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Non-Hermitian Kitaev chain with complex periodic and quasiperiodic potentials*

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We study the topological properties of the one-dimensional non-Hermitian Kitaev model with complex either periodic or quasiperiodic potentials. We obtain the energy spectrum and the phase diagrams of the system by using the transfer matrix method as well as the topological invariant. The phase transition points are given analytically. The Majorana zero modes in the topological nontrivial regimes are obtained. Focusing on the quasiperiodic potential, we obtain the phase transition from the topological superconducting phase to the Anderson localization, which is accompanied with the Anderson localization–delocalization transition in this non-Hermitian system. We also find that the topological regime can be reduced by increasing the non-Hermiticity.

Keywords: non-Hermitian physics, Majorana zero modes, transfer matrix

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1. Introduction

Exploring topological phases of matter in condensed matter physics has become an active topic of research over the last decade.[1–3] Among various novel phases, the topological superconducting phases (TSCs) characterized by bound Majorana edge modes have been intensively studied and been predicted in several compounds.[4,5] They are of great interest from the perspective of topological quantum computing because of their non-Abelian braiding statistics and the natural basis for topological qubits. The prototypical model for studying one-dimensional (1D) TSCs related with the effective spinless p-wave superconducting wire system is the Kitaev chain.[6] With the suitable model parameters, the Majorana zero modes (MZMs) arise at the ends of the chain under open boundary condition (OBC) and the system is in the topological nontrivial phase, which can be characterized by a bulk topological invariant. This is the result of bulk–edge correspondence which indicates that a nontrivial topological invariant in the bulk must correspond a localized edge mode that only appears at the boundaries in the thermodynamic limit.

Beyond the topological aspects, the study of Anderson localization in the 1D systems is also an interesting topic.[7–12] Although the TSCs are protected by the particle–hole symmetry, the topological phases could be destroyed by the strong disorders and change into the topological trivial Anderson localized phases. Besides the random disorder, it is found that the quasiperiodic potentials or the incommensurate structures can also induce the Anderson localization.[13,14] In the 1D Anderson model, the infinitesimal random disorders can localize all the states. While in the 1D incommensurate Aubry–André–Harper (AAH) model, the Anderson localization requires that the strength of the quasiperiodic potential is finite which is a direct result of the self-duality of the system.

After that, the competition between the Anderson localization and the topological superconducting phase draws many concerns.[15–20] For example, it is found that the MZMs are robust in the 1D p-wave SC systems with correlated or uncorrelated disorders. The transition from the TSC phase to the Anderson localized phase is obtained and the corresponding critical values are derived analytically. Meanwhile, the transition is also accompanied with the Anderson localization–delocalization processes.

Recently, there has been growing interest in the non-Hermitian (NH) topological phases.[21–25] Generally, the non-Hermiticity is achieved by introducing the nonreciprocal hopping processes or the gain and loss terms. The non-Hermiticity can induce many exotic phenomena such as the complex-energy gaps, non-Hermitian skin modes, and breakdown of the bulk–boundary correspondence based on the traditional Bloch band theory. All these pictures do not have the Hermitian counterparts. Moreover, when the non-Hermiticity is in-

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volved in the topological phases, the standard 10-fold Altland–Zirnbauer (AZ) symmetry class of the topological insulators and superconductors is generalized to the 38-fold Bernard–LeClair (BL) symmetry class.[22,23] These 38 BL symmetries can completely describe the intrinsic non-Hermitian topological phases. Obviously, it is very important to study the physics of non-Hermiticity meeting the 1D TSCs, and many interesting works such as $\mathcal{PT}$-symmetric TSCs,[26,27] Kitaev chain with gain and loss terms,[28,29] nonreciprocal hopping and p-wave pairing[30,31] have been done. All these results show that the MZMs in the topological phases are stable even for the NH systems.

