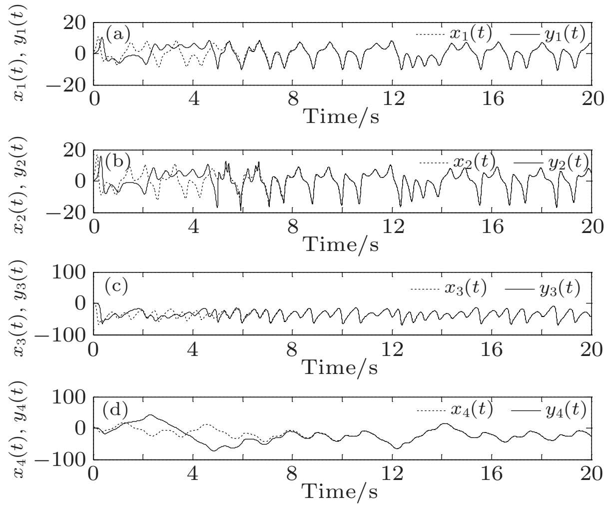

Fig. 8. Synchronization between two identical hyperchaotic systems. (a) $x_1(t)$ and $y_1(t)$; (b) $x_2(t)$ and $y_2(t)$; (c) $x_3(t)$ and $y_3(t)$; (d) $x_4(t)$ and $y_4(t)$.

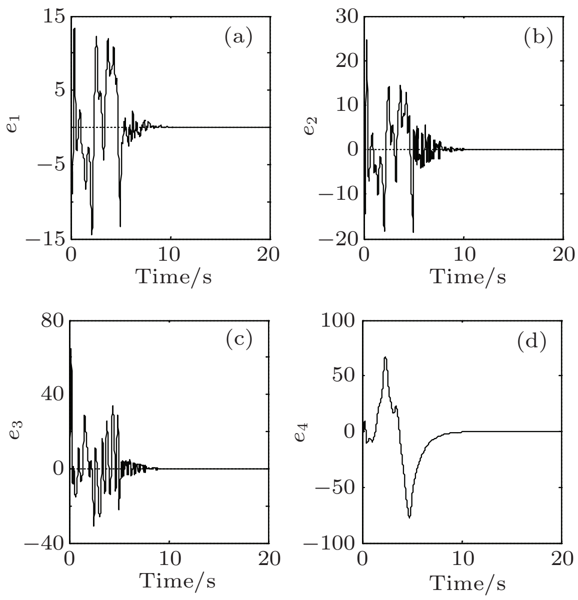

Fig. 9. Synchronization errors of the drive system (19) and response system (20). Time evolution of (a) e_1, (b) e_2, (c) e_3, and (d) e_4.

7. Conclusions

In this paper, we have proposed a new hyperchaotic system and analysed its basic properties. Then, the passive theory has been introduced. Based on this theory, we have designed a feedback controller to make the new system equivalent to a passive system. Furthermore, with the help of the feedback controllers, the new system can be globally asymptotically stabilized at its zero equilibrium. The two identical new hyperchaotic systems which synchronize with each other are achieved via passive feedback controller. Finally, the control method is tested through numerical simulation. It is noteworthy that the proposed hyperchaotic system can be used for constructing electronic oscillator in electronics.

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