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Citation: Chin. Phys. B, 2021, 30 (**1**): 010304. DOI: 10.1088/1674-1056/abc542

Journal homepage: http://cpb.iphy.ac.cn; http://iopscience.iop.org/cpb

Chin. Phys. B Vol. 30, No. 1 (2021) 010304

Steady and optimal entropy squeezing for three types of moving three-level atoms coupled with a single-mode coherent field*

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(Received 15 September 2020; revised manuscript received 16 October 2020; accepted manuscript online 28 October 2020)

The entropy squeezing properties of different types of moving three-level atoms coupled with a single-mode coherent field are studied. The influences of the moving velocity and initial states of the three-level atom on the entropy squeezing are discussed. The results show that, the entropy squeezing properties of the three-level atom depend on its initial state, moving velocity, and the type. A stationary three-level atom can not obtain a steady entropy squeezing whatever initial conditions are chosen, while a moving three-level atom can achieve a steady and optimal entropy squeezing through choosing higher velocity and appropriate initial state. Our result provides a simple method for preparing squeezing resources with ultra-low quantum noise of the three-level atomic system without additional any complex techniques.

Keywords: entropy squeezing, moving three-level atom, single-mode coherent field

PACS: 03.67.−a, 42.50.−p, 03.65.Yz **DOI:** 10.1088/1674-1056/abc542

1. Introduction

The quantum uncertainty and noise of quantum system restrict the precision measurement of observable in quantum physical experiments. It is important to break this limit and minimize the influence of the noise on the quantum system. Generating squeezing effect of observable is one of the best ways to restrain quantum uncertainty and noise in quantum system, and has been studied for many years.[1–3] Different types of measures have also been proposed for squeezing. Among them, there are the variance squeezing based on the Heisenberg uncertainty relation (HUR), and entropy squeezing based on entropy uncertainty relation (EUR),[4–7] which is proposed by one of our authors in Ref. [8]. These two uncertainty relations have the following differences: firstly, the HUR contains only the second-order statistical moments of the quantum state density matrix of the system, while the EUR contains moments to all orders; secondly, there is no direct connection between quantum information and the quantum fluctuations of atomic observables in the HUR, while there is direct connection in the EUR; thirdly, the lower bound of HUR is highly dependent on the atomic states, when the average value of the commutator for some quantum states is 0, the HUR is trivially satisfied and fails to provide any useful information on squeezing, while EUR does not have this defect. Therefore, entropy squeezing is a more precise measure of squeezing than the variance squeezing. The entropy squeezing properties of two-level atom and field of the Jaynes–Cummings model under various conditions were studied by Fang,[9,10] Gea-Banacloche,[11,12] and Phoenix.[13] Besides, in quantum communication and quantum computing, how to prepare stable and optimal entropy squeezing state is an important topic. In many previous works, the available entropy squeezing effects at most have either optimal squeezing degree or long squeezing time, but not both. It is so hard to prepare the squeezing resource which is not only optimal but also long-lasting. Yu[14] has provided a scheme generating steady and optimal entropy squeezing of a two-level atom by using quantum-jump-based feedback and classical driving, while Wang[15] has present another strategy by using non-Hermitian operation. However, these schemes are provided only for the case that the two-level atom is considered. Meanwhile, the quantum-jump-based feedback control and non-Hermitian operation are more complex techniques experimentally. So can we find an easier method to do this, even in the three-level atomic system?

With the development of laser cooling and trapping atom technology, the spatial motion of atoms has been considered to achieve cooling atoms and supercooled atoms. The effects of atomic motion and field mode structure on atomic dynamics were studied by Schlicher, Joshi, and Lawande.[16] Fang, Zhu, and Zhang have studied the effects of the atomic motion and the field-mode structure on field entropy and Schrödinger-cat states in Jaynes–Cummings model.[17] And the entropy squeezing properties of a moving two-level atom have been researched by Liu,[18] the results show that a moving two-level atom has better entropy squeezing properties than the static one. However, all the above studies on entropy squeezing only involve a two-level atom. As we know, compared with the two-level atom system, the three-level atom system has more advantages.[19–21] For example, it provides higher channel ca-

*Project supported by the National Natural Science Foundation of China (Grant Nos. 12064012 and 11374096).

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pacities, more secure cryptography as well as superior quantum gates. Liu[22] proposed the entropy squeezing of a three-level atom interacting with a single-mode field by using the quantum information entropy theory, but no one studied the entropy squeezing properties of a moving three-level atom. In this paper, we investigate the entropy squeezing properties of three types of moving three-level atoms coupled with a single-mode coherent field, and explore the possibility of achieving steady and optimal entropy squeezing of three-level atom by using atomic motion and without additional any complex techniques.

