



Collective modes of type-II Weyl fermions with repulsive S-wave interaction

Xun-Gao Wang(王勋高), Yuan Sun(孙远), Liang Liu(刘亮), and Wu-Ming Liu(刘伍明)

Citation: Chin. Phys. B, 2022, 31 (2): 026701. DOI: 10.1088/1674-1056/ac3d81

Journal homepage: <http://cpb.iphy.ac.cn>; <http://iopscience.iop.org/cpb>

What follows is a list of articles you may be interested in

Polaron and molecular states of a spin-orbit coupled impurity in a spinless Fermi sea

Hong-Hao Yin(尹洪浩), Tian-Yang Xie(谢天扬), An-Chun Ji(纪安春), and Qing Sun(孙青)

Chin. Phys. B, 2021, 30 (10): 106702. DOI: 10.1088/1674-1056/ac1b85

Superexchange-mediated magnetization dynamics with ultracold alkaline-earth atoms in an optical lattice

Shaobing Zhu(朱少兵), Jun Qian(钱军), Yuzhu Wang(王育竹)

Chin. Phys. B, 2017, 26 (4): 046702. DOI: 10.1088/1674-1056/26/4/046702

Improvement of variational approach in an interacting two-fermion system

Liu Yan-Xia, Ye Jun, Li Yuan-Yuan, Zhang Yun-Bo

Chin. Phys. B, 2015, 24 (8): 086701. DOI: 10.1088/1674-1056/24/8/086701

Condensation of Fermions in the double-well potential

Chen Xin-Wei, Liu Zhong-Qiang, Kong Xiang-Mu

Chin. Phys. B, 2014, 23 (2): 026701. DOI: 10.1088/1674-1056/23/2/026701

Collective modes of type-II Weyl fermions with repulsive S-wave interaction

Xun-Gao Wang(王勋高)^{1,2,3}, Yuan Sun(孙远)³, Liang Liu(刘亮)³, and Wu-Ming Liu(刘伍明)^{1,2,4,†}

¹Beijing National Laboratory for Condensed Matter Physics, Institute of Physics, Chinese Academy of Sciences, Beijing 100190, China

²School of Physical Sciences, University of Chinese Academy of Sciences, Beijing 100190, China

³Key Laboratory of Quantum Optics, Shanghai Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, Shanghai 201800, China

⁴Songshan Lake Materials Laboratory, Dongguan 523808, China

(Received 8 November 2021; revised manuscript received 23 November 2021; accepted manuscript online 26 November 2021)

Three-dimensional type-II Weyl fermions possess overtilted cone-like low-energy band dispersion. Unlike the closed ellipsoidal Fermi surface for type-I Weyl fermions, the Fermi surface is an open hyperboloid for type-II Weyl fermions. We evaluate the spin and density susceptibility of type-II Weyl fermions with repulsive S-wave interaction by means of Green's functions. We obtain the particle-hole continuum along the tilted momentum direction and perpendicular to the tilted momentum direction respectively. We find the zero sound mode in some repulsive interaction strengths by numerically solving the pole equations of the susceptibility within the random-phase approximation.

Keywords: Weyl fermions, zero sound mode, random-phase approximation

PACS: 67.85.Lm, 72.15.Nj

DOI: 10.1088/1674-1056/ac3d81

1. Introduction

Since the discover of topological insulator,^[1,2] searching for novel topological states has been paid much attention.^[3–6] Recently, some novel states, including Dirac semimetals^[7–9] and Weyl semimetals,^[10–14] have been confirmed in experiment. Weyl semimetal has emerged as novel topological states of matter, hosting low-energy excitations of massless Weyl fermions. Weyl fermions can be classified into type-I, which requires Lorentz invariance, and type-II Weyl fermions with broken Lorentz symmetry.^[15,16] These two classes of Weyl fermions are characterized by the tilted parameter α , with $\alpha < 1$ being type-I Weyl fermions, and $\alpha > 1$ being type-II Weyl fermions in our Hamiltonian model Eq. (1). Unlike the closed ellipsoidal Fermi surface for type-I Weyl fermions, the Fermi surface is an open hyperboloid for type-II Weyl fermions, as shown in Fig. 2, and the geometry of Fermi surface plays a crucial role in many material properties. Although Weyl nodes usually lie above the Fermi level E_F , we choose the type-II Weyl fermions system with Weyl nodes below E_F ,^[17,18] as illustrated in Fig. 1.