In this paper, we study the topological properties of the 1D NH Kitaev chain with either the periodic or the quasiperiodic potentials by using the transfer matrix derived from the equations of motion. We obtain the energy spectrum and the spatial distributions of the wavefunctions. Based on them, we obtain the phase transition from the TSCs to the topological trivial phase as well as the Anderson localization phase in this NH system and give the boundaries of different phases analytically. We also discuss the Majorana edge modes induced by the non-Hermiticity.

The rest of the paper is organized as follows. In Section 2, we introduce the model Hamiltonian and calculate the transfer matrix. The definition of the related topological invariant is also given. In Section 3, we study the energy spectrum of the system with NH periodic potential under the OBC. The MZMs in the topological nontrivial phase are obtained explicitly. In Section 4, we consider the system with NH quasiperiodic potential. The topological phase and Anderson localization are investigated. The corresponding phase boundaries are computed. Section 5 devotes to a summary.

2. The model Hamiltonian and transfer matrix approach

Turning to our starting point, we consider a finite 1D wire of spinless electrons exhibiting p-wave superconductivity, which is described by the following Hamiltonian:[15,16]

$$H = \sum_{n=1}^{N-1}\left[-t(f_n^\dagger f_{n+1}+f_n f_{n+1}^\dagger)+\Delta(f_n f_{n+1}+f_{n+1}^\dagger f_n^\dagger)\right] + \sum_{n=1}^{N}\mu_n(f_n^\dagger f_n-1/2), \tag{1}$$

where N is the number of sites, t is the nearest-neighbor hopping and set as 1 in this paper, f_n and $f_n^\dagger$ are the annihilation and creation operators of electrons on the site n, respectively, Δ is the superconducting pairing parameter, and μ_n is the on-site chemical potential. If μ_n is complex, Hamiltonian (1) is NH. The boundary condition is the open one.

Hamiltonian (1) with $\Delta=0$ commutes with the total number of fermions $\sum_{n=1}^N f_n^\dagger f_n$. Hence, the MZMs can only exist in the case of $\Delta\neq 0$. In order to study the Majorana modes in the system, we combine the fermionic operators f_n and $f_n^\dagger$ into the Majorana operators $a_n=f_n+f_n^\dagger$ and $b_n=(f_n+f_n^\dagger)/\mathrm{i}$. The Majorana operators a_n and b_n are Hermitian and satisfy the anticommutation rules $\{a_n,a_m\}=\{b_n,b_m\}=2\delta_{nm}$ and $\{a_n,b_m\}=0$. Then the arbitrary Majorana states can be expressed by the operators a_n and b_n as $\mathcal{M}_a=\sum_{n=1}^N\alpha_n a_n$ and $\mathcal{M}_b=\sum_{n=1}^N\beta_n b_n$, respectively, where α_n and β_n are the expansion coefficients. From Eq. (1), we obtain the equations of motion for the time-dependent Majorana modes in the Heisenberg picture $a_n=\alpha_n\,\mathrm{e}^{-\mathrm{i}wt}$ and $b_n=\beta_n\,\mathrm{e}^{\mathrm{i}wt}$ as[15,32]

$$\begin{aligned}&(1-\Delta)\alpha_{n-1}+(1+\Delta)\alpha_{n+1}-\mu_n\alpha_n=-\mathrm{i}w\beta_n,\\ &-(1+\Delta)\beta_{n-1}-(1-\Delta)\beta_{n+1}+\mu_n\beta_n=-\mathrm{i}w\alpha_n.\end{aligned} \tag{2}$$

Obviously, in order to identify the MZMs, we only need to consider the case of $w=0$, which leads to the fact that the equations in Eq. (2) are decoupled. Then the matrix form of Eq. (2) reads

$$\begin{pmatrix}\alpha_{n+1}\\ \alpha_n\end{pmatrix}=A_n\begin{pmatrix}\alpha_n\\ \alpha_{n-1}\end{pmatrix},\quad A_n=\begin{pmatrix}\frac{\mu_n}{\Delta+1} & \frac{\Delta-1}{\Delta+1}\\ 1 & 0\end{pmatrix}. \tag{3}$$