This paper is structured as follows. In Section 2, the definition entropy squeezing of a three-level atom is reviewed. In Section 3, the model is introduced. In Section 4, by numerical calculation, we analyze the entropy squeezing properties of three types of moving three-level atoms interacting with a single-mode coherent field. And we propose a method to generate steady and optimal entropy squeezing of the moving three-level atom. Finally, we provide a summery and conclusion in Section 5.

2. Entropy squeezing of a three-level atom

The information entropies satisfy the entropy uncertainty relation for three observables of the system with dimension $d=3$ derived by Alberto *et al.*[23]

$$H(S_x)+H(S_y)+H(S_z)\geq 2, \tag{1}$$

with

$$H(S_\alpha)=-\sum_{i=1}^{3}P_i(S_\alpha)\ln P_i(S_\alpha),\ (\alpha=x,y,z). \tag{2}$$

For an arbitrary quantum state ρ, $P_i(S_\alpha)=\langle\psi_{\alpha i}|\rho|\psi_{\alpha i}\rangle$ $(i=1,2,3)$ is the probability distribution of three possible measurement outcomes of operator S_α. And $|\psi_{\alpha i}\rangle$ is the i-th eigenstate of S_x, S_y, S_z. To compare the entropy uncertainty relation with the Heisenberg uncertainty relation, by introducing the exponential of entropy, the entropy uncertainty relation is written as the product form

$$\delta H(S_x)\delta H(S_y)\geq\frac{\mathrm{e}^2}{\delta H(S_z)}, \tag{3}$$

where $\delta H(S_\alpha)$ takes the form

$$\delta H(S_\alpha)\equiv\exp\left[H(S_\alpha)\right]. \tag{4}$$

When the entropy squeezing factor $E(S_\alpha)$ in component S_α satisfies the condition

$$E(S_\alpha)=\delta H(S_\alpha)-\frac{\mathrm{e}}{\sqrt{\delta H(S_z)}}<0,\ (\alpha=x\text{ or }y), \tag{5}$$

the quantum fluctuation in component S_α is said to be squeezed in entropy.

From Eqs. (7) and (5), we find that the value of the optimal entropy squeezing factor $E(S_\alpha)$ is $1-\mathrm{e}/\sqrt{\mathrm{e}}\approx-0.649$, the corresponding quantum noise is minimal. For a three-dimensional system, we choose the universally adapted spin operators in this paper, which are shown as

$$S_x=\begin{pmatrix}0&\frac{\sqrt{2}}{2}&0\\ \frac{\sqrt{2}}{2}&0&\frac{\sqrt{2}}{2}\\ 0&\frac{\sqrt{2}}{2}&0\end{pmatrix};$$
$$S_y=\begin{pmatrix}0&-\frac{\sqrt{2}}{2}\mathrm{i}&0\\ \frac{\sqrt{2}}{2}\mathrm{i}&0&-\frac{\sqrt{2}}{2}\mathrm{i}\\ 0&\frac{\sqrt{2}}{2}\mathrm{i}&0\end{pmatrix};$$
$$S_z=\begin{pmatrix}1&0&0\\ 0&0&0\\ 0&0&-1\end{pmatrix}. \tag{6}$$

Their information entropies are given by

$$\begin{aligned}H(S_x)&=H(S_{x_1})+H(S_{x_2})+H(S_{x_3}),\\ H(S_y)&=H(S_{y_1})+H(S_{y_2})+H(S_{y_3}),\\ H(S_z)&=H(S_{z_1})+H(S_{z_2})+H(S_{z_3}),\end{aligned} \tag{7}$$

where $H(S_{\alpha_i})=-P_i(S_\alpha)\ln P_i(S_\alpha)$ $(\alpha=x,y,z;\ i=1,2,3)$ is the information entropy corresponding to different eigenvalues of operators S_α.

3. Physical model

In this paper, the system consists of a moving three-level atom and a single-mode coherent field. Through the cavity quantum electrodynamics experiment, an atomic beam can interacts with different modes after passing through the cavity, which means that the situation considered in this paper is experimentally possible. The total Hamiltonian of the system in the rotating-wave approximation can be written as $(\hbar=1)$

$$H=H_0+H_\mathrm{I}, \tag{8}$$

with

$$H_0=\omega a^\dagger a+\sum_{i=1}^{3}\omega_i|i\rangle\langle i|,\ \ (i=1,2,3), \tag{9}$$

$$H_\mathrm{I}=\lambda(t)(a|l_1\rangle\langle l_2|+a|l_3\rangle\langle l_4|+\mathrm{H.C.}), \tag{10}$$

