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Strong Interaction Effect on Jet Energy Loss with Detailed Balance *

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The strong force effect on gluon distribution of quark-gluon plasma and its influence on jet energy loss with detailed balance are studied. We solve the possibility equation and obtain the value of non-extensive parameter q. In the presence of strong interaction, more gluons stay at low-energy state than the free gluon case. The strong interaction effect is found to be important for jet energy loss with detailed balance at intermediate jet energy. The energy gain via absorption increases with the strong interaction. This will affect the nuclear modification factor $R_{\rm AA}$ and the parameter of $\hat{q}$ at intermediate jet energy.

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It is known from statistical quantum chromodynamics (QCD) that strongly interacting matter undergoes a deconfinement transition to a new state, quark-gluon plasma (QGP). One of the main probes of QGP is jets (energetic partons). Gluon radiation induced by multiple scattering of the energetic partons propagating in a dense medium[1] leads to jet quenching. As discovered in high-energy heavy-ion collisions at RHIC, jet quenching is manifested in both the suppression of single inclusive hadron spectrum at high transverse momentum $p_{\rm T}$ region[2,3] and the disappearance of the typical back-to-back jet structure in dihadron correlations.[4] Extensive theoretical investigation of jet quenching has been widely carried out in recent years.[5−11] Most of them were focused on the radiative jet energy loss;[12] gluon absorption is not considered at high transverse momentum. However, for the intermediate jet energy region, it was shown that the gluon absorption plays an important role.[13] The gluon distribution considered here is the distribution of ideal gluon gas. However, quark-gluon plasma is strongly coupled. The strong interaction will induce the gluon distribution to a more complicated case. In this Letter, we report a study on mean occupation number distribution of gluons with strong interaction and analyze the interaction effect on radiative and absorptive jet energy loss.

If the potential energy behaves like $V(r) = -A/r^{\alpha}$, $\alpha < 3$, the interactions will exhibit singularities at the origin in Boltzmann–Gibbs (B-G) statistics.[14] The value of α in the interaction potential (the Cornell potential) among gluons in quark-gluon plasma is less than three. This made B-G statistics an inefficient theory to describe gluons in QGP. Non-extensive statistics is a statistical theory which aims at solving this problem. Recent developments in astrophysical scenarios,[15] heavy ion collisions[16] and so on show a quantitative agreement between experimental data and theoretical models based on non-extensive statistics. In non-extensive statistics, the parameter q describes the non-extensiveness of the system. In the limit of $q \to 1$, non-extensive statistics comes to B-G statistics. Thus the entropic index q in S_q can be regarded as the physical effect on a standard B-G system which causes the system's non-extensiveness and non-additive entropy. In this work, we study the strong interaction effect on the gluons by analyzing the non-extensive parameter's departure from unit and determine the mean occupation number distribution of gluons with strong interaction. Then we analyze its influence on the radiative and absorptive jet energy loss and compare our result with that under ideal gluon gas distribution.

The singularity for $V(r) = -A/r^{\alpha}$, $\alpha < 3$ shows non-extensiveness of the energy with interactions among particles.[14] Non-extensive statistics is a statistical theory which aims at describing non-extensiveness. It is applied in many fields, such as astrophysical self-gravitating systems,[15] heavy ion collisions,[16] anomalous diffusion.[17] It is helpful for achieving a better understanding of the phenomenon.

Non-extensive statistical mechanics is based on the generalized functional form of the entropy (in natural unit with $k = \hbar = 1$)

$$S_q = \frac{1-\sum_{i=1}^{W} p_i^q}{q-1} \Big(\sum_{i=1}^{W} p_i = 1; q \in R\Big), \tag{1}$$

where W is the total number of microstates, and the parameter q describes the non-extensiveness of the system. The B-G entropy can be obtained from Eq. (1) in the limit of $q \to 1$.