The experimental realization of the type-II Weyl semimetal has been widely studied.^[19–24] We have investigated the collective modes near the single Weyl node model with repulsive S-wave interaction for type-I Weyl fermions.^[25] In order to lay the foundation for important applications of Weyl fermions to quantum simulation as well as information technologies in future, it is necessary to study the collective excitations for type-II Weyl fermions in ultracold atom gases.

The superiority of ultracold atom gases than the semiconductor or semimetal is the tunable interaction strength by the technique of Feshbach resonance.^[26,27] Collective excitations have received intensive studies.^[28–31] In contrast to the partially tilted type-I Weyl fermions, whose lower band is full of particles and upper band is partially occupied, both the bands of the overtilted type-II Weyl fermions are partially occupied. In this paper we will focus on solving the poles equation of type-II Weyl fermions with repulsive S-wave interaction to obtain the collective modes within the random-phase approximation (RPA).

Our paper is organized as follow. In Section 2 we present the effective low-energy model for the type-II Weyl Fermi atomic gases with repulsive S-wave interaction in the continuum limit approximation. We show the energy dispersion of our model and the open hyperboloidal Fermi surface, which is the so-called double drumheadlike surfaces. We calculate the bare spin and density susceptibility by the Green's functions theory and obtain collective modes by solving the poles equation of the susceptibility. The dispersion relations for the modes are shown by numerically solving the RPA equations in different interaction strengthes in Section 3. At last we will give the summation of our paper.

2. The model and calculation

We start from considering a low-energy effective model for a single Weyl node with repulsive S-wave interaction in three-dimensional fermionic system in the continuum limit ap-

[†]Corresponding author. E-mail: wliu@iphy.ac.cn

proximation near the Weyl node. The effective Hamiltonian with the tilted term is given by

$$\mathcal{H} = \sum_{\mathbf{k}, \alpha, \beta} c_{\mathbf{k}, \alpha}^\dagger h(\mathbf{k})_{\alpha\beta} c_{\mathbf{k}, \beta} + 2g \sum_{\mathbf{k}, \mathbf{k}', q} c_{\mathbf{k}+q, \uparrow}^\dagger c_{\mathbf{k}'-q, \downarrow}^\dagger c_{\mathbf{k}', \downarrow} c_{\mathbf{k}, \uparrow}, \quad (1)$$

$$h = v_0 \mathbf{k} \cdot \boldsymbol{\sigma} + (v_1 k_z - \mu) \hat{I}, \quad (2)$$

where μ is the chemical potential where we consider only the Fermi surface above the Weyl node. α and β are the spin (pseudo-spin) indexes. Here and in the following, we treat \hbar as a system of units. The first term h is the noninteracting term of the Hamiltonian, whose eigenstates, the same as the eigenstates of the Weyl Hamiltonian without the tilted term, can be obtained as

$$|\mathbf{k}, +\rangle = \begin{pmatrix} \cos \frac{\theta}{2} \\ \sin \frac{\theta}{2} e^{i\phi} \end{pmatrix}, \quad |\mathbf{k}, -\rangle = \begin{pmatrix} \sin \frac{\theta}{2} \\ -\cos \frac{\theta}{2} e^{i\phi} \end{pmatrix}, \quad (3)$$

where θ, ϕ are the polar angle and the azimuth angle in wave vector space, respectively. The energy eigenvalues are $\xi_{\mathbf{k}, \pm} = \pm v_0 \sqrt{k_x^2 + k_y^2 + k_z^2} + v_1 k_z - \mu$, which are shown in Fig. 1. The second term in Eq. (1) represents the S-wave interaction in ultracold atomic gases. Here, we focus on the repulsive interaction case.

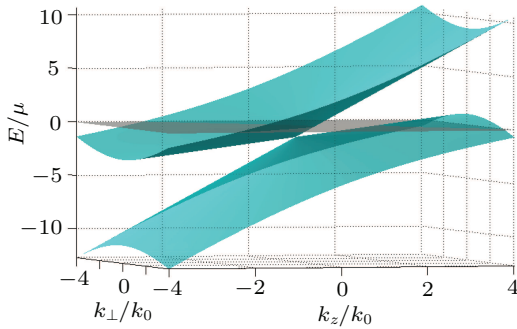


Fig. 1. Band dispersion for type-II Weyl fermions in the k_\perp - k_z plane, where $k_\perp = \sqrt{k_x^2 + k_y^2}$. The grey plane represents the Fermi level, which is above the Weyl node.