where a $(a^\dagger)$ is the annihilation (creation) operator of the field of frequency ω. ω_i is the eigenfrequency of each level and $|i\rangle$ represents the i-th level of the three-level atom. For different types of three-level atoms, the corresponding l_1,l_2,l_3,l_4 are different. As shown in Fig. 1, for a Ξ-type three-level atom, the transitions $|1\rangle\rightleftharpoons|2\rangle$ and $|2\rangle\rightleftharpoons|3\rangle$ are allowed, and the single photon transition $|1\rangle\rightleftharpoons|3\rangle$ is electrical dipole forbidden. So, for a Ξ-type three-level atom, $l_1=3,l_2=2,l_3=2,l_4=1$; for a Λ-type three-level atom, transition $|1\rangle\rightleftharpoons|2\rangle$ is forbidden, $l_1=3,l_2=1,l_3=3,l_4=2$; and for a V-type three-level atom, transition $|2\rangle\rightleftharpoons|3\rangle$ is forbidden, $l_1=2,l_2=1,l_3=3,l_4=1$. $\lambda(t)$ is the coupling parameter adjusted to be explicitly time-dependent and it takes the form[24]

$$\lambda(t) = g\sin[kz(t)] = g\sin\left(\frac{p\pi vt}{L} + \frac{\pi}{2}\right). \tag{11}$$

Here g is the coupling constant of the three-level atom interacting with the field, while k is the wave number and $z(t)$ is the instantaneous position of the three-level atom inside the cavity. p denotes the half-wavelengths number of the field mode in the cavity of length L, v represents the velocity of the atom. And $v = 0$ corresponds to the case of the stationary three-level atom.

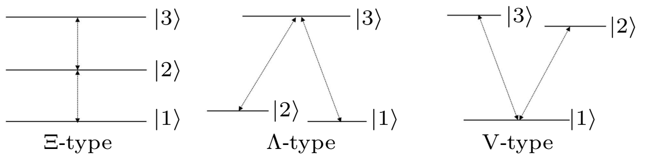

Fig. 1. Different types of three-level atoms.

Suppose the moving three-level atom is initially in a coherent superposition state

$$\left|\Psi_{\rm A}^{\rm I}(0)\right\rangle = \cos\theta|3\rangle + \sin\theta\sin\varphi|2\rangle + \sin\theta\cos\varphi|1\rangle. \tag{12}$$

And the field is initially in a coherent state

$$\left|\Psi_{\rm F}^{\rm I}(0)\right\rangle = |\alpha\rangle = \sum_n A_n|n\rangle = \exp\left(-\frac{1}{2}|\alpha|^2\right)\sum_n \frac{\alpha^n}{\sqrt{n!}}|n\rangle, \tag{13}$$

where $\alpha = \bar{n}^{1/2}\exp(\mathrm{i}\beta)$, $\bar{n}$ is the average number of photon, and β is the phase of the field. Consider the resonant case for three types of moving three-level atoms: Ξ-type : $\omega_3 - \omega_1 = 2\omega_2 = 2\omega$; V-type : $\omega_2 - \omega_1 = \omega_3 - \omega_1 = \omega$; Λ-type : $\omega_3 - \omega_1 = \omega_3 - \omega_2 = \omega$. For the initial conditions (12) and (13), the solution of the Schrödinger equation is given by

$$\left|\Psi_{\rm AF}^{\rm I}(t)\right\rangle = \sum_{n=0}^{\infty}\left[a_n(t)|3,n\rangle + b_n(t)|2,n\rangle + c_n(t)|1,n\rangle\right], \tag{14}$$

the correlation coefficients of the three types of moving three-leve atoms are given in Appendix A. The reduced density matrix of the moving three-level atom can be written as

$$\begin{aligned}\rho_{\rm A}^{\rm I}(t) &= {\rm Tr}_{\rm F}\left|\Psi_{\rm AF}^{\rm I}(t)\right\rangle\left\langle\Psi_{\rm AF}^{\rm I}(t)\right| \\ &= \begin{pmatrix} \sum_n a_n a_n^* & \sum_n a_n b_n^* & \sum_n a_n c_n^* \\ \sum_n b_n a_n^* & \sum_n b_n b_n^* & \sum_n b_n c_n^* \\ \sum_n c_n a_n^* & \sum_n c_n b_n^* & \sum_n c_n c_n^* \end{pmatrix} \\ &= \begin{pmatrix} \rho_{11} & \rho_{12} & \rho_{13} \\ \rho_{21} & \rho_{22} & \rho_{23} \\ \rho_{31} & \rho_{32} & \rho_{33} \end{pmatrix}. \end{aligned} \tag{15}$$

Employing the reduced density matrix of the moving three-level atom given by Eq. (15), we investigate the entropy squeezing properties of the component of the three types of moving three-level atoms.

4. Results and discussion

In general, the series in Eq. (15) can not be summed exactly. The numerical results of the entropy squeezing of the three types of three-level atoms are shown in Figs. 2–5 for different components and initial conditions.

