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0–π transition induced by the barrier strength in spin superconductor Josephson junctions*

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The Andreev-like levels and the free energy of the spin superconductor/insulator/spin superconductor junction are obtained by using the Bogoliubov–de Gennes equation. The phase dependence of the spin supercurrents exhibits a 0–π transition by changing the barrier strength. The dependences of the critical current on the barrier strength and the temperature are also presented.

Keywords: spin superconductor, Josephson junction

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1. Introduction

In the conventional superconductor, the electrons are condensed into the Cooper pairs, forming a Bardeen–Cooper–Schrieffer (BCS) ground state.[1] Each Cooper pair carries two units of the electron charge and no spin polarization for spin-singlet pairing. Being the counterpart of the charge superconductor, there also exists a quantum state, called a spin superconductor (SSC), where the condensed pair is composed of an electron and a hole rather than two electrons.[2–5] The electronlike spin-up carriers and the holelike spin-up carriers attract each other due to the Coulomb interaction[6] and form the Cooper pairs in the SSC, which are the charge neutral spin-triplet pairs carrying two units of the electron spin. About four decades ago,[7,8] electron–hole Cooper pairs were predicted in a double-layer system, where the electron in one layer and the hole in the other layer are separated by an insulator. The insulating layer is proposed to be thin enough to allow for the strong Coulomb interaction between the electrons and the holes leading to an effective attraction but thick enough to prevent the electron–hole recombination. Experimentally, the existence of the exciton condensate in the semiconductor double quantum well systems was also reported.[9,10] In addition, the spin superconductivity can exist in various systems, such as the spin-polarized triplet exciton systems of graphene,[2,3] Bose–Einstein condensate of magnetic atoms,[11–13] Bose–Einstein condensate of magnons and spinons,[14–16] and so on. Very recently, the signature of spin superconductivity has also been observed experimentally in canted antiferromagnet Cr_2O_3.[17]

The conventional Josephson effect is the phenomenon of the supercurrent across a device known as the Josephson junction, which consists of two superconductors coupled by a weak link.[18–23] The charge supercurrent across the junction is mediated by the Andreev bound states formed by the electrons and holes inside the normal segment of the junction.[24–31] As an analogy of the conventional Josephson junction, the SSC Josephson junction can be achieved by a weak link between two SSCs. There are both the spin-conserved and the spin-flip reflections at the normal-metal/SSC interface,[3] which are the counterparts of the normal reflection and the Andreev reflection at the normal-metal/superconductor interface, respectively. The repeated spin-flip reflections in the SSC Josephson junction, just like the Andreev reflection in the conventional Josephson junction, will lead to the formation of Andreev-like bound states and a dissipationless pure spin current.[32]

In this paper, we investigate an SSC Josephson junction by employing the Bogoliubov–de Gennes (BdG) approach, which is completely in parallel to the treatment in the conventional Josephson junction. The Andreev-like levels and the free energy of the junction are obtained. It is shown that there is a pure spin supercurrent driven by the phase difference of the junction. The current phase relation can exhibit a 0–π transition by tuning the barrier strength. We note that there are two differences between the SSC Josephson junction and the conventional one. First, the supercurrent in the SSC Josephson junction is a pure spin current rather than a charge current. Second, the 0–π transition is tuned by the barrier strength rather than the magnetic middle layer.[33–36] The dependences of the critical current on the barrier strength and the temperature are also discussed.

The rest of this paper is organized as follows. The formalism is presented in Section 2. The numerical results are

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discussed in Section 3. Finally, the main results are summarized in Section 4.

2. Formalism

The model under consideration is a one-dimensional junction oriented along the x axis. The junction consists of two SSCs sandwiched by an insulating layer at $x = 0$. The SSC can be described by a minimal two-band model,[6] where an electron in the spin-up conduction band and a hole in the spin-down valence band can be condensed into a Cooper pair due to the Coulomb interaction. Both the electrons in the spin-up band and the holes in the spin-down band possess the spin of $+\hbar/2$, thus the Cooper pair is charge neutral and has a spin of $\hbar$. The dispersion of the spin-up conduction band and that of the spin-down valence band are assumed as $\varepsilon_+ = p^2/2m - M_0$ and $\varepsilon_- = -p^2/2m + M_0$, respectively, with p being the momentum, m the mass of the electron, and M_0 the exchange energy. The spin-down conduction band and the spin-up valence band are far from the Fermi level and are irrelevant to the transport.