Here A_n is the transfer matrix. The similar expressions can be obtained for the set of $\{\beta_n\}$ and the corresponding transfer matrix B_n is related with A_n as $B_n=A_n^{-1}$. The existence of MZMs requires that the α_n (or β_n) should be normalizable, i.e., $\sum_n|\alpha_n|^2$ (or $\sum_n|\beta_n|^2$) should be finite. The behaviors of MZMs at the boundaries of a finite chain are determined by the full transfer matrix $\mathcal{A}=\prod_{n=1}^N A_n$, which has two eigenvalues λ_1 and λ_2. If the periodicity of the system is p, then the properties of the edge modes are determined by $\mathcal{A}=A_pA_{p-1}\cdots A_1$. Denote the number of eigenvalues of the matrix $\mathcal{A}$ less than 1 in the magnitude as n_f. If $n_f=2$, there will be an a-mode localized at the left end and a b-mode at the right end of the lattice. If $n_f=0$, the two modes will be localized at the opposite ends. If $n_f=1$, there do not exist the MZMs because the α_n (or β_n) can not be normalized.

The topological invariant related with the system (1) is

$$\nu=(-1)^{n_f+1}. \tag{4}$$

From the above discussions, we know that the system is in the topological phase (T phase) if $\nu=-1$ while is in the nontopological phase (N phase) if $\nu=1$. The topological invariant can also be given as $\nu=-\mathrm{sgn}[f(1)f(-1)]$, where $f(z)=\det(\mathcal{A}-zI)$ is the characteristic polynomial of full transfer matrix $\mathcal{A}$. The topology of the system depends on the magnitude of Δ and we take Δ to be positive. From Eq. (3), we know that $\det|\mathcal{A}|<1$. Thus the two eigenvalues of the transfer matrix $\mathcal{A}$ satisfy $|\lambda_1\lambda_2| < 1$, which means that if $|\lambda_1| < |\lambda_2|$, then $|\lambda_1| < 1$ and n_f is completely determined by the quantity $|\lambda_2|$. This enable us to write the topological invariant (4) as $\nu = \mathrm{sgn}(\ln|\lambda_2|)$. Based on this topological invariant, we can study the topological properties of the NH Kitaev chain with complex periodic (Section 3) or quasiperiodic (Section 4) potentials.

3. Non-Hermitian periodic potentials

In this section, we focus on the Kitaev chain with NH periodic potentials. The non-Hermiticity is introduced by the complex chemical potential μ_n. We consider following four typical patterns. (1) $\mu_n = \mathrm{i}V$, that is the chemical potential is pure imaginary. (2) $\mu_{2j-1} = \mathrm{i}V$ and $\mu_{2j} = -\mathrm{i}V$, where $j = 1,2,\ldots,N/2$ and N is even, which means that the chemical potential takes the alternate values and the corresponding period is 2. (3) $\mu_{4l-3} = \mu_{4l-2} = \mu_{4l-1} = \mathrm{i}V$ and $\mu_{4l} = -\mathrm{i}V$. (4) $\mu_{4l-3} = \mu_{4l-2} = \mathrm{i}V$ and $\mu_{4l-1} = \mu_{4l} = -\mathrm{i}V$. Here $l = 1,2,\ldots,N/4$ and N is the multiple of 4. Thus the period of the chemical potential in cases (3) and (4) is 4.

We first consider the $\mu_n = \mathrm{i}V$ pattern, where all the sites of the NH Kitaev chain are added with an uniform imaginary potential. Usually, the eigenenergies of the system are complex. The absolute values of eigenenergies $|E|$ of the system with $\Delta = 0.5$ versus the strength V of the NH potential are shown in Fig. 1(a). From it, we see that the MZMs denoted as the red points indeed exist in this NH system and the system is in the topological nontrivial phase when $|V| < 1$. A pair of Majorana edge states emerges and satisfies the relations $[H,\mathcal{M}_a] = [H,\mathcal{M}_b] = 0$. Due to the non-Hermiticity, $\mathcal{M}_a^\dagger \neq \mathcal{M}_a$ and $\mathcal{M}_b^\dagger \neq \mathcal{M}_b$. These anomalous statistics contrast with the conventional ones for the Majorana fermions in the Hermitian counterpart, which are originated from the distinction between right and left eigenstates of the NH system.