Through choosing an eigenstate $|\Psi_{S_x}(1)\rangle = \frac{\sqrt{2}}{2}(-|1\rangle + |3\rangle)$ of S_x as the initial state of the three-level atom, the field in a coherent state with average photon number $\bar{n} = 4$, and phase $\beta = 0$, we plot the time evolution of the x-component entropy squeezing factor $E(S_x)$ against time for two cases of stationary three-level atom and moving three-level atom in Figs. 2–4, respectively.

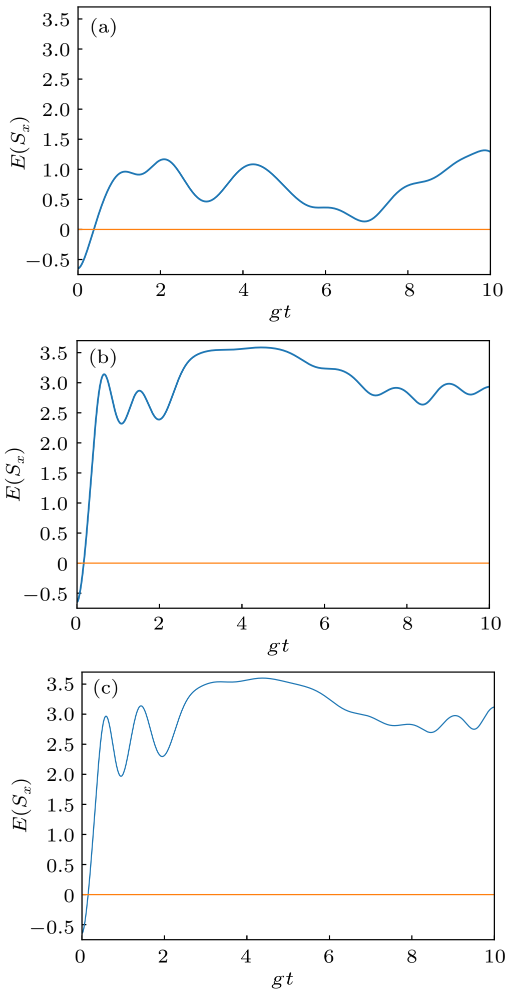

Fig. 2. The evolution of $E(S_x)$ of different stationary three-level atoms. (a) Ξ-type three-level atom; (b) V-type three-level atom; (c) Λ-type three-level atom.

From Fig. 2, it can be seen that when the stationary three-level atom is initially in the eigenstate of S_x, there is obvious entropy squeezing of x component in the initial stage for all three types of three-level atoms, but the duration of entropy squeezing is very short. By comparing Fig. 2(a) with Figs. 2(b) and 2(c), it is obvious that, the entropy squeezing disappears more slowly for the Ξ-type three-level atom (see Fig. 2(a)) than for the V-type three-level atom (see Fig. 2(b)) and Λ-type three-level atom (see Fig. 2(c)). In addition, it is clear that the evolution behavior of entropy squeezing factor $E(S_x)$ is almost the same for the V-type and Λ-type three-level atoms by comparing Fig. 2(b) with Fig. 2(c). It can be concluded from the above discussion that a stationary three-level atom coupled with a single-mode coherent field cannot produce steady and optimal entropy squeezing whatever initial conditions are chosen.

The results for the time evolution of the entropy squeezing factor $E(S_x)$ of the Ξ-type three-level atom in considering the movement of the atom along the z-axis of the cavity are plotted in Figs. 3(a)–3(d). From these figures, we can see the influence of velocity of a moving Ξ-type three-level atom on the time evolution of entropy squeezing factor $E(S_x)$.

In Fig. 3(a), the velocity of the Ξ-type three-level atom is $1g$, which corresponds to the case of the atomic motion with slow velocity. It can be seen that, the motion of the three-level atom leads to the periodic oscillation of entropy squeezing factor $E(S_x)$ and periodic alternating optimal entropy squeezing. With the increase of the three-level atomic velocity (e.g., $v = 5g$ in Fig. 3(b), $v = 10g$ in Fig. 3(c)), the amplitude and period of $E(S_x)$ decrease, and the steady entropy squeezing appears. When the velocity of the three-level atom is increased to $20g$, it is remarkable that the amplitude and period of $E(S_x)$ obviously decrease, and the steady and optimal entropy squeezing of x component of the moving Ξ-type three-level atom can be generated (see Fig. 3(d)) without any additional complex control techniques.