In the basis of $(\psi_+, \psi_-)^{\mathrm{T}}$, with $\psi_\pm$ being the field operators of the electrons in band $\varepsilon_\pm$, the BdG Hamiltonian of the SSC reads

$$H = \begin{pmatrix} \varepsilon_+ & \Delta \\ \Delta^* & \varepsilon_- \end{pmatrix}, \tag{1}$$

where Δ is the pair potential between the electrons in band ε_+ and the holes in band ε_-. The pair potentials in the left and right SSCs have a phase difference φ. The insulating layer is modeled by a barrier potential $V\delta(x)$. The eigenfunction of the BdG Hamiltonian can be obtained as

$$\psi(x) = \begin{cases} a\begin{pmatrix} v \\ u \end{pmatrix} \mathrm{e}^{\mathrm{i}kx} + b\begin{pmatrix} u \\ v \end{pmatrix} \mathrm{e}^{-\mathrm{i}kx}, & x<0, \\ c\begin{pmatrix} u\mathrm{e}^{\mathrm{i}\varphi/2} \\ v\mathrm{e}^{-\mathrm{i}\varphi/2} \end{pmatrix} \mathrm{e}^{\mathrm{i}kx} + d\begin{pmatrix} v\mathrm{e}^{\mathrm{i}\varphi/2} \\ u\mathrm{e}^{-\mathrm{i}\varphi/2} \end{pmatrix} \mathrm{e}^{-\mathrm{i}kx}, & x>0, \end{cases} \tag{2}$$

where

$$u = \sqrt{(E + \sqrt{E^2 - |\Delta|^2})/2E},$$

$$v = \sqrt{(E - \sqrt{E^2 - |\Delta|^2})/2E},$$

with E being the energy of the quasiparticles. The wave vector is written as $k = \sqrt{2m(M_0 + \sqrt{E^2 - |\Delta|^2})/\hbar^2}$. For the states with $E > |\Delta|$, the wave vectors are real, while for the subgap states, the wave vectors have a small imaginary part so that the wave functions properly decay at the infinity. The coefficients a, b, c, and d can be determined by matching the boundary conditions

$$\psi(0^+) = \psi(0^-), \quad \psi'(0^+) - \psi'(0^-) = \frac{2mV}{\hbar^2}\sigma_z \psi(0), \tag{3}$$

where σ_z is the Pauli matrix.

The Pauli matrix in Eq. (3) indicates that the boundary condition in an SSC junction is quite different from the one in a conventional superconductor junction. For the conventional superconductor, the two diagonal blocks of the BdG Hamiltonian describe the electrons and the holes, respectively. Both the kinetic energy and the barrier potential possessed by the electrons are opposite to those by the holes so that there is no Pauli matrix in the boundary condition for a nonmagnetic junction. For the SSC, the two diagonal blocks of the BdG Hamiltonian describe the electrons in the conduction band and the electrons in the valence band, respectively. The conduction electrons have opposite kinetic energy to the valence electrons but they feel the same barrier potentials, which results in the Pauli matrix in the boundary condition (3). Therefore, the barrier potential in the SSC junction may play a role like the magnetic scattering in the conventional superconductor junction, and may induce a 0–π transition in the SSC Josephson junction as discussed below.

Boundary conditions (3) are the linear equations of the coefficients a, b, c, and d. The nontrivial solution demands a secular equation, which in turn gives four Andreev-like levels in the junction as

$$\varepsilon_i(\varphi, z) = \pm\Delta_0 \sqrt{\frac{(8-2z^2)\cos\varphi \pm 8z\sqrt{z^2+2-2\cos\varphi}\cos(\varphi/2) + 8 + 2z^2 + z^4}{(4+z^2)^2}}, \tag{4}$$

where Δ_0 is the magnitude of the pair potential and $z = 2mV/\hbar^2 k_0$ is the dimensionless barrier strength with $k_0 = \sqrt{2mM_0/\hbar^2}$. The φ-dependent part of the free energy of the junction is given by[33]

$$F(\varphi) = -\frac{1}{\beta}\ln\left[\prod_i \left(1 + \mathrm{e}^{-\beta\varepsilon_i}\right)\right], \tag{5}$$

where $\beta = 1/k_{\mathrm{B}}T$ with temperature T and Boltzmann constant k_{B}. The dissipationless current carried by the Cooper pair in the junction is determined by[26]

$$I(\varphi) = \frac{\partial F(\varphi)}{\partial \varphi} = \sum_i f(\varepsilon_i)\frac{\partial \varepsilon_i(\varphi)}{\partial \varphi}, \tag{6}$$

where $f(\varepsilon_i)$ is the Fermi–Dirac distribution function. Since the Cooper pair in the SSC is charge neutral and has a spin of $\hbar$, the current in Eq. (6) is a pure spin supercurrent.