It can be straightforwardly verified that for systems A_1 and A_2 if the joint probability satisfies $p_{ij}^{A_1+A_2} = p_i^{A_1} p_j^{A_2}$,

$$\begin{aligned} S_q(A_1+A_2) = {} & S_q(A_1) + S_q(A_2) \\ & + (1-q) S_q(A_1) S_q(A_2). \end{aligned} \tag{2}$$

This shows the non-extensiveness of the entropy.

In non-extensive statistical mechanics, the mean

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value of a variable is

$$\langle x\rangle_q \equiv \int_0^\infty dx x P(x), \tag{3}$$

where $P(x)$ is the escort distribution, which is defined as an arbitrary, possibly fractal, probability distribution. In thermostatistics of multifractals, the probability distribution $p(x)$ is sought on the basis of incomplete knowledge, an occurrent event will induce a set of further probability distribution[18]

$$P(x) \equiv \frac{[p(x)]^q}{\int_0^\infty dx'[p(x')]^q}. \tag{4}$$

We can verify that $P(x)$ is normalized, $\int_0^\infty dx P(x) = 1$.

If the system is in equilibrium, with the principle of maximum entropy, then Fermi–Dirac and Bose–Einstein (escort) mean occupation number distributions could be generalizable as follows:[19]

$$\bar{n}_i = \frac{1}{[1+(q-1)\beta(\epsilon_i-\mu)]^{\frac{q}{q-1}} \pm 1}. \tag{5}$$

This distribution is not always a positive real number. To guarantee the real and positive character of the mean occupation number distribution, it is necessary to introduce a cut-off prescription, and $\bar{n}_i$ is set to zero for the negative case of the distribution.[20]

In the previous work,[13] detailed balance effect of jet energy loss is considered under distribution of ideal gluon gas. However, the gluons are strongly coupled. The gluon distribution with strong force interaction needs to be reconsidered.

The theory of jet energy loss was developed under a class of multiple scattering diagrams. The average separation between the scattering centers is specified by the mean free path λ. The jet energy loss is controlled by the opacity-number of scatterings $\bar{n} = N\sigma_{\rm el}/A_\perp = L/\lambda$, where N, L and $A_\perp$ are, respectively, the number, the thickness, and the transverse area of the targets. Thus we will focus on gluon absorption during each scattering.

In the area of each scattering with a size of mean free path λ, the number of gluons in this area can be estimated by

$$N = \frac{dN_{\rm g}}{dy}\cdot\Big(\frac{\lambda}{D}\Big)^2 \approx 8, \tag{6}$$

where $dN_{\rm g}/dy$ is the expected rapidity density of the gluons, which is approximately 1000 at RHIC energies at $\sqrt{s} \approx 200$ AGeV for $A = 208$,[21] and D is the transverse size of quark-gluon plasma, approximately 11 fm.[22] The mean free path is

$$\lambda_{\rm g}^{-1} = \langle\sigma_{\rm qg}\rho_{\rm g}\rangle + \langle\sigma_{\rm gg}\rho_{\rm g}\rangle \approx \frac{2\pi\alpha_{\rm s}^2}{\mu^2} 9\times 7\zeta(3)\frac{T^3}{\pi^2}, \tag{7}$$

where μ is the Debye screening mass with $\mu^2 = 4\pi\alpha_{\rm s}T^2$ from the perturbative QCD at finite temperature.[23] The temperature of quark-gluon plasma is set to be 250 MeV at RHIC, then $\lambda_{\rm g} = 1$ fm.

We consider these gluons to compose a gluon system. Since the diameter of this system, i.e., the mean free path λ, is much larger than $1/\mu$, we ignore the interactions with other areas in other scatterings, then the possibility of this system is

$$P = \frac{\exp(-\beta E)}{Z}, \tag{8}$$

where Z is the corresponding partition function $Z = \int \exp(-\beta E) d^{3N}p d^{3N}r/(2\pi)^{3N}$.

