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# Fusionable and fissionable waves of $(2 + 1)$ -dimensional shallow water wave equation

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We investigate a  $(2 + 1)$ -dimensional shallow water wave equation and describe its nonlinear dynamical behaviors in physics. Based on the  $N$ -soliton solutions, the higher-order fissionable and fusionable waves, fissionable or fusionable waves mixed with soliton molecular and breather waves can be obtained by various constraints of special parameters. At the same time, by the long wave limit method, the interaction waves between fissionable or fusionable waves with higher-order lumps are acquired. Combined with the dynamic figures of the waves, the properties of the solution are deeply studied to reveal the physical significance of the waves.

**Keywords:** fissionable wave, fusionable wave, breather wave, higher-order lump

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## 1. Introduction

The interaction solutions of nonlinear partial differential equations are a topic of general interest in nonlinear systems.<sup>[1–4]</sup> Among them, shallow water wave equation has been one of the hottest issues in recent years,<sup>[5–11]</sup> such as marine engineering, hydrodynamics, mathematical physics in other fields. Because its exact solution is a special solution existing stably in space,<sup>[12]</sup> it has very important practical significance for many complex physical phenomena<sup>[13]</sup> and some nonlinear engineering problems. So far, the phenomena of soliton fission wave and fusion wave<sup>[14]</sup> and the interaction solutions of  $(2 + 1)$ -dimensional shallow water wave equation (SWW) have been studied in a small amount.

The main purpose of this article is to study the fusion and fission waves and some interaction solutions of the  $(2 + 1)$ -dimensional SWW equation,<sup>[5]</sup> which is usually written as

$$u_{yt} - u_{xxx} - 3u_{xx}u_y - 3u_xu_{xy} + u_{xx} = 0. \quad (1)$$

Equation (1) can be obtained by taking  $z = x$  into the following  $(3 + 1)$ -dimensional shallow water wave equation:

$$u_{yt} - u_{xxx} - 3u_{xx}u_y - 3u_xu_{xy} + u_{xz} = 0, \quad (2)$$

this equation has been used with tsunamis, atmospheric circulation, river transport, virtual reality, and other issues widely. Many authors obtained various forms of solutions to Eq. (2) by using the Hirota bilinear method, Darboux transformation method, etc.<sup>[6–11]</sup> Equation (1) can be transformed into the following form by using the Hirota bilinear method:

$$B(f \cdot f) = (D_y D_t - D_x^3 D_y + D_x^2)(f \cdot f)$$

$$= 2f_{xx}f - 2f_x^2 - 2f_{xxx}f + 6f_{xy}f_x - 6f_{xy}f_{xx} + 2f_{yt}f - 2f_yf_t = 0. \quad (3)$$

Recently, some scholars have paid continuous attention to the phenomena of fission and fusion.<sup>[14–16]</sup> Wang *et al.* took the Burgers equation and the Sharma–Tasso–Olver equation as two concrete examples to show the fission and fusion of the solitary wave and the soliton solutions respectively which are studied by means of the Hirota’s direct method and the Bäcklund transformation.<sup>[17]</sup> Later, from the Levi spectral problem, two basic Darboux transformations of the Sharma–Tasso–Olver equation have been obtained. Then from the trivial seed solution, the authors set the multi-kink solutions and soliton fission and fusion solutions into the following form:<sup>[18]</sup>

$$u = \alpha \ln \left( 1 + \sum_{i=1}^N \rho_i e^{\kappa_i x + p_i y + \omega_i t} \right)_{xx}. \quad (4)$$

However, this approach is too special to get the interactions between fissionable or fusionable waves and other types of waves. In order to generate the hybrid solutions mentioned before, Chen *et al.* changed  $N$ -soliton solutions into the following form:

$$f = \alpha \ln \left( \sum_{\mu=0,1} \exp \left( \sum_{j=1}^N \mu_j \xi_j + \sum_{j>1}^N A_{1j} \mu_1 \mu_j + \sum_{j>2}^N A_{2j} \mu_2 \mu_j \right) \right)_{xx}, \quad (5)$$

where  $\xi_j, A_{ij}$  have been given in Ref. [19]. Taking this method, some hybrid solutions are obtained, such as an interaction between a first-order lump wave and  $N$ -fissionable waves. On the basis of Eq. (1), no matter how constrained it is, a hybrid of fissionable waves and fusionable waves cannot be obtained.