The Fermi surface of our model is not a closed curved surface because of the over-tilted term. We obtain that the Fermi surface is governed by the equation

$$\pm v_0 \sqrt{k_x^2 + k_y^2 + k_z^2} + v_1 k_z = \mu. \quad (4)$$

Finally, we obtain the standard conic section of Fermi surface

$$\frac{(k_z - k_{z0})^2}{a^2} \pm \frac{k_\perp^2}{b^2} = 1, \quad (5)$$

where $k_{z0} = \frac{\alpha k_0}{\alpha^2 - 1}$, the tilted parameter $\alpha = \frac{v_1}{v_0}$, $a^2 = \frac{k_0^2}{(\alpha^2 - 1)^2}$, $b^2 = \frac{k_0^2}{|\alpha^2 - 1|}$, $k_\perp^2 = k_x^2 + k_y^2$ and $k_0 = \frac{\mu}{v_0}$, which corresponds to the Fermi momentum for type-I Weyl fermions without the tilted term. Here, \pm correspond to ellipsoidal Fermi surface

for $\alpha < 1$ and hyperboloidal Fermi surface for $\alpha > 1$, respectively.

It is interesting that the origin of coordinate is just a focus of a hyperboloid. Coincidentally, it is apparent that the origin of coordinate is also a focus of an ellipsoid for type-I Weyl fermions,

$$a^2 \mp b^2 = k_{z0}^2, \quad (6)$$

where \mp represent the case of $\alpha < 1$ and $\alpha > 1$, respectively. Both the Fermi surfaces are depicted in Fig. 2.

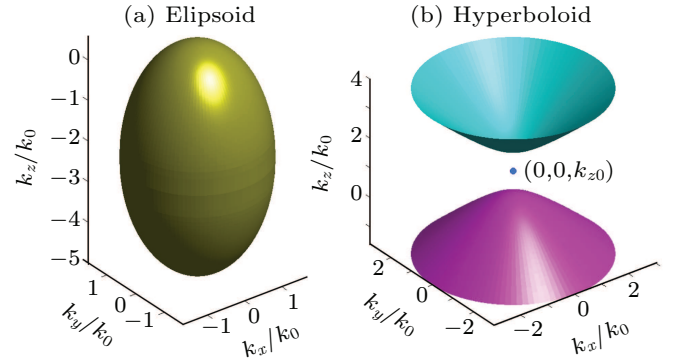


Fig. 2. Schematic illustration of Fermi surface. (a) Schematic showing the ellipsoid Fermi surface for type-I Weyl fermions. (b) Schematic showing the hyperboloid Fermi surface for type-II Weyl fermions. The cyan branch of hyperboloid represents the Fermi surface of the lower band, and the other purple branch corresponds to the upper band.

In order to investigate the collective behavior in the system, we start from the spin and density susceptibility in Matsubara formalism, which can be written as

$$\chi^{\mu\nu}(q, i\omega_m) = \sum_{k,p} \sum_{\alpha\beta\gamma\delta} \int_0^\beta d\tau e^{i\omega_m \tau} \times \langle T_\tau c_{k,\alpha}^\dagger \sigma_{\alpha\beta}^\mu c_{k+q,\beta}(\tau) c_{p+q,\gamma}^\dagger \sigma_{\gamma\delta}^\nu c_{p,\delta}(0) \rangle, \quad (7)$$

where $\sigma^\mu = (1, \sigma_x, \sigma_y, \sigma_z)$, and $\sigma^\pm = \sigma_x \pm i\sigma_y$ represent the raising and lowering operators.

By use of the Green's function theory and the relationship between Matsubara Green's function and the retarded Green's function, the spin and density susceptibility without interaction can be written as

$$\chi_b^{\mu\nu}(q, \omega) = - \sum_{k,s,r} F_{sr}^{\mu\nu} \frac{n_F(\xi_{k-q/2,r}) - n_F(\xi_{k+q/2,s})}{\xi_{k-q/2,r} - \xi_{k+q/2,s} + \omega + i0^+}, \quad (8)$$

where $n_F(\xi)$ is the Fermi distribution function and $F_{sr}^{\mu\nu}$ is an introduced overlap factor for calculation conveniently. At zero temperature, the distribution function is just the Heaviside step function $n_F(\xi) = \Theta(-\xi)$. The overlap factor, which contains two contributions from the intrasubband and the intersubband, has the form

$$F_{sr}^{\mu\nu} = \text{tr}[P_s(\mathbf{k} + \mathbf{q}/2) \sigma^\mu P_r(\mathbf{k} - \mathbf{q}/2) \sigma^\nu], \quad (9)$$

where s, r denote the helicity indexes and P_s is the projection operator with respect to the helicity eigenstates.