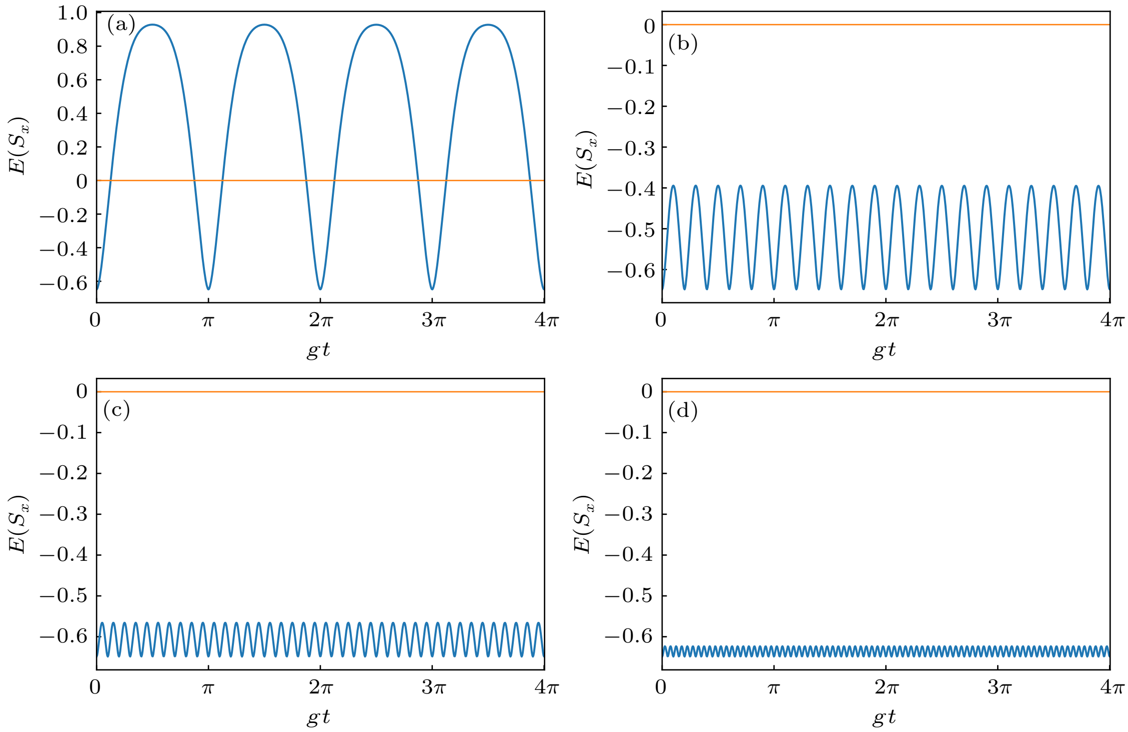

Fig. 3. Influence of velocity of a moving Ξ-type three-level atom on $E(S_x)$ with $p = L/\pi$: (a) $v = 1g$; (b) $v = 5g$; (c) $v = 10g$; (d) $v = 20g$.

The influences of the atomic velocity on the entropy squeezing of x component of the V-type and Λ-type moving three-level atoms are shown by dotted line and solid line in Fig. 4 under the same parameter conditions as those in Fig. 3, respectively. Comparing Figs. 3 and 4, it is obvious that, when different types of three-level atoms move at the same velocity, the entropy squeezing properties of the moving Ξ-type three-level atom are better than one of the other two types moving three-level atoms. By comparing Figs. 3(d) and 4(d), it is clear that, all three types of moving three-level atoms along the axis of the cavity can produce steady and optimal entropy squeezing without any additional control techniques, but let the three-level atom move at higher velocity and in a proper initial state.

In the above case, we choose an eigenstate of S_x as the initial state of the moving three-level atom, the minimum value of $E(S_x)$ is -0.649. But we know that S_x has other two eigenstates

$$|\Psi_{S_x}(2)\rangle = \frac{1}{2}|1\rangle - \frac{\sqrt{2}}{2}|2\rangle + \frac{1}{2}|3\rangle;$$

$$|\Psi_{S_x}(3)\rangle = \frac{1}{2}|1\rangle + \frac{\sqrt{2}}{2}|2\rangle + \frac{1}{2}|3\rangle. \quad (16)$$

We would like to point out that, for a three-level atom, if different eigenstates are selected as its initial state, the calculated value of $H(S_z)$ will be different, which will lead to a difference in the minimum value of $E(S_x)$, in other words, it produces entropy squeezing with different squeezing depth. As discussed above, when the three-level atom is initially in $|\Psi_{S_x}(1)\rangle$, the entropy squeezing depth is -0.649, which is optimal, while when the three-level atom is initially in the other two eigenstates of S_x given by Eq. (16), the entropy squeezing depth is -0.284, namely, the entropy squeezing properties of the three-level atom become bad. The situation discussed above is not going to happen for a two-level atom. Similarly, the S_y operator also has three eigenstates

$$|\Psi_{S_y}(1)\rangle = \frac{\sqrt{2}}{2}(|1\rangle + |3\rangle);$$
$$|\Psi_{S_y}(2)\rangle = -\frac{1}{2}|1\rangle + \frac{\sqrt{2}}{2}\mathrm{i}|2\rangle + \frac{1}{2}|3\rangle;$$
$$|\Psi_{S_y}(3)\rangle = -\frac{1}{2}|1\rangle - \frac{\sqrt{2}}{2}\mathrm{i}|2\rangle + \frac{1}{2}|3\rangle. \quad (17)$$

