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Controlling dipole squeezing of two atoms inside a cavity via manipulating an atom outside the cavity*

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Considering that three two-level atoms are initially in the GHZ single state and two of the atoms are simultaneously put into a cavity initially in the coherent state, we investigate the dipole squeezing properties of the two atoms inside the cavity under the condition of resonant interaction. It is shown that dipole squeezing properties of the two atoms inside the cavity are strongly affected by rotation manipulating of the atom outside the cavity.

Keywords: GHZ state, atomic dipole squeezing, rotationally manipulating, controlling
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1. Introduction

The concept of atomic dipole squeezing was first introduced by Walls and the co-worker [1] in studying the fluctuation of single atom resonance fluorescent light in 1981. When the fluctuation of some dipole moment component of the atom is lower than the quantum noise limit, it is said that the atom is in a squeezed state. Experimentally, it has been demonstrated that atoms in a squeezed state can radiate squeezed light.[2] Soon afterwards, the concept of atomic dipole square squeezing was put forward in the study of two-atom resonant fluorescence.[3] The studies on the effects of atomic dipole square squeezing have been an attractive field in quantum optics and are limited to the cases that the initial states are single-atom states or product states of two atoms interacting with light field.[3−11] In recent years, the properties of the atomic entangled state have aroused huge interest. Entangled states indicate the non-local properties of quantum mechanics and are widely applied in quantum computation, quantum cryptography, quantum teleportation and so on.[12,13] Recently, Yang and Guo [14] have considered a pair of two-level atoms initially in the EPR single state, then put one atom into the resonant cavity that is in the vacuum state. The result shows that the emission properties of the atom inside the cavity are much affected by the manipulation of the atom outside the cavity. We [15] have considered entanglement swapping and disentanglement schemes via a Greenberger–Horne–Zeilinger (GHZ) entangled state of three two-level atoms interacting with a coherent field. When a two-level atom is injected into a high-Q cavity and other two atoms are far away form the cavity, the entanglement swapping and disentanglement can be realized by carrying out a measurement on the atom inside the cavity and by selecting appropriate interaction time of atom inside the cavity with the coherent field. However, we note that the subject of how the manipulation of the atom outside the cavity affects the dipole squeezing of the atom inside a cavity has not been studied so far. In this paper, we consider three two-level atoms A, B, and C initially in the GHZ single state, then we simultaneously put two atoms B and C into a cavity initially in coherent state and let the atom A be far away from the cavity. It will be seen that dipole squeezing properties of the two atoms B and C inside the cavity are strongly affected by manipulating the atom A outside

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the cavity. However, if we perform a rotation operation on the atom A and then perform a measurement on it, then different measurements produce different dipole squeezing effect of the atoms inside the cavity.

2. Theoretical model

We first assume that three two-level atoms A, B, and C are initially prepared in a single GHZ entangled state, the atoms B and C are simultaneously injected into a cavity in coherent state, and atom A is far away from the cavity (as shown in Fig.1). Under the resonant condition($\omega_f - \omega_a = 0$), the Hamiltonian of the system in the rotating-wave approximation can be written as ($\hbar = 1$)

$$\begin{aligned} \hat{H} &= \hat{H}_0 + \hat{H}_I, \\ \hat{H}_0 &= \omega_f(\hat{a}^+\hat{a} + \hat{S}_{zA} + \hat{S}_{zB} + \hat{S}_{zC}), \\ \hat{H}_I &= g(\hat{a}^+\hat{S}_B^- + \hat{a}^+\hat{S}_C^- + \hat{S}_B^+\hat{a} + \hat{S}_C^+\hat{a}), \end{aligned} \tag{1}$$

where $\hat{a}^+$ and $\hat{a}$ are, respectively, the creation and annihilation operators of the field mode with frequency ω_f; ω_a is the atomic transition frequency of B and C; g is the coupling constant between the atoms B and C with the cavity field; $\hat{S}_{zi}$ and $\hat{S}_i^{\pm}$ (the cavity field interacting only with the atoms B and C) are the inversion and transition operators for the ith (i=A, B, C) atom, respectively.