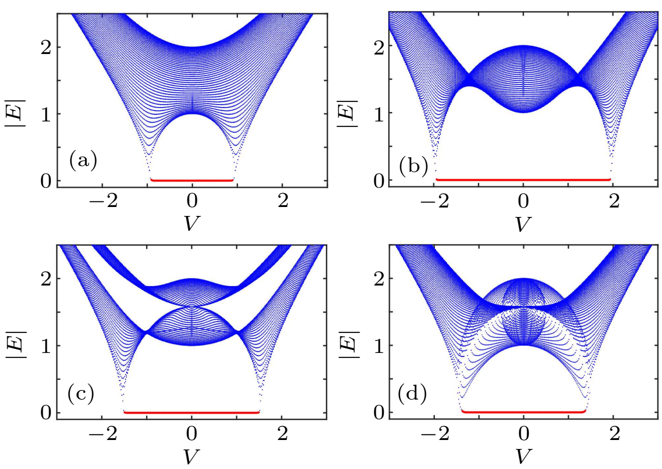

Fig. 1. The absolute values of eigenenergies $|E|$ of the system versus the strength V of NH potential. The red points represent the MZMs. The system is in the topological phase. (a) Uniform potential $\mu_n = \mathrm{i}V$. (b) Period-2 potential $(\mathrm{i}V,-\mathrm{i}V)$. (c) Period-4 potential $(\mathrm{i}V,\mathrm{i}V,\mathrm{i}V,-\mathrm{i}V)$. (d) Period-4 potential $(\mathrm{i}V,\mathrm{i}V,-\mathrm{i}V,-\mathrm{i}V)$. Here $N = 100$ and $\Delta = 0.5$.

From the analysis of the eigenvalues of the transfer matrix A_n (3) and according to the topological invariant (4), we obtain that the system is in the topological nontrivial phase when the strength of the NH potential satisfies $|V| < 2\Delta$, while the system is in the topological trivial phase and the boundary localized MZMs disappear if $|V| > 2\Delta$. The phase diagram is shown in Fig. 2(a). The critical value of the topological phase transition is $|V_c| = 2\Delta$. The MZMs appear if the p-wave pairing $\Delta \neq 0$.

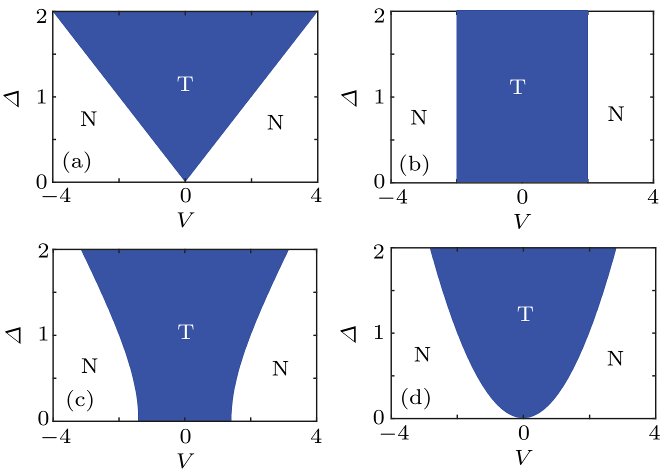

Fig. 2. Phase diagrams of the system, where T means the topological nontrivial phase and N means the topological trivial phase. (a) Uniform potential $\mathrm{i}V$. (b) Period-2 potential $(\mathrm{i}V,-\mathrm{i}V)$. (c) Period-4 potential $(\mathrm{i}V,\mathrm{i}V,\mathrm{i}V,-\mathrm{i}V)$. (d) Period-4 potential $(\mathrm{i}V,\mathrm{i}V,-\mathrm{i}V,-\mathrm{i}V)$. The topological phase boundaries are (a) $\Delta = |V|/2$, (b) $|V| = 2$, (c) $\Delta^2 = V^2/2-1$, (d) $\Delta = V^2/4$.