Here E is the energy of the gluon system

$$E = \sum_{i=1}^{N}\frac{p_i^2}{2\mu} + \sum_{j=1}^{C_N^2} V_j(r), \tag{9}$$

with $V_j(r)$ being the interaction potential between gluons. The interaction potential can be parameterized as[24]

$$V(r) = C\sigma r - \frac{3\alpha_{\rm s}}{r} - D, \tag{10}$$

in terms of the string tension σ and a $1/r$ contribution containing both transverse string and the Coulombic effects. Here $\sigma = 0.19\,{\rm GeV}^2$ is the fundamental quark–antiquark flux tube energy, $C = 9/4$ indicates the scaling of the energy density, $\alpha_{\rm s}$ is the strong coupling constant, and $\alpha_{\rm s} = 0.09$. The constant D is used to fit the height of potential. We set the zero potential energy at the distance of the Debye length $1/\mu$, then we obtain $D = 0.855$ GeV. Since the plasma is strongly coupled, we concentrate on the strong coupling form and ignore the Coulombic term to avoid the infrared divergence.

The probability of the gluon system can also be considered as the joint probability of the gluons composing this system. Then

$$P = \frac{\exp(-\beta E)}{Z} = (P_q)^N, \tag{11}$$

where P_q is the possibility of each gluon.

If the gluon is free, its probability can be written as

$$P_\epsilon = \frac{\exp(-\beta\epsilon)}{Z_1}, \tag{12}$$

where $\epsilon = p^2/2\mu$, Z_1 is the corresponding partition function $Z_1 = \int\int \exp(-\beta\epsilon) d^3\boldsymbol{p} d^3\boldsymbol{r}/(2\pi)^3$.

However, gluons are not free, they are strongly coupled. This will induce the probability changing into

$$P_q = \frac{(P_\epsilon)^q}{\sum(P_\epsilon)^q} = \frac{e^{-\beta q\epsilon}}{\frac{V}{h^3}(\frac{2m\pi}{\beta q})^{\frac{3}{2}}}, \tag{13}$$

according to the non-extensive statistical theory in Eq. (4).

We simulate the energy of the system Eq. (9) at the static case, substitute Eq. (13) into Eq. (11), and solve this probability equation. We can then obtain the average q and find that it changes with the gluon number N as shown in Fig. 1. With increasing the gluon number, the interaction on each gluon is stronger, thus the value of q departs more from unit. Based on the evaluation of the gluon number in Eq. (6), the value of q is 0.4 for QGP at RHIC.

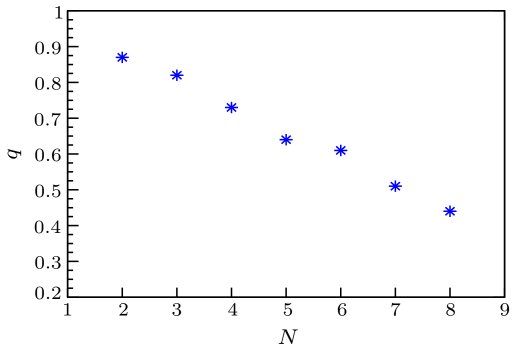

Fig. 1. The value of the non-extensive parameter q as a function of gluon number N.

With the value of $q = 0.4$, the mean occupation number distribution of the strongly coupled gluons $\overline{n}_i$ at the RHIC temperature $T = 250\,\mathrm{MeV}$ is obtained, as shown in Fig. 2. It shows that with the strong force interaction, more gluons will stay at low-energy state, while fewer will jump to high-energy state if compared with the free gluon case.