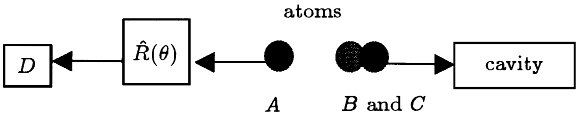

Fig.1. The proposed experiment of three two-level toms A, B, and C interacting with a cavity initially in coherent state.

We next assume that the cavity mode is initially prepared in the coherent state:

$$|\Psi(0)\rangle_F = \sum_{n=0}^{\infty} f_n |n\rangle = \mathrm{e}^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle, \tag{2}$$

where α is a complex number, $|\alpha|^2 = \bar{n}$ is the average photon number. The three two-level atoms A, B, and C are prepared in the GHZ state

$$|\Psi(0)\rangle_{ABC} = \frac{1}{\sqrt{2}}(|e_A, e_B, e_C\rangle - |g_A, g_B, g_C\rangle). \tag{3}$$

Thus the initial state vector of the system takes the form

$$\begin{aligned} |\Psi(0)\rangle_{ABC+F} =& \frac{1}{\sqrt{2}} \sum_{n=0}^{\infty} f_n |n\rangle \\ & \times (|e_A, e_B, e_C\rangle - |g_A, g_B, g_C\rangle). \end{aligned} \tag{4}$$

Then, with the initial condition, the solution (the state vector of system at time $t > 0$) of the Schrödinger equation in the interaction picture is given by

$$\begin{aligned} |\Psi(t)\rangle_{ABC+F} =& \sum_{n=0}^{\infty} f_n A(t) |e_A, e_B, e_C, n\rangle \\ & + \sum_{n=0}^{\infty} f_n B(t) |e_A, e_B, g_C, n+1\rangle \\ & + \sum_{n=0}^{\infty} f_n C(t) |e_A, g_B, e_C, n+1\rangle \\ & + \sum_{n=0}^{\infty} f_n D(t) |e_A, g_B, g_C, n+2\rangle \\ & + \sum_{n=2}^{\infty} f_n E(t) |g_A, e_B, e_C, n-2\rangle \\ & + \sum_{n=1}^{\infty} f_n F(t) |g_A, g_B, e_C, n-1\rangle \\ & + \sum_{n=1}^{\infty} f_n G(t) |g_A, e_B, g_C, n-1\rangle \\ & + \sum_{n=0}^{\infty} f_n H(t) |g_A, g_B, g_C, n\rangle. \end{aligned} \tag{5}$$

Here

$$A(t) = -\frac{n+1}{\sqrt{2}(2n+3)}[1 - \cos(\sqrt{2(2n+3)}gt)] + \frac{1}{\sqrt{2}}, \tag{6}$$

$$B(t) = C(t) = -\mathrm{i}\frac{1}{2}\frac{\sqrt{n+1}}{\sqrt{2n+3}} \sin(\sqrt{2(2n+3)}gt), \tag{7}$$

$$D(t) = -\frac{\sqrt{n+1}\sqrt{n+2}}{\sqrt{2}(2n+3)}[1 - \cos(\sqrt{2(2n+3)}gt)], \tag{8}$$

$$E(t) = \frac{\sqrt{n-1}\sqrt{n}}{\sqrt{2}(2n-1)}[1 - \cos(\sqrt{2(2n-1)}gt)], \tag{9}$$

$$F(t) = G(t) = \mathrm{i}\frac{1}{2}\frac{\sqrt{n}}{\sqrt{2n-1}} \sin(\sqrt{2(2n-1)}gt), \tag{10}$$

$$H(t) = \frac{n}{\sqrt{2}(2n-1)}[1 - \cos(\sqrt{2(2n-1)}gt)] - \frac{1}{\sqrt{2}}. \tag{11}$$