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The reason is that equation (1) has fewer terms than the classic  $N$ -soliton solutions.

Based on the classic  $N$ -soliton solutions, we introduce a new constraint to get a variety of hybrid solutions containing fission and waves in this paper. In order to illustrate our ideas clearly, we will consider a specific example: the (2 + 1)-dimensional shallow water wave equation.

$$f = \sum_{\mu=0,1} \exp \left( \sum_{j<s}^N \mu_j \mu_s A_{js} + \sum_{j=1}^N \mu_j \eta_j \right), \tag{7}$$

$$\eta_j = \kappa_j x + p_j y + \omega_j t + \phi_j, \quad e^{A_{js}} = -\frac{(\kappa_j - \kappa_s)^3 (p_j - p_s) - (\kappa_j - \kappa_s)^2 - (\omega_j - \omega_s)(p_j - p_s)}{(\kappa_j - \kappa_s)^3 (p_j - p_s) - (\kappa_j + \kappa_s)^2 - (\omega_j + \omega_s)(p_j + p_s)}, \tag{8}$$

with

$$\kappa_j^3 p_j - \kappa_j^2 - p_j \omega_j = 0. \tag{9}$$

In order to obtain the fissionable and fusionable waves, we add the  $\exp(x)$  range to remove some items in Eq. (7). The  $\exp(x) = 0$  is true if and only if  $x = \ln(0)$ . The  $\exp(x + \ln(0)) = 0 \exp(x) = 0$ , if all  $A_{js} = \ln(0)$ , then equation (7) can be converted into Eq. (4). If  $A_{js} = \ln(0)$ ,  $3 \leq j < s \leq N$ , then equation (7) converts into Eq. (5). This leads to the following interesting conclusion.

Based on the  $N$ -soliton solutions, the  $M$ -fissionable waves and  $L$ -fusionable waves can be derived through the following constraints:

$$e^{A_{js}} = 0, \quad (1 \leq j < s \leq 2M, M < j < s \leq N, N = 2M + 2L), \tag{10}$$

$$P_s = \frac{-B \pm \sqrt{B^2 - 4CD}}{2C}, \tag{11}$$

where

$$C = 3k_j k_s^2 p_j - 3k_j^2 k_s p_j + k_j^2, \tag{12}$$

$$B = k_j k_s p_j (3k_j p_j - 3k_s p_j - 2), \quad D = k_s^2 p_j^2.$$

It is noted that the  $M$ -fissionable waves do not simply refer to the fissionable phenomenon, but rather to fissionable or fusionable phenomenon produced by  $M$  linear waves.  $M$ -fissionable waves are described for ease of writing only. Equation (11) has two cases: one case corresponds to fissionable phenomenon and other to fusionable phenomenon.

When  $M = 1, L = 1$ , the one-fissionable or fusionable wave can be obtained

$$u(x, y, t) = 2(\ln f)_{xx}, \tag{13}$$

with

$$f = 1 + e^{\xi_1} + e^{\xi_2}, \tag{14}$$

$$\kappa_1 = -\frac{1}{2}, \quad \kappa_2 = \frac{1}{3}, \quad p_1 = 1,$$

## 2. Fissionable and fusionable waves

The  $N$ -soliton solutions of Eq. (3) can be easily found by using bilinear method:

$$u(x, y, t) = 2(\ln f)_{xx}, \tag{6}$$

where

$$p_2 = 9/4 - 5\sqrt{33}/12, \quad \phi_1 = 0, \quad \phi_2 = 0, \tag{15}$$

the one-fusionable wave can be obtained by substituting Eq. (15) into Eq. (13)

$$\kappa_1 = 2, \quad \kappa_2 = 1, \quad p_1 = -1, \tag{16}$$

$$p_2 = -1/2 + \sqrt{15}/10, \quad \phi_1 = 0, \quad \phi_2 = 0,$$

the one-fissionable wave can be obtained by substituting Eq. (21) into Eq. (13).

The form in Fig. 1(a) shows one-fusionable wave, as can be seen from panels (b) and (c), the solitons gradually converge to form the fusion wave. Panel (d) shows one-fissionable wave, with the change of time from panels (e) and (f), the soliton gradually breaks up with time.