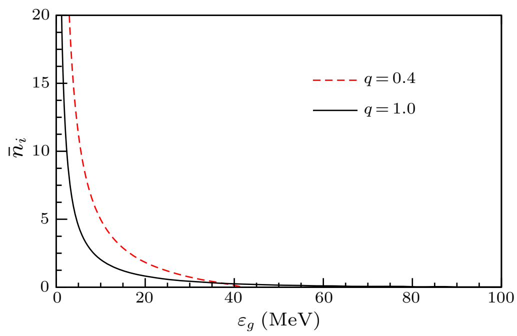

Fig. 2. The mean occupation number distribution of strongly coupled gluons $\overline{n}_i$ as a function of gluon energy ε_{g} at temperature $T = 250\,\mathrm{MeV}$.

Jet energy loss with thermal absorption is found to be important at intermediate jet energy. The thermal absorption originates from gluons in quark-gluon plasma, whose distribution is influenced by the interaction among them. Here we will study the strong interaction effect on the jet energy loss with detailed balance and analyze the effect on the nuclear modification factor R_{AA}.

The quark-gluon plasma medium is assumed in thermal equilibrium shortly after the production of the jet. Taking into account both stimulated emission and thermal absorption in a thermal medium with finite temperature T, one has the probability of gluon radiation with energy ω,[13]

$$\frac{dP^{(0)}}{d\omega} = \frac{\alpha_{\mathrm{s}} C_{\mathrm{F}}}{2\pi} \int \frac{dz}{z} \int \frac{d\boldsymbol{k}_\perp^2}{\boldsymbol{k}_\perp^2} \Big\{ N(zE)\delta(\omega + zE) + [1 + N(zE)]\delta(\omega - zE)\theta(1-z) P\Big(\frac{\omega}{E}\Big) \Big\}, \tag{14}$$

where $k = (\omega, \boldsymbol{k})$ is the four-momentum of the radiated gluon, $z = \omega/E$, C_{F} is the Casimir of the quark jet in the fundamental representation, and the splitting function $P_{\mathrm{gq}}(z) \equiv P(z)/z = [1 + (1-z)^2]/z$ for $q \to gq$. The first term is obtained from thermal absorption and the second term from gluon emission with the Bose–Einstein enhancement factor. Here the thermal gluon distribution with strong interaction is

$$N(|\boldsymbol{k}|) = \frac{1}{[1 + (q-1)(|\boldsymbol{k}|/T)]^{\frac{q}{q-1}} - 1}, \tag{15}$$

according to Eq. (5) in non-extensive statistics.

To define the effective parton energy loss, we consider only gluon radiation outside a cone with $|\boldsymbol{k}_\perp| > \mu$, where μ is the Debye screening mass in the finite temperature. Assuming the scale of the hard scattering as $Q^2 = 4E^2$, then the kinematic limits of the gluon's transverse momentum will be $\mu^2 \leq \boldsymbol{k}_{\perp\,\max}^2 \leq 4|\omega|(E-\omega)$.[13]

Subtracting the gluon radiation spectrum in the vacuum, one then obtains the energy loss due to final-state absorption and stimulated emission

$$\begin{aligned} \Delta E_{\mathrm{abs}}^{(0)} &= \int d\omega\omega \Big(\frac{dP^{(0)}}{d\omega} - \frac{dP^{(0)}}{d\omega}\,|_{T=0} \Big) \\ &= \frac{\alpha_{\mathrm{s}} C_{\mathrm{F}}}{2\pi} E \int dz \int \frac{d\boldsymbol{k}_\perp^2}{\boldsymbol{k}_\perp^2} [-P(-z)N(zE) \\ &\quad + P(z)N(zE)\theta(1-z)]. \end{aligned} \tag{16}$$

Even though the stimulated emission cancels part of the contribution from absorption, the net medium effect without rescattering is still dominated by the final-state thermal absorption, resulting in a net energy gain as shown in Fig. 3(b).