To describe the dipole squeezing of the two atoms, we now introduce two Hermitian operators of the two atoms; they are orthogonal to each other:

$$\hat{S}_1 = \frac{1}{2}(\hat{S}_B^+ + \hat{S}_C^+ + \hat{S}_B^- + \hat{S}_C^-), \tag{12}$$

$$\hat{S}_2 = \frac{1}{2\mathrm{i}}(\hat{S}_B^+ + \hat{S}_C^+ - \hat{S}_B^- - \hat{S}_C^-). \tag{13}$$

They satisfy the commutation relation

$$[\hat{S}_1, \hat{S}_2] = \mathrm{i}\hat{S}_Z, \tag{14}$$

where $\hat{S}_Z = \hat{S}_{B,Z} + \hat{S}_{C,Z}$. The uncertainty relation is

$$\left\langle (\Delta \hat{S}_1)^2 \right\rangle \left\langle (\Delta \hat{S}_2)^2 \right\rangle \geq \frac{1}{4} \left\langle \hat{S}_Z \right\rangle^2 , \qquad (15)$$

where $\left\langle (\Delta \hat{S}_i)^2 \right\rangle = \left\langle \hat{S}_i^2 \right\rangle - \left\langle \hat{S}_i \right\rangle^2$ are the variances in $\hat{S}_i$. It follows that

$$\left\langle (\Delta \hat{S}_i)^2 \right\rangle < \frac{1}{2} \left| \left\langle \hat{S}_Z \right\rangle \right| , \qquad (16)$$

or

$$Q_i = \left\langle (\Delta \hat{S}_i)^2 \right\rangle - \frac{1}{2} \left| \left\langle \hat{S}_Z \right\rangle \right| , \quad (i = 1, 2), \qquad (17)$$

squeezing is said to exist when $Q_i < 0$.

3. Results and discussion

Firstly, we consider the dipole squeezing of the atoms B and C inside the cavity, before a rotation is manipulated for atom A. According to Eq.(5), after the atoms B and C are interacting with the cavity field for a time t, if the atom A is detected in the ground state by a detector D, then the state of the system is reduced. It is easy to obtain the function Q_2 that exhibits the dipole squeezing effect by Eq.(17). Figure 2 shows the evolution of the function Q_2 for the reduced state, in different initial average photon number. On the other hand, one can easily find that the function Q_2 does not exhibit the dipole squeezing effect, in the same parameter cases, when the atom A is detected in the excited state.

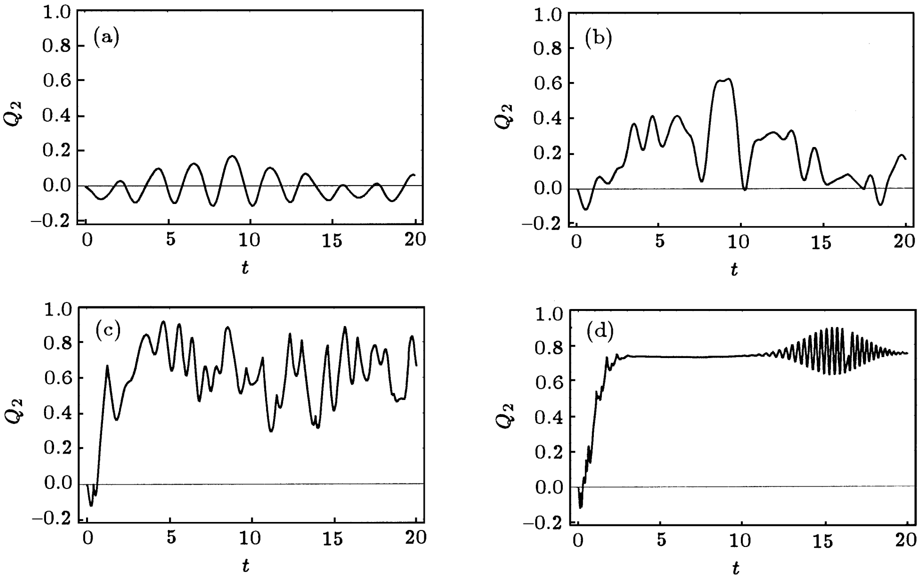

Fig.2. Time evolution of Q_2 for atom A detected in the ground state as function of $\bar{n} \cdot g$=1 and (a) $\bar{n}$=0.5; (b) $\bar{n}$=1; (c) $\bar{n}$=2; (d) $\bar{n}$=5.