With  $M = 2, L = 2$ , an interaction between two-fissionable or fusionable waves can be described by the following expression:

$$u(x, y, t) = 2(\ln f)_{xx}, \tag{17}$$

with

$$f = 1 + e^{\xi_1} + e^{\xi_2} + e^{\xi_3} + e^{\xi_4} + e^{\xi_1 + \xi_3 + A_{13}} \tag{18}$$

$$+ e^{\xi_1 + \xi_4 + A_{14}} + e^{\xi_2 + \xi_3 + A_{23}} + e^{\xi_2 + \xi_4 + A_{24}}.$$

The relevant parameters  $A_{js}, P_s$  are given in Eqs. (10) and (11) respectively. With appropriate constraints, equation (17) can be described as three basic types of interaction between fissionable waves and fusionable waves: the interaction between two-fusionable waves, the hybrid of one-fissionable wave and one-fusionable wave, and the nonlinear superposition between 2-fissionable waves. The specific parameters are given as follows:

### Case I

$$\kappa_1 = \frac{1}{2}, \quad \kappa_2 = \frac{1}{3}, \quad p_1 = 1, \tag{19}$$

$$p_2 = 9/4 - 5\sqrt{33}/12, \quad \phi_1 = 2, \quad \phi_2 = 2,$$

$$\kappa_3 = -1, \quad \kappa_4 = \frac{1}{2}, \quad p_3 = 1,$$

$$p_4 = 13/10 - 3\sqrt{21}/10, \quad \phi_3 = 20, \quad \phi_4 = 0, \quad (19)$$

the two-fusionable waves can be acquired by substituting Eq. (19) into Eq. (17).

**Case II**

$$\begin{aligned} \kappa_1 &= 2, \quad \kappa_2 = 1, \quad p_1 = -1, \\ p_2 &= -1/2 + \sqrt{15}/10, \quad \phi_1 = 0, \quad \phi_2 = 30, \\ \kappa_3 &= -\frac{1}{2}, \quad \kappa_4 = \frac{1}{3}, \quad p_3 = 1, \\ p_4 &= 9/4 - 5\sqrt{33}/12, \quad \phi_3 = 0, \quad \phi_4 = 0, \end{aligned} \quad (20)$$

the two-fissionable waves can be obtained by substituting Eq. (20) into Eq. (17).

**Case III**

$$\begin{aligned} \kappa_1 &= 2, \quad \kappa_2 = 1, \quad p_1 = -1, \\ p_2 &= -1/2 + \sqrt{15}/10, \quad \phi_1 = 5, \quad \phi_2 = 0, \\ \kappa_3 &= 1, \quad \kappa_4 = \frac{1}{2}, \quad p_3 = -1, \\ p_4 &= -1/2 + \sqrt{21}/14, \quad \phi_3 = 20, \quad \phi_4 = 10, \end{aligned} \quad (21)$$

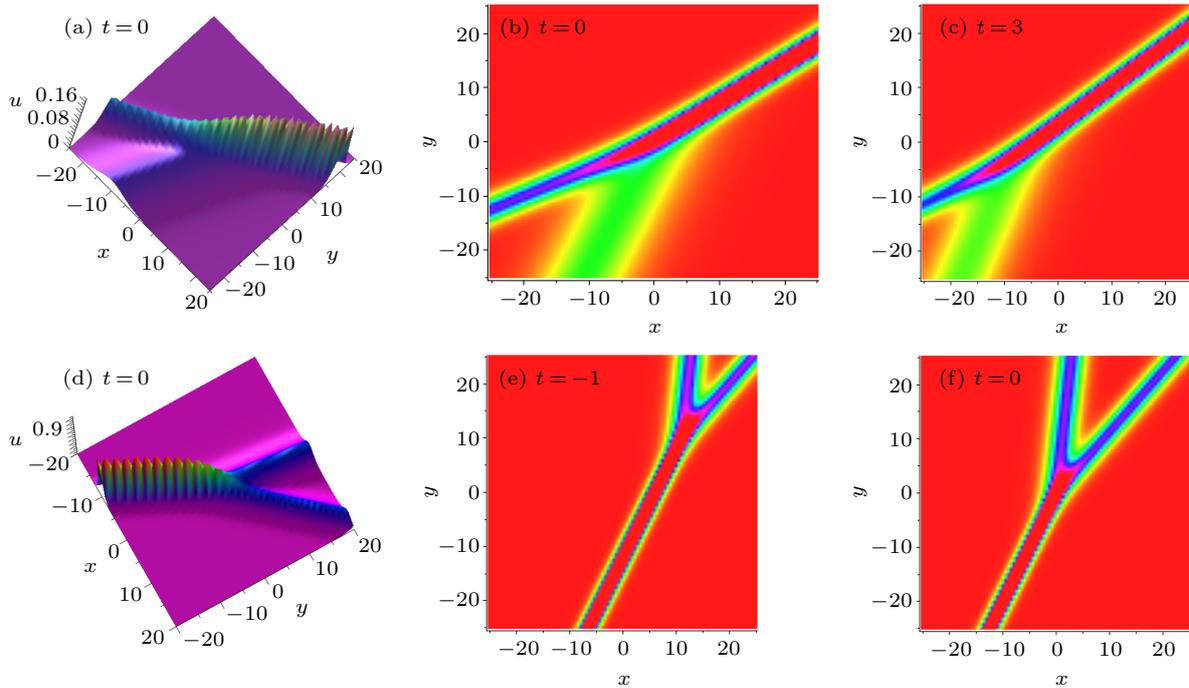
an interaction between one-fusionable wave and one-fissionable wave can be got by substituting Eq. (21) into Eq. (17).