The $\xi_{k+q/2,s} - \xi_{k-q/2,r}$ in the denominator of Eq. (8) represents the particle-hole excitation energy, which consists of the contributions from the intrasubband and the intersubband. The particle-hole continuum, where the imaginary part of the susceptibility is nonzero, corresponding to the finite lifetime of collective modes, is shown in Fig. 3 with q along the tilted direction and in Fig. 4 with q perpendicular to the tilted direction.

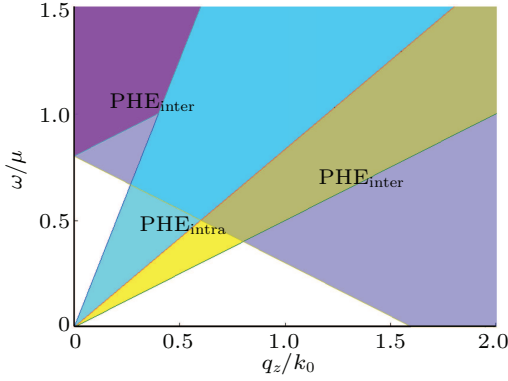


Fig. 3. Particle-hole continuum with q along the tilted direction z -axis at the tilted parameter $\alpha = 1.5$. The purple and the light purple regions indicate the intersubband contribution to particle-hole excitations ($\text{PHE}_{\text{inter}}$). The blue and the yellow regions indicate the intrasubband contribution to particle-hole excitations ($\text{PHE}_{\text{intra}}$).

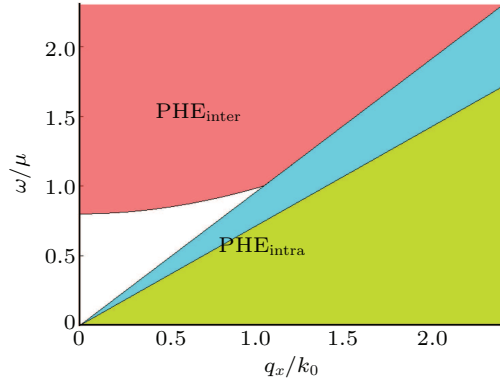


Fig. 4. Particle-hole continuum with q perpendicular to the tilted direction z -axis at the tilted parameter $\alpha = 1.5$. Supposing q is along x -axis direction. The light red region indicates the intersubband contribution to particle-hole excitations ($\text{PHE}_{\text{inter}}$). The blue and the green regions indicate the intrasubband contribution to particle-hole excitations ($\text{PHE}_{\text{intra}}$).

We take advantage of the random-phase approximation to deal with the interaction term, and express the RPA susceptibility in terms of the bare susceptibility

$$\chi_{\text{RPA}}^{\mu\nu}(q, i\omega_m) = \chi_b^{\mu\rho}(q, i\omega_m) \left(\frac{1}{1 + g\eta\chi(q, i\omega_m)} \right)^{\rho\nu}, \quad (10)$$

where $\eta = \text{diag}(1, -1, -1, -1)$.

3. Theoretical and numerical results

The open hyperboloid Fermi surface without the spherical symmetry for occupation of particles in the momentum space gives rise to a formidable integrand in the long-wavelength

limit. Consequently, we focus on the collective behavior beyond the long-wavelength limit by numerical calculation. For the sack of simplicity, we only consider the case of q along the tilted direction. The detailed overlap factor has introduced in our previous article.^[25] From the expression of the overlap factor, we can find that the 4×4 matrix can be written as four 2×2 block matrices, where all elements of the two off-diagonal 2×2 matrices have e^{ϕ} or $e^{-\phi}$. The integrals of these elements equal zero with respect to the momentum \mathbf{k} as the azimuth angle ϕ runs from 0 to 2π . So the 4×4 matrix form of the spin and density susceptibility can be simplified into two uncoupled 2×2 submatrices. Then we can get two uncoupled poles equations of the spin and density susceptibility. Of course, the overlap factor does not have this symmetry with respect to the azimuth angle ϕ for q perpendicular to the tilted direction, and the 4×4 susceptibility cannot be grouped into two uncoupled 2×2 submatrices.