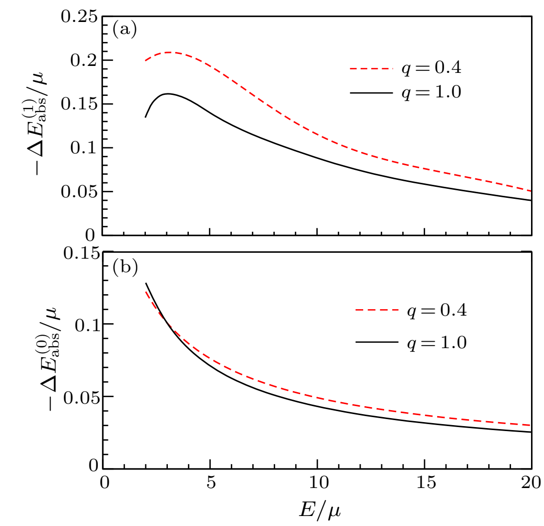

Fig. 3. Energy gain via absorption with $(-\Delta E_{\mathrm{abs}}^{(1)})$ and without $(-\Delta E_{\mathrm{abs}}^{(0)})$ rescattering at different values of q with consideration of strong interaction among gluons.

During the propagation of the hard parton after its production, it will suffer multiple scattering with the medium. The multiple scattering can also induce gluon radiation. Here we will investigate the stimulated emission and thermal absorption associated with multiple scattering in a hot QCD medium with strong interaction.

We also follow the framework of opacity expansion developed by Gyulassy, Levai, and Vitev (GLV).[8] It was shown by GLV that the higher order corrections contribute slightly to the radiative energy loss. Thus here we consider only contributions to the first order in the opacity expansion.

Similarly to the case of final-state absorption, one can also include stimulated emission and thermal absorption at the first order in opacity, the corresponding energy loss can be expressed as

$$\begin{aligned}\Delta E^{(1)}_{\rm abs} &= \int d\omega\omega\Big(\frac{dP^{(1)}}{d\omega} - \frac{dP^{(1)}}{d\omega}|_{T=0}\Big)\\ &= \frac{\alpha_{\rm s}C_{\rm F}}{\pi}\frac{L}{\lambda_{\rm g}}E\int dz\int\frac{d\boldsymbol{k}_\perp^2}{\boldsymbol{k}_\perp^2}\int d^2\boldsymbol{q}_\perp|v(\boldsymbol{q}_\perp)|^2\\ &\cdot\frac{\boldsymbol{k}_\perp\cdot\boldsymbol{q}_\perp}{(\boldsymbol{k}_\perp-\boldsymbol{q}_\perp)^2}N(zE)\\ &\times[-P(-z)\langle{\rm Re}(1-e^{iw_1y_{10}})\rangle\\ &+P(z)\langle{\rm Re}(1-e^{iw_1y_{10}})\rangle\theta(1-z)],\end{aligned}\tag{17}$$

which mainly comes from thermal absorption with partial cancelation by stimulated emission in the medium.

Here

$$\begin{aligned}\Delta E^{(1)}_{\rm rad} &= \frac{\alpha_{\rm s}C_{\rm F}}{\pi}\frac{L}{\lambda_{\rm g}}E\int dz\int\frac{d\boldsymbol{k}_\perp^2}{\boldsymbol{k}_\perp^2}\int d^2\boldsymbol{q}_\perp|v(\boldsymbol{q}_\perp)|^2\\ &\frac{\boldsymbol{k}_\perp\cdot\boldsymbol{q}_\perp}{(\boldsymbol{k}_\perp-\boldsymbol{q}_\perp)^2}P(z)\langle{\rm Re}(1-e^{iw_1y_{10}})\rangle\theta(1-z).\end{aligned}\tag{18}$$