When we choose the eigenstate $|\Psi_{S_y}(1)\rangle$ as the initial state of the moving three-level atom, the steady and optimal entropy squeezing properties of y direction for the three types of three-level atoms are shown in Fig. 5 with $v = 20g$. It can be seen from Fig. 5 that, under the same parameter conditions, the V-type three-level atom and Λ-type three-level atom have better entropy squeezing properties than the Ξ-type three-level atom. Our numerical results also show that, when the three-level atom is initially in the other two eigenstates of S_y, the entropy squeezing depth of y direction is decreased to −0.284, the entropy squeezing properties of the three-level atom are worse.

From the discussion above, it can be seen that, we can regulate the direction and depth of entropy squeezing of moving three-level atom by selecting appropriate initial state of the three-level atom. If we want to enhance the entropy squeezing in the x direction, we should choose Ξ-type three-level atom with higher velocity and the initial state $|\Psi_{S_x}(1)\rangle$; if we want to strengthen the entropy squeezing in the y direction, we should select V-type or Λ-type three-level atom with higher velocity and the initial state $|\Psi_{S_y}(1)\rangle$ given by Eq. (17). Therefore, according to the actual situation, the steady and optimal entropy squeezing in the x direction or y direction can be obtained by selecting the three-level atoms with different types, different initial states, and higher velocity.

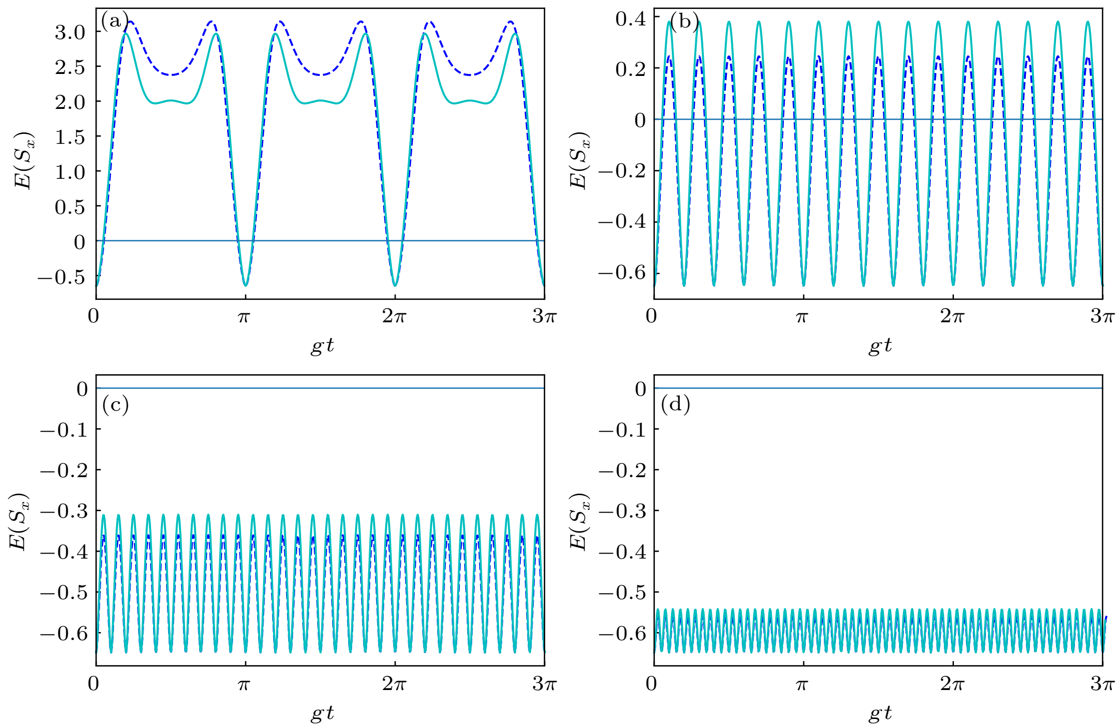

Fig. 4. Influence of velocity of a moving V-type three-level atom (blue dotted line) and Λ-type three-level atom (green solid line) on $E(S_x)$ with $p = L/\pi$: (a) $v = 1g$; (b) $v = 5g$; (c) $v = 10g$; (d) $v = 20g$.