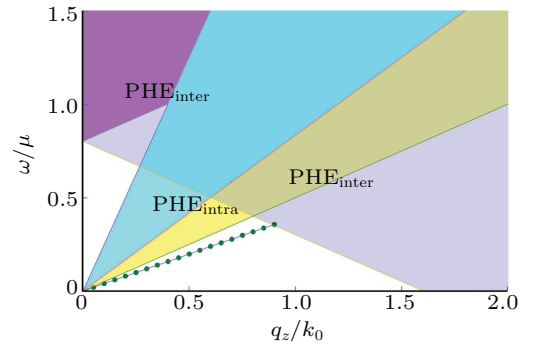


Fig. 5. The collective zero sound mode at $g = 0.1$. Here we take the tilted parameter $\alpha = 1.5$ and the ultraviolet cut-off momentum $\Lambda = 15k_0$ in numerical calculation.

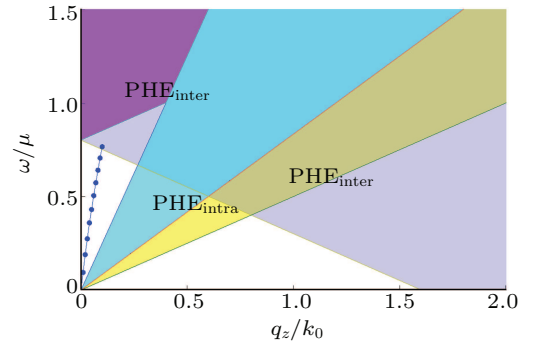


Fig. 6. The collective zero sound mode at $g = 1$. Here we take the tilted parameter $\alpha = 1.5$ and the ultraviolet cut-off momentum $\Lambda = 15k_0$ in numerical calculation.

We obtain the dispersion relations beyond the long-wavelength limit by solving the poles equation using the numerical method. Because of the open hyperboloid Fermi surface for both bands, we define an unified ultraviolet cut-off momentum Λ for both bands, setting $\Lambda = 15k_0$ in our calculations. We find that there is one zero sound mode at $g = 0.1$, $g = 1$ as well as $g = 5$ for the tilted parameter $\alpha = 1.5$, as shown in Figs. 5–7, respectively. Here, the interaction strength parameter g has the energy dimension, and we make g in units

of chemical potential μ for calculations. We find that the collective modes fall into the particle-hole continuum at some interaction strengths, such as $g = 0.2$ and $g = 0.5$.

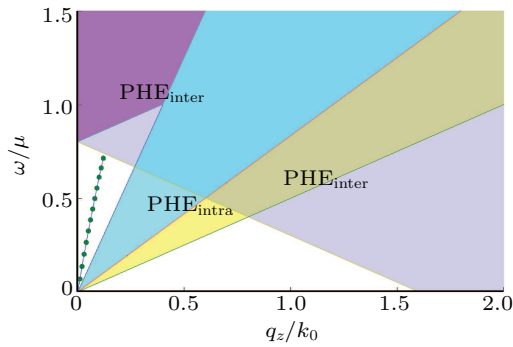


Fig. 7. The collective zero sound mode at $g = 5$. Here we take the tilted parameter $\alpha = 1.5$ and the ultraviolet cut-off momentum $\Lambda = 15k_0$ in numerical calculation.

4. Conclusion

We evaluate the collective modes in three-dimensional fermion system governed by type-II Weyl Hamiltonian with repulsive S-wave interaction. In contrast to the fixed Coulomb interaction in condensed matter, we can find collective modes depending on the interaction strength in ultracold atomic gases, which can be tuned into a large range of values by Feshbach resonance in ultracold atoms. By contrast, there exist one zero sound mode, one gapped mode or two gapped modes for different repulsive interaction strengths for the type-I Weyl fermions without the tilted term, and we find that only one zero sound mode appears for some interaction strength, such as $g = 0.1$, $g = 1$ and $g = 5$.

Acknowledgements

Project supported by the National Key R&D Program of China (Grant No. 2016YFA0301500), the National Natural Science Foundation of China (Grants No. 61835013), and the Strategic Priority Research Program of the Chinese Academy of Sciences (Grant Nos. XDB01020300 and XDB21030300). We are grateful to Sun FaDi for useful discussions.