For the period-2 NH potential $(\mathrm{i}V,-\mathrm{i}V)$, the gain and loss in the system are balanced because the chemical potential takes the alternative values. From the transfer matrix $\mathcal{A} = A_2A_1$, where A_1 and A_2 take the forms of Eq. (3) with the replacing of μ_n by $\mathrm{i}V$ and $-\mathrm{i}V$ for the first matrix element, respectively, we obtain the topological invariant as $\nu = -\mathrm{sgn}(4-V^2)$. The system is in the topological phase and the corresponding topological invariant is $\nu = -1$ if $|V| < 2$ for arbitrary $\Delta \neq 0$. The absolute values of the eigenenergies of the system with $\Delta = 0.5$ are shown in Fig. 1(b) and the phase diagram is shown in Fig. 2(b). These results are consist with the previous ones obtained by using different methods.[27,29]

Next, we consider the more complicated pattern $(\mathrm{i}V,\mathrm{i}V,\mathrm{i}V,-\mathrm{i}V)$, where the period of the NH potential is 4. The absolute values of the energy spectrum are shown in Fig. 1(c) and the corresponding phase diagram is shown in Fig. 2(c). From them, we see that the MZMs exist in the topological phase. The full transfer matrix is $\mathcal{A} = A_4A_3A_2A_1$, where the value of chemical potential in $A_1 = A_2 = A_3$ is $\mathrm{i}V$ and that in A_4 is $-\mathrm{i}V$. According to the above discussion in Section 2, the topological invariant is $\nu = -\mathrm{sgn}(4(1+\Delta^2)^2 - V^4)$. Then the regime of the topological phase is determined from the full transfer matrix in one period and the result is

$\Delta^2 > V^2/2-1$. The transition from the topological nontrivial phase to the normal phase happens at the critical values $\Delta^2 = V^2/2-1$. Thus, the strong non-Hermiticity will destroy the MZMs .

For the pattern $(\mathrm{i}V, \mathrm{i}V, -\mathrm{i}V, -\mathrm{i}V)$, the absolute values of the energy spectrum are shown in Fig. 1(d) and the corresponding phase diagram is shown in Fig. 2(d). The full transfer matrix is $\mathcal{A} = A_4A_3A_2A_1$, where the value of chemical potential in $A_1 = A_2$ is $\mathrm{i}V$ and that in $A_3 = A_4$ is $-\mathrm{i}V$. Thus the topological invariant is $\nu = -\mathrm{sgn}(16\Delta^2 - V^4)$. The topological regime is $\Delta^2 > V^2/4$ and the boundaries of different phases are $\Delta^2 = V^2/2-1$. The above results are summarized in Table 1.

Table 1. Criteria for the topological phases for a given NH periodic potential, where $\Delta > 0$.

Period	Pattern of μ_n	Topological regime
1	$\mathrm{i}V$	$\Delta > \lvert V\rvert/2$
2	$(\mathrm{i}V, -\mathrm{i}V)$	$\lvert V\rvert < 2$
4	$(\mathrm{i}V, \mathrm{i}V, \mathrm{i}V, -\mathrm{i}V)$	$\Delta^2 > V^2/2-1$
4	$(\mathrm{i}V, \mathrm{i}V, -\mathrm{i}V, -\mathrm{i}V)$	$\Delta > V^2/4$

4. Non-Hermitian quasiperiodic potential

Now, we consider the Kitaev chain with the NH quasiperiodic potential

$$\mu_n = V\cos(2\pi\alpha n + \phi), \tag{5}$$

where V is the amplitude, $\alpha = (\sqrt{5}-1)/2$ is an irrational number for the incommensurate lattice, and ϕ is the phase factor. The non-Hermiticity is induced if the phase factor is pure imaginary, i.e., $\phi = \mathrm{i}h$. If $h = 0$ and $\Delta = 0$, the system (1) is Hermitian and degenerates into the well-known AAH model,[13,14] which has a metal–insulator transition at the critical point $V_c = 2$. If $h = 0$ and $\Delta \neq 0$, the system has a quantum phase transition from the TSCs to the Anderson localization.[15–17] Here, we consider the general $h \neq 0$ case.