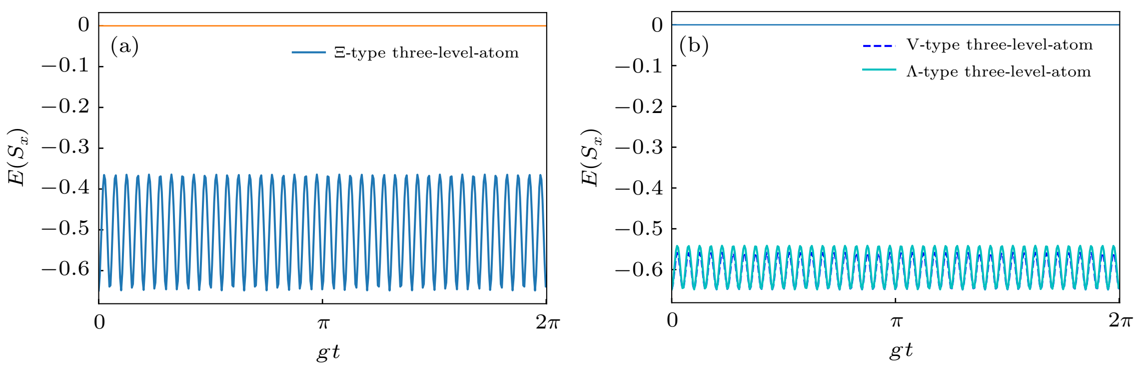

Fig. 5. When the initial state of the three-level-atom is $|\Psi_{S_y}(1)\rangle = \frac{\sqrt{2}}{2}(|1\rangle + |3\rangle)$: (a) Ξ-type three-level atom; (b) V-type three-level atom and Λ-type three-level atom. $p = L/\pi$ and $v = 20g$.

5. Summary and conclusion

We have studied the entropy squeezing properties of different types of moving three-level atoms coupled with a single-mode coherent field. The influences of the moving velocity and initial states of the three-level atom on the entropy squeezing are discussed. The results show that, the entropy squeezing properties of the three-level atom depend on its initial state, moving velocity, and the type. A stationary three-level atom cannot obtain a steady entropy squeezing whatever initial conditions are chosen, while a moving three-level atom can achieve a steady and optimal entropy squeezing through choosing higher velocity and appropriate initial state of the three-level atom. The direction and depth of entropy squeezing of the moving three-level atom can be regulated by selecting appropriate initial state of the three-level atom. When we choose Ξ-type three-level atom with higher velocity and the initial state $|\Psi_{S_x}(1)\rangle$, we can produce a steady and optimal entropy squeezing in the x direction; while when we choose V-type or Λ-type three-level atom with higher velocity and appropriate initial state, we can prepare a steady and optimal entropy squeezing in the y direction. As a conclusion, for generating a steady and optimal entropy squeezing of three-level atom, our method only needs to control the moving velocity and choose the initial state of the three-level atom, and do not need to add any complex control techniques. Therefore, our result provides a simple method for preparing squeezing resources of three-level atom with ultra-low quantum noise.

Appendix A

We present the corresponding coefficients of three types of moving three-level atoms:

Ξ-type:

$$\begin{aligned} a_n(t) =& -\mathrm{i}\lambda(t)\sqrt{n+1}\left[\frac{Y_{n+1}}{Z_{n+1}}\sin(Z_{n+1}t)+\frac{X_{n+1}}{(Z_{n+1})^2}\right.\\ &\left.-\frac{X_{n+1}}{(Z_{n+1})^2}\cos(Z_{n+1}t)\right]+A_n\cos\theta;\\ b_n(t) =& Y_n\cos(Z_nt)+\frac{X_n}{Z_n}\sin(Z_nt);\\ c_n(t) =& -\mathrm{i}\lambda(t)\sqrt{n}\left[\frac{Y_{n-1}}{Z_{n-1}}\sin(Z_{n-1}t)+\frac{X_{n-1}}{(Z_{n-1})^2}\right.\\ &\left.-\frac{X_{n-1}}{(Z_{n-1})^2}\cos(Z_{n-1}t)\right]+A_n\sin\theta\cos\varphi, \end{aligned} \tag{A1}$$

where

$$\begin{aligned} X_n &= -\mathrm{i}\lambda(t)\left[\sqrt{n}A_{n-1}\cos\theta+\sqrt{n+1}A_{n+1}\sin\theta\cos\varphi\right];\\ Y_n &= A_n\sin\theta\sin\varphi;\\ Z_n &= \lambda(t)\sqrt{2n+1}. \end{aligned} \tag{A2}$$