In the following, we discuss the case when we insert a rotation operation $\hat{R}(\theta)$ on the atom A, before the atom is detected (see Fig.1). The rotation is defined as

$$\begin{aligned} \hat{R}(\theta) \left| g \right\rangle_A &= \cos\theta \left| g \right\rangle_A + \sin\theta \left| e \right\rangle_A , \\ \hat{R}(\theta) \left| e \right\rangle_A &= -\sin\theta \left| g \right\rangle_A + \cos\theta \left| e \right\rangle_A . \end{aligned} \qquad (18)$$

After the rotation operation, Eq.(5) is changed accordingly. For comparison convenience, we plot in Fig.3 in a similar way as in Fig.2. Again the function Q_2 exhibits dipole squeezing effect at some time, but the function Q_1 does not show dipole squeezing effect at all. It is also seen from Fig.3 that rotation operation angle influences the dipole squeezing obviously.

Furthermore, we change the view angle and keep the rotation operation angle fixed, so that we can discuss only the influence of the initial field conditions. When we vary the initial average photon number, the results are shown in Fig.4. Again, when atom A is detected in the ground state by a detector D, the function Q_2 exhibits dipole squeezing effect.

4. Conclusion

We have studied the atomic dipole squeezing properties of three two-level atoms initially in the GHZ single state interacting with the coherent field state. The results show that the interaction of a cavity field with three entangled two-level atoms can give more interesting properties of atomic dipole squeezing than those with a disentangled atom or two disentangled atoms. Furthermore, there are more parameters that we can use to control the atomic dipole squeezing properties.

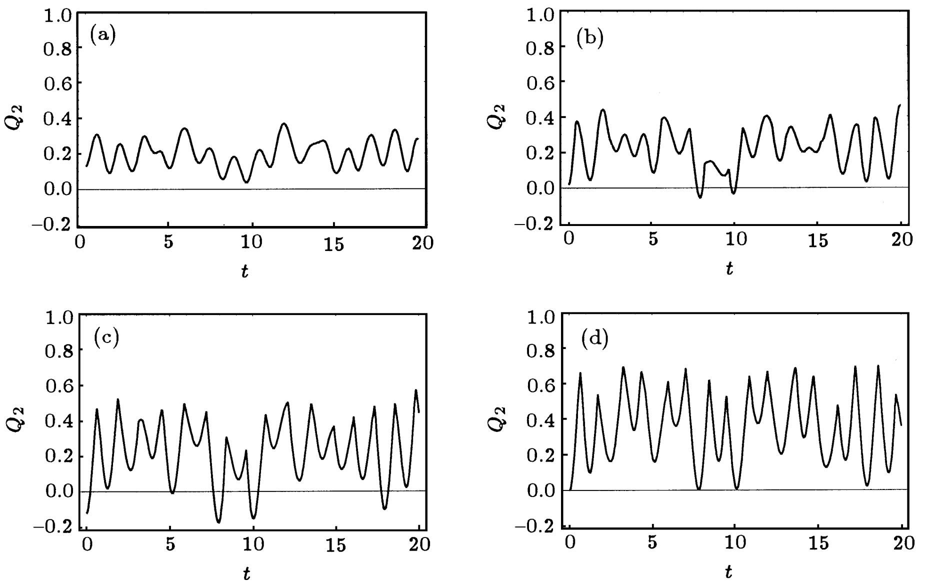

Fig.3. Time evolution of Q_2 for atom A detected in the ground state as function of rotation angle $\theta \cdot g$=1, $\bar{n}$=0.5 and (a) $\theta = \pi/6$; (b) $\theta = \pi/4$; (c) $\theta = \pi/3$; (d) $\theta = \pi/2$.