The absolute values of the eigenenergies $|E|$ of the system versus different potential strength V with the model parameters $\Delta = 0.5$, $h = 1$, and $N = 100$ are shown in Fig. 3(a). We see that the MZMs indeed exist and the system is in the topological non-trivial phase in the regime of $V < V_c$. Thus the MZMs are robust against the existence of the NH quasiperiodic potential in certain parameter regime. The distributions of right and left MZM wavefunctions in the topological phase with $V = 0.5$ are shown in Fig. 3(b). From it, we see that the Majorana edge states are located at the ends of the chain. If V is larger than the critical value V_c, the MZMs disappear and the system is in the Anderson localization phase. At the critical point V_c, the topological phase transition from the superconducting to the Anderson localization arises. The boundaries of different phases can be analytically calculated from the introduced transfer matrix as well as the topological invariant.

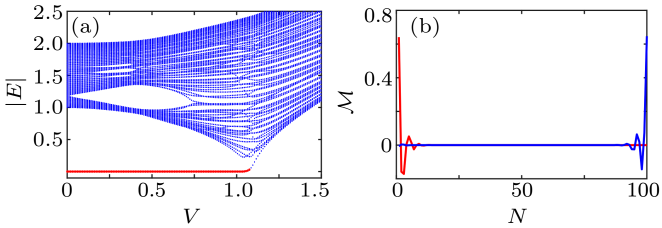

Fig. 3. (a) The absolute values of eigenenergies $|E|$ versus the strengths of quansiperiodic potential V. The MZMs denoted by the red points exist in the topological nontrivial phase regime $V < V_c$, where V_c is the critical value. If $V > V_c$, the MZMs vanish and the system enters into the topological trivial phase. (b) The spatial distributions of wavefunctions $\mathcal{M}$ for the MZMs in the topological phase with $V = 0.5$. We see that the Majorana edge states are located at the ends of the chain. Here $N = 100$, $\Delta = 0.5$, and $h = 1$.

The transfer matrix A_n given by Eq. (3) with the constraint $0 < \Delta < 1$ can be written as

$$A_n = \sqrt{\delta}S\tilde{A}_nS^{-1}, \quad \tilde{A}_n = \begin{pmatrix} \frac{\mu_n}{\sqrt{1-\Delta^2}} & -1 \\ 1 & 0 \end{pmatrix}, \tag{6}$$

where $\delta = (1-\Delta)/(1+\Delta)$ and $S = \mathrm{diag}(\delta^{1/4}, 1/\delta^{1/4})$. Thus the full transfer matrix reads

$$\mathcal{A} = \left(\sqrt{\frac{1-\Delta}{1+\Delta}}\right)^N S\tilde{\mathcal{A}}S^{-1}, \quad \tilde{\mathcal{A}} = \prod_{n=1}^{N}\tilde{A}_n. \tag{7}$$

The matrix $\mathcal{A}$ has two eigenvalues λ_1 and λ_2. Suppose $|\lambda_1| < |\lambda_2|$, and the Lyapunov exponent can be determined as[9,15]

$$\gamma(V,h,\Delta) = \max\left\{0, \lim_{N\to\infty}\frac{1}{N}\ln|\lambda_2|\right\}. \tag{8}$$