If  $L = 0$ , the  $N$ -soliton solutions can only be simplified to an  $N$ -fissionable wave. Most studies have only obtained pure fusion or fission waves. By selecting the following parameters, an interaction between fission prior and the fusion waves can be obtained:

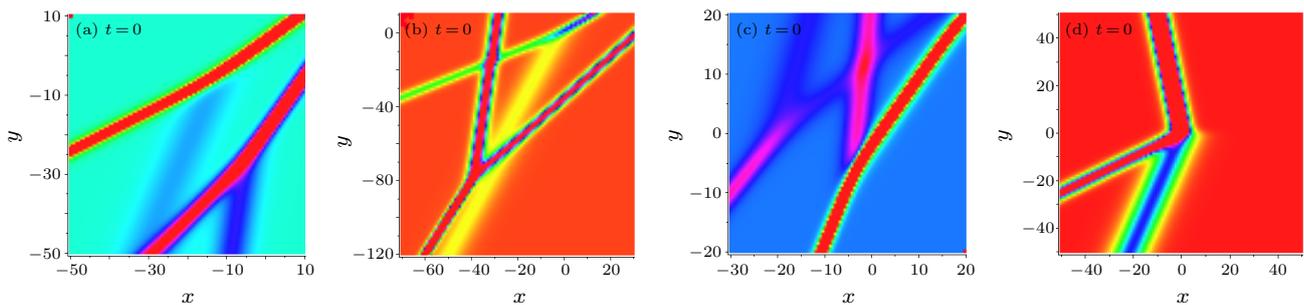
$$\begin{aligned} \kappa_1 &= -1/2, \quad \kappa_2 = 1/3, \quad p_1 = 1, \quad p_2 = 9/4 - 5\sqrt{33}/12, \\ \kappa_3 &= 1/3, \quad p_3 = 7/6, \quad \phi_1 = 0, \quad \phi_2 = 0, \quad \phi_3 = 0, \end{aligned} \quad (22)$$

an interaction wave between fusion first and then fission can be obtained by substituting Eq. (22) into Eq. (17).

The four pictures in Fig. 2 vividly and intuitively illustrate the interaction of these four types of fusionable or fissionable phenomenon. The interaction between line waves is elastic, so it is reasonable to hold the view that the interaction waves obtained from the  $N$ -soliton solutions shown in Fig. 2 is also elastic.



**Fig. 1.** (a)–(c) The one-fusionable wave can be obtained by substituting Eq. (15) into Eq. (13); (d)–(f) the one-fissionable wave can be obtained by substituting Eq. (15) into Eq. (13).



**Fig. 2.** (a) Two-fusionable waves, (b) two-fissionable, (c) an interaction between one-fusionable wave and one-fissionable wave, (d) an interaction wave between fission first and then fusion.