References

- [1] Hasan M Z and Kane C L 2010 *Rev. Mod. Phys.* **82** 3045
- [2] Qi X L and Zhang S C 2011 *Rev. Mod. Phys.* **83** 1057
- [3] Bernevig B A, Hughes T L and Zhang S C 2006 *Science* **314** 613
- [4] Yang B J and Nagaosa N 2014 *Nat. Commun.* **5** 4898
- [5] Potter A C, Kimchi I and Vishwanath A 2014 *Nat. Commun.* **5** 5161
- [6] Goldman N, Budich J C and Zoller P 2016 *Nat. Phys.* **12** 639
- [7] Sachdeva R, Thakur A, Vignale G and Agarwal 2015 *Phys. Rev. B* **91** 205426
- [8] Carbotte J P 2016 *Phys. Rev. B* **94** 165111
- [9] Armitage N P, Mele E J and Vishwanath A 2018 *Rev. Mod. Phys.* **90** 015001
- [10] Wan X G, Turner A M, Vishwanath A and Savrasov S Y 2011 *Phys. Rev. B* **83** 205101
- [11] Xu S Y, Belopolski I, Alidoust N, *et al.* 2015 *Science* **349** 613
- [12] Lu L, Wang Z Y, Ye D X, Ran L X, Fu L, Joannopoulos J D and Soljacic M 2015 *Science* **349** 622
- [13] Lv B Q, Weng H M, Fu B B, Wang X P, Miao H, Ma J, Richard P, Huang X C, Zhao L X, Chen G F, Fang Z, Dai X, Qian T and Ding H 2015 *Phys. Rev. X* **5** 031013
- [14] Dubcek T, Kennedy C, Lu L, Ketterle W, Soljacic M and Buljan H 2015 *Phys. Rev. Lett.* **114** 225301
- [15] Wang C L, Zhang Y, Huang J W, *et al.* 2016 *Phys. Rev. B* **94** 241119
- [16] Park M J, Basa B and Gilbert M J 2017 *Phys. Rev. B* **95** 094201
- [17] Autes G, Gresch D, Troyer M, Soluyanov A A and Yazyev O V 2016 *Phys. Rev. Lett.* **117** 066402
- [18] Yao M Y, Xu N, Wu Q S, Autes G, Kumar N, Strocov V N, Plumb N C, Radovic M, Yazyev O V, Felser C, Mesot J and Shi M 2019 *Phys. Rev. Lett.* **122** 176402
- [19] Jia G Y, Huang Z X, Ma Q Y and Li G 2020 *Nanophotonics* **9** 715
- [20] Lin C L, Arafune R, Minamitani E, Kawai M and Takagi N 2018 *J. Phys.: Condens. Matter* **30** 105703
- [21] Soluyanov A A, Gresch D, Wang Z J, Wu Q S, Troyer M, Dai X and Bernevig B A 2015 *Nature* **527** 495
- [22] Jiang J, Liu Z K, Sun Y, Yang H F, Rajamathi C R, Qi Y P, Yang L X, Chen C, Peng H, Wang C C, Sun S Z, Mo S K, Vobornik I, Fujii J, Parkin S S P, Felser C, Yan B H and Chen Y L 2017 *Nat. Commun.* **8** 13973
- [23] Deng K, Wan G L, Deng P, *et al.* 2016 *Nat. Phys.* **12** 1105
- [24] Wang Z J, Gresch D, Soluyanov A A, Xie W W, Kushwaha S, Dai X, Troyer M, Cava R J and Bernevig B A 2016 *Phys. Rev. Lett.* **117** 056805
- [25] Wang X G, Wang H Y, Zhang J M and Liu W M 2020 *Chin. Phys. B* **29** 117201
- [26] Kohler T, Goral K and Julienne P S 2006 *Rev. Mod. Phys.* **78** 1311
- [27] Chin C, Grimm R, Julienne P and Tiesinga E 2010 *Rev. Mod. Phys.* **82** 1225
- [28] Raghu S, Chung S B, Qi X L and Zhang S C 2010 *Phys. Rev. Lett.* **104** 116401
- [29] Zhang S S, Yu X L, Ye J W and Liu W M 2013 *Phys. Rev. A* **87** 063623
- [30] Detassis F, Fritz L and Grubinskas S 2017 *Phys. Rev. B* **96** 195157
- [31] Santos-Cottin D, Martino E, Le Mardel F, Witteveen C, von Rohr F O, Homes C C, Rukelj Z and Akrap A 2020 *Phys. Rev. Mater.* **4** 021201