V-type:

$$\begin{aligned} a_n(t) =& -\mathrm{i}\lambda(t)\sqrt{n+1}\left[\frac{Y_{n+1}}{Z_{n+1}}\sin(Z_{n+1}t)+\frac{X_{n+1}}{(Z_{n+1})^2}\right.\\ &\left.-\frac{X_{n+1}}{(Z_{n+1})^2}\cos(Z_{n+1}t)\right]+A_n\cos\theta;\\ b_n(t) =& -\mathrm{i}\lambda(t)\sqrt{n+1}\left[\frac{Y_{n+1}}{Z_{n+1}}\sin(Z_{n+1}t)+\frac{X_{n+1}}{(Z_{n+1})^2}\right.\\ &\left.-\frac{X_{n+1}}{(Z_{n+1})^2}\cos(Z_{n+1}t)\right]+A_n\sin\theta\sin\varphi;\\ c_n(t) =& Y_n\cos(Z_nt)+\frac{X_n}{Z_n}\sin(Z_nt), \end{aligned} \tag{A3}$$

where

$$\begin{aligned} X_n &= -\mathrm{i}\lambda(t)\left[\sqrt{n}A_{n-1}\cos\theta+\sqrt{n}A_{n-1}\sin\theta\sin\varphi\right];\\ Y_n &= A_n\sin\theta\cos\varphi;\\ Z_n &= \lambda(t)\sqrt{2n}. \end{aligned} \tag{A4}$$

Λ-type:

$$\begin{aligned} a_n(t) =& Y_n\cos(Z_nt)+\frac{X_n}{Z_n}\sin(Z_nt);\\ b_n(t) =& -\mathrm{i}\lambda(t)\sqrt{n}\left[\frac{Y_{n-1}}{Z_{n-1}}\sin(Z_{n-1}t)+\frac{X_{n-1}}{(Z_{n-1})^2}\right.\\ &\left.-\frac{X_{n-1}}{(Z_{n-1})^2}\cos(Z_{n-1}t)\right]+A_n\sin\theta\sin\varphi;\\ c_n(t) =& -\mathrm{i}\lambda(t)\sqrt{n}\left[\frac{Y_{n-1}}{Z_{n-1}}\sin(Z_{n-1}t)+\frac{X_{n-1}}{(Z_{n-1})^2}\right.\\ &\left.-\frac{X_{n-1}}{(Z_{n-1})^2}\cos(Z_{n-1}t)\right]+A_n\sin\theta\cos\varphi, \end{aligned} \tag{A5}$$

where

$$\begin{aligned} X_n &= -\mathrm{i}\lambda(t)\sqrt{n+1}\left[A_{n+1}\sin\theta\cos\varphi+A_{n+1}\sin\theta\sin\varphi\right];\\ Y_n &= A_n\cos\theta;\\ Z_n &= \lambda(t)\sqrt{n+1}. \end{aligned} \tag{A6}$$

References

[1] Kitagawa M and Ueda M 1993 *Phys. Rev. A* **47** 5138
[2] Srensen J L, Hald J and Polzik E S 1998 *Phys. Rev. Lett.* **80** 3487
[3] Srensen A and Mlmer K 1999 *Phys. Rev. Lett.* **83** 2274
[4] Berta M, Christandl M, Colbeck R, Renes J M and Renner R 2010 *Nat. Phys.* **6(9)** 659
[5] Hu M L and Fan H 2012 *Phys. Rev. A* **86** 032338
[6] Hu M L and Fan H 2013 *Phys. Rev. A* **87** 022314
[7] Hu M L, Hu X Y, Wang J C, Peng Y, Zhang Y R and Fan H 2018 *Phys. Rep.* **762** 1
[8] Fang M F, Zhou P and Swain S 2000 *Journal of Modern Optics* **47** 1043
[9] Liu X and Fang M F 2002 *Chin. Phys.* **11** 926
[10] Li C X and Fang M F 2003 *Chin. Phys.* **12** 294
[11] Gea-Banacloche J 1990 *Phys. Rev. Lett.* **65** 3385
[12] Gea-Banacloche J 1991 *Phys. Rev. A* **44** 5913
[13] Phoenix S J D and Knight P L 1991 *Phys. Rev. A* **44** 6023
[14] Yu M and Fang M F 2016 *Quantum Information Processing* **15** 4175
[15] Wang Y Y and Fang M F 2018 *Chin. Phys. B* **27** 114207
[16] Rainer R Schlicher 1989 *Opt. Commun.* **70** 97
[17] Fang M F 1998 *Physica A* **259** 193
[18] Liu X J, Fang M F and Zhou Q P 2005 *Acta Phys. Sin.* **54** 703
[19] Kaszlikowski D, Gnacinski P, Zukowski M, Miklaszewski W and Zeilinger A 2000 *Phys. Rev. Lett.* **85** 4418
[20] Walborn S P, Lemelle D S, Almeida M P and Ribeiro P H S 2006 *Phys. Rev. Lett.* **96** 090501
[21] Xu X and Fang M F 2020 *Chin. Phys. B* **29** 057305
[22] Liu F F, Fang M F and Xu X 2019 *Chin. Phys. B* **28** 060304
[23] Riccardi A, Macchiavello C and Maccone L 2017 *Phys. Rev. A* **95** 032109
[24] Sargent M III, Scully M O and Lamb W E Jr 1974 *Laser Physics* (Reading: Addison-Wesley)