The factor $1-\exp(i\omega_1 y_{10})$ reflects the destructive interference arising from the non-Abelian LPM effect, and $\langle\ldots\rangle = \int dy\rho(y)\ldots$ is the longitudinal target profile. The target distribution is assumed to be an exponential form $\rho(y) = 2\exp(-2y/L)/L$, $\lambda_{\rm g} = C_{\rm F}\lambda/C_{\rm A}$ is the mean free path of the gluon, and $|\bar{v}(\boldsymbol{q}_\perp)|^2$ is the normalized distribution of momentum transfer from the scattering centers

$$|\bar{v}(\boldsymbol{q}_\perp)|^2 \equiv \frac{1}{\sigma_{\rm el}}\frac{d^2\sigma_{\rm el}}{d^2\boldsymbol{q}_\perp} = \frac{1}{\pi}\frac{\mu_{\rm eff}^2}{(\boldsymbol{q}_\perp^2+\mu^2)^2},\tag{19}$$

$$\frac{1}{\mu_{\rm eff}^2} = \frac{1}{\mu^2} - \frac{1}{q_{\perp\,\max}^2+\mu^2},\ q_{\perp\,\max}^2 \approx 3E\mu.\tag{20}$$

Substituting the gluon number distribution function Eq. (15) at $q = 1, 0.4$ into Eq. (17), the energy gain can be obtained, as shown in Fig. 2. Figures 3(a) and 3(b) show the energy gain via gluon absorption with $(-\Delta E^{(1)}_{\rm abs})$ and without $(-\Delta E^{(0)}_{\rm abs})$ rescattering at different values of $q = 1$, 0.4 with considering strong interaction among gluons. It is found that the energy gain increases with the departure of q from unit. This is because more gluons stay at low-energy state with the strong interaction. That will induce the increase of the possibility of thermal absorption.

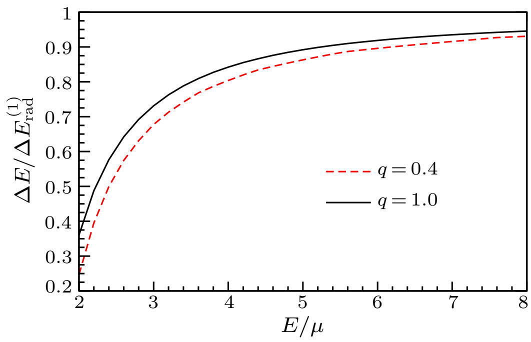

Fig. 4. The ratio of effective parton energy loss with $(\Delta E = \Delta E^{(0)}_{\rm abs} + \Delta E^{(1)}_{\rm abs} + \Delta E^{(1)}_{\rm rad})$ and without $(\Delta E^{(1)}_{\rm rad})$ absorption as a function of E/μ at different values of q.

Figure 4 shows the ratios of the calculated radiative energy loss with and without thermal absorption as functions of E/μ for $L/\lambda_{\rm g} = 5$ at different values of q considering the strong interaction among gluons. It is found that the thermal absorption reduces the effective parton energy loss for intermediate values of parton energy. For $q = 1$, which corresponds to the free gluon case, the ratio is the same as that in the previous work.[13] However, when considering the strong interaction among gluons with the departure of q from unit, the ratio is only 60% of that in the free gluon case at intermediate jet energy. This is because more gluons stay at low-energy state, and the ratio increases with the gluon absorption. It indicates the importance of the strong interaction effect on the jet energy loss with detailed balance at intermediate jet energy. The decrease of the ratio will increase the nuclear modification factor $R_{\rm AA}$ at the intermediate jet energy. That will induce the change of parameter of $\hat{q}$. For partons with very high energy the effect of the gluon absorption is small and can be ignored.

In summary, we have considered the effect of strong interaction on the mean occupation number distribution of gluons. Since this effect induces more gluons to stay at low-energy state than the free gluon case, it is important for jet energy loss with detailed balance at intermediate jet energy. The energy gain via absorption increases with considering the strong interaction. The ratio of energy loss with and without thermal absorption is only 60%, half of the free gluon case. This will affect the nuclear modification factor $R_{\rm AA}$ and the parameter of $\hat{q}$ at intermediate jet energy.

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