3. Results and discussion

The free energy of the junction is plotted in Fig. 1, which is a function of the phase difference and the barrier strength. When the minimum of the free energy is at $\varphi = 0$ or $\varphi = \pi$, the junction is called a 0-junction or a π-junction, respectively. One finds from Fig. 1(a) that the junction is a 0-junction with the small barrier and a π-junction with the large barrier, resulting in a 0–π transition induced by the barrier strength. This feature is different from the conventional Josephson junction, where the 0–π transition is induced by the magnetic scattering.[37]

The 0–π transition point z_t can be found as the cross point of the curves $F(z,\varphi = 0)$ and $F(z,\varphi = \pi)$ as shown in Fig. 1(b). For the barrier strength far from the transmission point, the free energy has only two extrema. The minimum point is at $\varphi = 0$ and the maximum point is at $\varphi = \pi$ or vice versa, as shown by the red solid line and the purple dash-dotted line in Fig. 1(a). With increasing barrier strength, the free energy at $\varphi = 0$ increases and the one at $\varphi = \pi$ decreases, until they are equal at $z = z_t$. During this process, there must be a new extreme point present in the region $0 < \varphi < \pi$ to prevent the free energy from being a flatline at $z = z_t$, as shown by the blue dashed line and the orange dotted line in Fig. 1(a), and the two curves of the free energy with the barrier strength near the transition point.

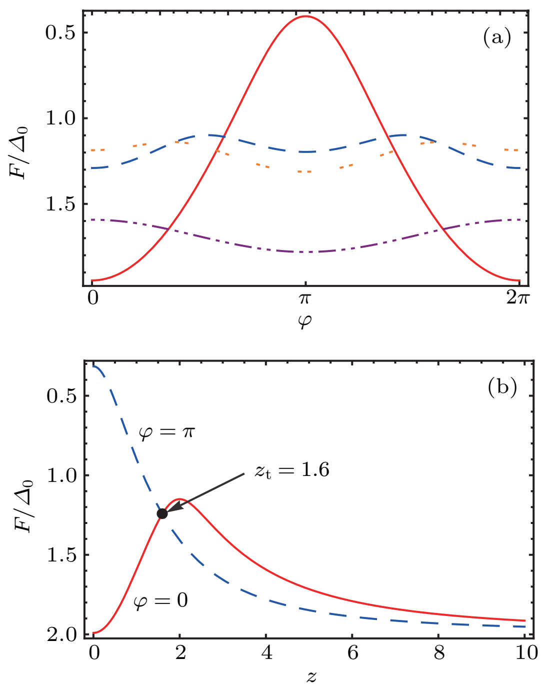

Fig. 1. (color online) (a) Free energy as a function of φ at $T = 0.2T_c$, where T_c is the superconducting critical temperature. The red solid line and the purple dash-dotted line are for $z = 0.3$ and $z = 4$, respectively. The blue dashed line and the orange dotted line are for $z = 1.5$ and $z = 1.7$, respectively. (b) Free energy as a function of z. The red solid line and the blue dashed line are for $\varphi = 0$ and $\varphi = \pi$, respectively.

The 0–π transition can also be indicated by the phase dependence of the Josephson spin current, which is depicted in Figs. 2(a) and 2(b). For the 0-junction with $z = 0.3$, the phase dependence of the Josephson spin current is a sinusoidal function with the maximum spin current occurring in the region of $0 < \varphi < \pi$. For the π-junction with $z = 4$, the current–phase relation is a negative sinusoidal function with the maximum spin current occurring in the region of $\pi < \varphi < 2\pi$. For the junctions with the barrier strength near the transition point z_t, the current–phase relation deviates from the sinusoidal form and has additional zero points besides $\varphi = 0$ and $\varphi = \pi$, which is a result of the extra extreme point in the free energy. However, the current–phase relation is still a 2π periodic function of φ. The maximum spin current still occurs in the region of $0 < \varphi < \pi$ for the 0-junction and in the region of $\pi < \varphi < 2\pi$ for the π-junction, as shown by the red solid curve and the blue dashed one in Fig. 2(b), respectively.