From Eq. (8), the Lyapunov exponent can also be expressed as

$$\gamma(V,h,\Delta) = \gamma\left(\frac{V}{\sqrt{1-\Delta^2}}, h, 0\right) - \frac{1}{2}\ln\left(\frac{1+\Delta}{1-\Delta}\right), \tag{9}$$

where $\gamma\left(\frac{V}{\sqrt{1-\Delta^2}}, h, 0\right)$ is the Lyapunov exponent of the AAH model with the quasiperiodic potential $V\cos(2\pi\alpha n + \mathrm{i}h)/\sqrt{1-\Delta^2}$ and its value is presented in Refs. [33–35]. Then the Lyapunov exponent of the system (1) with the NH quasiperiodic potential (6) is

$$\gamma = \begin{cases} 0, & \text{if} \quad V\mathrm{e}^h < 2(1+\Delta), \\ \ln\left[\frac{V}{2(1+\Delta)}\right] + h, & \text{if} \quad V\mathrm{e}^h > 2(1+\Delta). \end{cases} \tag{10}$$

If $\gamma(V,h,\Delta) > 0$, the system is localized and in the topological trivial phase while if $\gamma(V,h,\Delta) = 0$, the system is extended and in the topological nontrivial phase.[36,37] Thus the topological properties of the system can be characterized by the Lyapunov exponent (9).

The Lyapunov exponents of the system with $\Delta = 0.5$ versus the different values of NH phase factor h are shown in Fig. 4. From it, we see that the Lyapunov exponent is zero and the system is in the topological phase if $V < V_c$. Meanwhile, with the increase of h, the values of the phase transition point V_c are decreased. These results are consist with those obtained from Fig. 3.

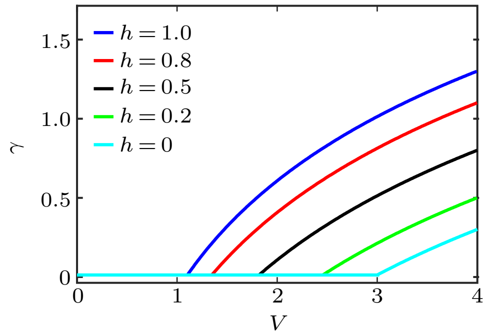

Fig. 4. The Lyapunov exponent $\gamma(V,h,\Delta)$ of the system with $\Delta = 0.5$.

Next, we shall determine the boundaries of different phases. From Eq. (10) and according to the above analysis, we obtain the critical values of phase transition as

$$V_c = 2(1+\Delta)\,e^{-h}. \tag{11}$$

The Lyapunov exponent and topological properties of the system with $\Delta > 1$ can be obtained by taking the transformation $\mu_n \to (-1)^n \mu_n/\Delta$ and $\Delta \to 1/\Delta$.[15] Thus, we obtain the complete phase diagram of the systems and show it in Fig. 5. The topological phase is in the regime of $\Delta > V\,e^h/2 - 1$ while the localization phase is in the regime of $\Delta < V\,e^h/2 - 1$.

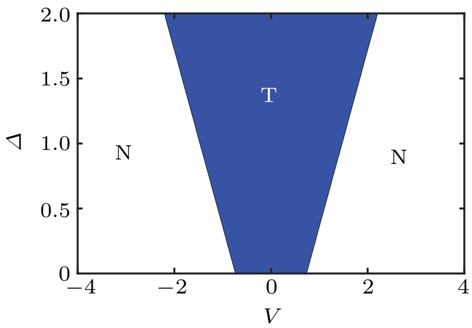

Fig. 5. The phase diagram of the Kitaev chain with NH quasiperiodic potential. Here $h = 1$ and the phase boundaries are determined by Eq. (11).

5. Summary

In summary, we investigate the topological properties of the 1D Kitaev model with NH periodic and quasiperiodic potentials. From the analysis of the energy spectrum and using the transfer matrix method, we find that the MZMs indeed exist in the system within certain model parameter regimes and are robust against the NH potentials. With the help of the distribution of the wavefunctions, we obtain that the Majorana edge states are located at the ends of the chain. We also calculate the topological invariant and obtain the phase diagram of the system. There exists a phase transition from the topological non-trivial state to the topological trivial state. The boundaries of different phases are determined analytically. For the NH incommensurate quasiperiodic potential, the topological phase transition is accompanied by the Anderson localization–delocalization transition.

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