### 3. Interactions between fusionable or fissionable waves and line wave molecule or higher-order breather waves

In order to find interactions between fusionable or fissionable waves and line wave molecule,<sup>[20]</sup> equation (6) satisfies

$$e^{A_{js}} = 0, \quad \frac{P_{M+2l-1}}{P_{M+2l}} = \frac{\kappa_{M+2l-1}}{\kappa_{M+2l}} = \frac{\omega_{M+2l-1}}{\omega_{M+2l}} \neq \pm 1, \\ (1 \leq j < s \leq 2M, 1 \leq l \leq L, N = 2M + 2L). \quad (23)$$

Taking parameters as follows:

$$\kappa_1 = 2, \quad \kappa_2 = -1, \quad p_1 = -1, \\ p_2 = -\frac{1}{2} + \frac{\sqrt{10}}{15}, \quad \phi_1 = 0, \quad \phi_2 = 0, \\ \kappa_3 = \frac{1}{2}, \quad \kappa_4 = 1, \quad p_3 = \frac{1}{2}, \\ p_4 = 1, \quad \phi_3 = 10, \quad \phi_4 = 0, \quad (24)$$

we can obtain the interaction between one-fissionable wave and line wave molecule by substituting Eq. (24) into Eq. (6).

If equation (6) satisfies

$$e^{A_{js}} = 0, \quad \xi_{M+2l-1} = \xi_{M+2l}^*, \\ (1 \leq j < s \leq 2M, 1 \leq l \leq L, N = 2M + 2L), \quad (25)$$

the interaction solutions of  $M$ -fusionable or fissionable waves and  $L$ -order breather waves can be obtained by appropriate parameter constraints.

Taking the following parameters into Eq. (6)

$$\kappa_1 = -\frac{1}{2}, \quad \kappa_2 = \frac{1}{3}, \quad p_1 = 1, \\ p_2 = \frac{9}{4} - \frac{5\sqrt{33}}{12}, \quad \phi_1 = 0, \quad \phi_2 = 0, \\ \kappa_3 = \frac{2}{7} - \frac{2}{7}i, \quad \kappa_4 = \frac{2}{7} + \frac{2}{7}i, \quad p_3 = \frac{1}{3} + \frac{1}{5}i, \\ p_4 = \frac{1}{3} - \frac{1}{5}i, \quad \phi_3 = 10, \quad \phi_4 = 10, \quad (26)$$

we can obtain the interactions between one-fusionable wave and one-order breather wave.

### 4. Interactions between fusionable or fissionable waves and higher-order lump waves

A nonlinear superposition between an  $M$ -fusionable or fissionable waves and  $L$ -order lump waves can be derived if the following constraint is applied to the  $N$ -soliton solution:

$$\kappa_{2m-1} = \kappa_{2m}^* = K_{2m-1}\varepsilon, \quad p_{2m-1} = p_{2m} = P_{2m-1}\varepsilon, \\ \phi_{2m-1} = \phi_{2m}^* = \pi i, \quad (1 \leq m \leq M), \quad \varepsilon \rightarrow 0, \\ N = 2M + 2L, \quad e^{A_{js}} = 0, \quad (M \leq j < s \leq 2M),$$

among them, the lump wave is controlled by the parameters  $K_{2m}, P_{2m}, K_{2m-1}, P_{2m-1}$ . Through reasonable parameter constraints and long wave limit method, the interaction between  $M$ -fusionable or fissionable waves and  $L$ -lumps can be obtained.

In particular, if  $M = 1, L = 1$ , we can derive the expression for a hybrid of one-fusionable wave and one-order lump wave:

$$u(x, y, t) = 2(\ln f)_{xx}, \quad (27) \\ f = 1 + e^{\xi_1} + e^{\xi_2} + e^{\xi_3} + e^{\xi_4} + e^{\xi_1 + \xi_3 + A_{13}} + e^{\xi_1 + \xi_4 + A_{14}} \\ + e^{\xi_2 + \xi_3 + A_{23}} + e^{\xi_2 + \xi_4 + A_{24}} + e^{\xi_3 + \xi_4 + A_{34}}. \quad (28)$$

Putting the following parameters into Eq. (27)

$$\kappa_1 = -\frac{1}{2}, \quad \kappa_2 = \frac{1}{3}, \quad p_1 = 1, \quad p_2 = \frac{9}{4} - \frac{5\sqrt{33}}{12}, \\ \phi_1 = 10, \quad \phi_2 = 0, \quad \kappa_3 = \frac{1}{2} + i, \quad \kappa_4 = \frac{1}{2} - i, \\ p_3 = \frac{1}{2} + \frac{i}{3}, \quad p_4 = \frac{1}{2} - \frac{i}{3}, \quad \phi_3 = i\pi, \quad \phi_4 = i\pi, \quad (29)$$

with the long wave limit method, we can obtain the hybrid wave of one-fusionable wave and one-order lump wave.