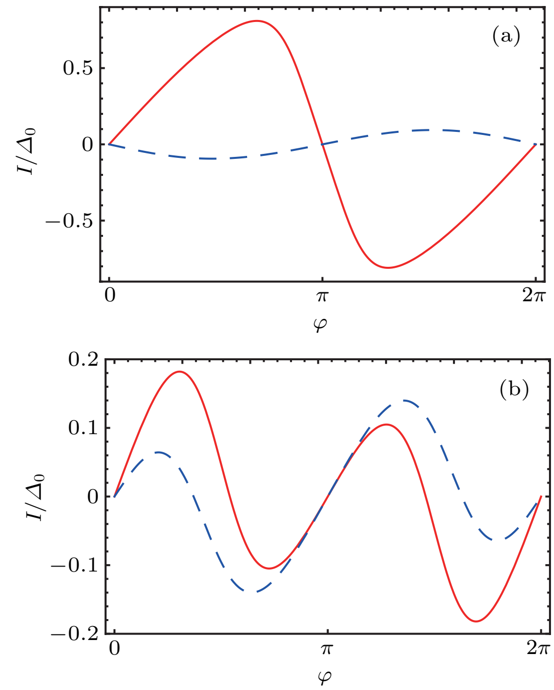

Fig. 2. (color online) The phase dependence of the Josephson spin current. (a) The red solid line and the blue dashed line are for $z = 0.3$ and $z = 4$, respectively. (b) The red solid line and the blue dashed line are for $z = 1.5$ and $z = 1.7$, respectively.

The critical spin current I_c, which is defined as the maximum Josephson spin current, is shown in Fig. 3 as a function of the barrier strength. One finds that there is a cuspidal point in the z dependence of I_c, which also indicates the 0–π transition. The current acquires its maximum value at the critical phase difference φ_0, which is a function of z. Therefore, we have $I_c = I(\varphi_0(z), z)$ with its derivative written as

$$\frac{\mathrm{d}I_c}{\mathrm{d}z} = \left.\frac{\partial I}{\partial \varphi}\right|_{\varphi_0} \frac{\mathrm{d}\varphi_0}{\mathrm{d}z} + \left.\frac{\partial I}{\partial z}\right|_{\varphi_0}. \quad (7)$$

Since φ_0 is the extreme point of the current, we have $(\partial I/\partial \varphi)|_{\varphi_0} = 0$ and $\mathrm{d}I_c/\mathrm{d}z = (\partial I/\partial z)|_{\varphi_0}$. For the 0-junction and the π-junction, φ_0 is less than π and greater than π, respectively, as shown in Fig. 2. Therefore, there is always a jump of

φ_0 and a discontinuity in the derivative of I_c around the transition point z_t, as shown in Fig. 3. Such a discontinuity is also shown in the 0–π transition in the conventional superconductor/ferromagnet/superconductor junctions, however, the transition is driven by the thickness of the junction there.[38]

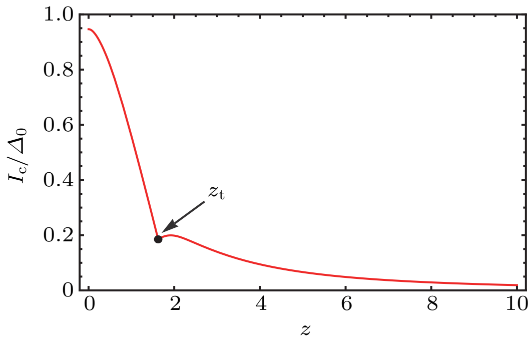

Fig. 3. (color online) The critical current as a function of z.

Finally, we present the temperature dependence of I_c in Fig. 4 by assuming the BCS-like temperature dependence of the pair potential. One finds that the critical spin current decreases naturally with temperature increasing and vanishes at the superconducting critical temperature.

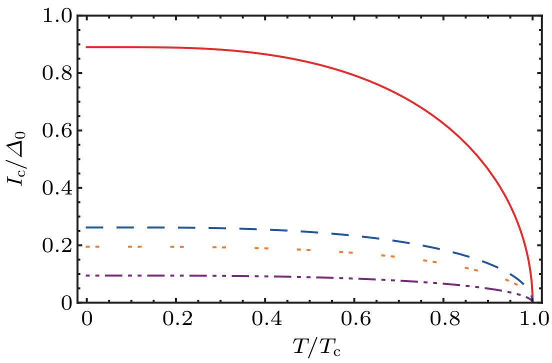

Fig. 4. (color online) The temperature dependence of I_c. The red solid line, the blue dashed line, the orange dotted line, and the purple dash-dotted line are for $z = 0.3$, $z = 1.5$, $z = 1.7$, and $z = 4$, respectively.

4. Summary

We investigate the dc Josephson effect in an SSC Josephson junction. In contrast to the conventional Josephson junction, the Josephson current is a pure spin current and the 0–π transition is driven by the barrier strength rather than the magnetic mechanism. The signals of the 0–π transition are shown in the free energy, in the current-phase relation, and in the critical Josephson current.

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