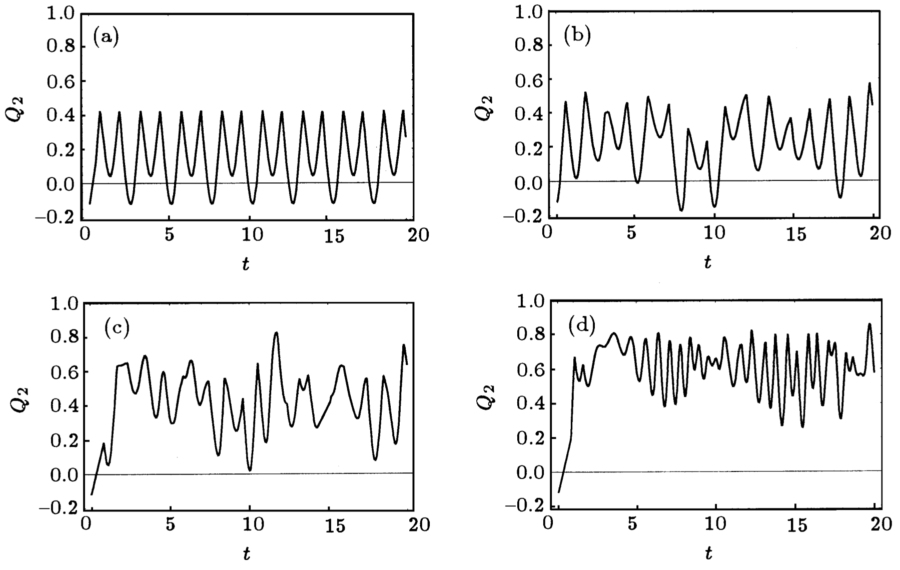

Fig.4. Time evolution of Q_2 for atom A detected in the ground state as function of $\bar{n} \cdot g$=1, $\theta = \pi/3$, and (a) $\bar{n}$=0.01; (b) $\bar{n}$=0.5; (c) $\bar{n}$=1; (d) $\bar{n}$=2.

References

[1] Walls D F and Zoller P 1981 *Phys. Rev. Lett.* **47** 709

[2] Wodkiewicz K, Knight P L, Buckle S J and Barnett S M 1987 *Phys. Rev.* A **35** 2567

[3] Li X S, Lin D L and George T F 1989 *Phys. Rev.* A **40** 2504

[4] Zhou P and Peng J S 1989 *Acta Phys. Sin.* **38** 2044 (in Chinese)

[5] Ashraf M M and Razmi M S K 1992 *Phys. Rev.* A **45** 8121

[6] Zhan Y B 1994 *Phys. Lett.* A **192** 60

[7] Liu T K and Peng J S 1997 *Acta Opt. Sin.* **17** 991 (in Chinese)

[8] Liu T K, Wang J S, Liu X J and Zhan M S 1999 *Acta Phys. Sin.* **48** 2051 (in Chinese)

[9] Wu Y and Yang X X 2001 *Phys. Rev.* A **63** 43816

[10] Huang C J, Heu H Y, Zhou M and Li J F 2002 *Acta Phys. Sin.* **51** 1049 (in Chinese)

[11] Zhang J T, Chen C Y and Tong C Y 1999 *Chin. Phys.* **8** 261

[12] Bennett C H, Brassard G and Mermin N D 1992 *Phys. Rev. Lett.* **68** 557

[13] Cirac J I and Zoller P 1995 *Phys. Rev. Lett.* **74** 4091

[14] Yang C P and Guo G C 1999 *Phys. Lett.* A **255** 129

[15] Liu T K, Wang J S, Feng J and Zhan M S 2002 *Chin. Phys. Lett.* **19** 1573