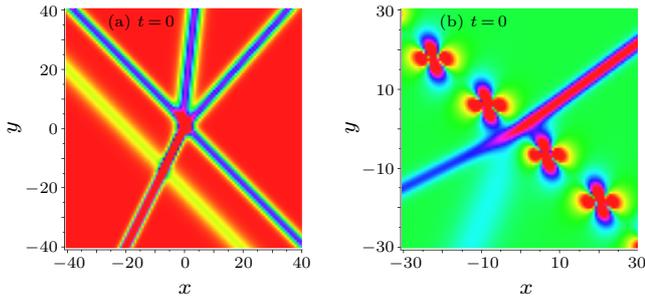
Similarly, when  $M = 1, L = 2$ , inserting the following parameters into Eq. (27):

$$\kappa_1 = -\frac{1}{2}, \quad \kappa_2 = \frac{1}{3}, \quad p_1 = 1, \quad p_2 = \frac{9}{4} - \frac{5\sqrt{33}}{12}, \\ \phi_1 = 20, \quad \phi_2 = 0, \quad \kappa_3 = \frac{1}{2} + i, \quad \kappa_4 = \frac{1}{2} - i, \\ p_3 = \frac{1}{2} + \frac{1}{3}i, \quad p_4 = \frac{1}{2} - \frac{1}{3}i, \quad \phi_3 = i\pi, \quad \phi_4 = i\pi, \\ \kappa_5 = \frac{1}{6} + \frac{1}{2}i, \quad \kappa_6 = \frac{1}{6} - \frac{1}{2}i, \quad p_5 = i, \quad p_6 = -i, \\ \phi_5 = i\pi, \quad \phi_6 = i\pi, \quad (30)$$

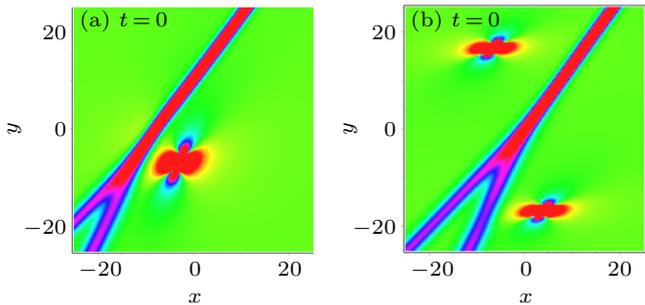
through the method of long wave limit, the hybrid of one-fusionable wave and two-order lump waves can be acquired.

### 5. Conclusion

In this paper, the fissionable waves, fusionable waves, breather waves, lump waves, and hybrid waves of the  $(2 + 1)$ -dimensional shallow water wave (SWW) equation are studied by Hirota's bilinear method and long wave limit method. Based on the  $N$ -soliton solutions and reasonable constraint parameters, the fusionable waves, fissionable waves, and higher-order breather waves of the SWW equation have been obtained. When  $M = 1, L = 1$ , one-fissionable and one-fusionable wave can be obtained by a series of special parameter constraints, the dynamic image is shown in Fig. 1. When  $L = 2, M = 2$ , we can get two-fissionable, two-fusionable



**Fig. 3.** (a) An interaction between one-fissionable wave and a line wave molecule, (b) an interaction between one-fusionable wave and one-order breather wave.



**Fig. 4.** (a) An interaction between the hybrid of one-fusionable wave and one-order lump wave, (b) an interaction between the hybrid of one-fusionable wave and two-order lump waves.

waves, and some mixed waves, which are exhibited by three-dimensional Figs. 2 and 3. The corresponding long wave limit method is used to construct high-order lump waves. At the same time, we have obtained the dynamic graphs of the hybrid of the fusionable or fissionable waves, one-order and two-order lumps displayed by Fig. 4. In addition, the method of deriving fissionable or fusionable waves proposed in this paper can be extended to other  $(2+1)$ -dimensional integrable equations. In the near future, based on this method, we will

discuss the moving path of lump in hybrid solutions, including different types of combinations in lump wave, line wave, and other types of waves.

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