CHIN. PHYS. LETT. Vol. 34, No. 6 (2017) 067101

Local Heating in a Normal-Metal–Quantum-Dot–Superconductor System without Electric Voltage Bias *

Li-Ling Zhou(周利玲)**, Xue-Yun Zhou(周雪云), Rong Cheng(程融),
Cui-Ling Hou(侯翠岭), Hong Shen(沈红)
Department of Physics, Jiujiang University, Jiujiang 332005

(Received 17 February 2017)

We investigate the heat generation Q in a quantum dot (QD), coupled to a normal metal and a superconductor, without electric bias voltage. It is found that Q is quite sensitive to the lead temperatures $T_{\rm L,R}$ and the superconductor gap magnitude Δ. At $T_{\rm L,R} \ll \omega_0$ (ω_0 is the phonon frequency), the superconductor affects Q only at $\Delta < \omega_0$, and the maximum magnitude of negative Q appears at some Δ slightly smaller than ω_0. At elevated lead temperature, contribution to Q from the superconductor arises at Δ, ranging from less than to much larger than ω_0. However, the peak value of Q is several times smaller than that in the case of $T_{\rm L,R} \ll \omega_0$. Interchanging lead temperatures $T_{\rm L}$ and $T_{\rm R}$ leads to quite different Q behaviors, while this makes no difference for a normal-metal–quantum-dot–normal-metal system, and the QD can be cooled much more efficiently when the superconductor is colder.

PACS: 71.38.−k, 73.63.Kv, 73.63.−b DOI: 10.1088/0256-307X/34/6/067101

A modern chip may have several billion transistors in an area the size of a human fingernail. With the constant miniaturization of semiconductor devices, the heat dissipation in integrated circuits (ICs) has attracted growing attention. As ICs become smaller, the quantity of heat that dissipates from them becomes much higher. For a nanoscale junction, a small amount of heat can cause a substantial temperature rise because of inefficient heat conduction and small heat capacity. Such a temperature increase is relevant since it may affect the normal behavior of semiconductor devices. To save electronic devices from imminent failure, researchers have carried out[1−18] investigations in recent years to find out general laws of heat generation in nanosystems and then to prevent heat generation.

For a nanoscale device, the Joule heating is a predominant heat mechanism for heat generation. An electron enters the device and leaves with less energy. The lost energy is transferred to ionic degrees of freedom via inelastic electron–phonon (e–p) collisions, i.e., heat is generated. Compared with the size of the nanodevice, the mean free path of inelastic scattering of electrons is large. As a result, the chance of e–p collision for an electron is quite small. The heat generation, however, is still substantial[19−22] due to extremely large density of electron current, by many orders of magnitude larger than that in a macroscopic metal. On the topic of heat generation, the hybrid system consisting of a quantum dot (QD) coupled to two normal leads has been investigated extensively.[1,3,5,6,10] Some interesting features unique in nanostructures, absent in the bulk, are found. For example, at zero temperature a threshold bias voltage is required for heat generation. For a large fixed voltage, the Joule heat in the QD is not proportional to the current. Very recently, local heating in a slightly different model, i.e., a normal-metal–quantum-dot–superconductor system (N–QD–S), is discussed.[2,4] In the discussions, the electron and phonon temperatures are taken as the same, and researchers focus mainly on how the heating in the QD is affected by bias voltage, gate voltage, and system temperature. The obtained results show that, due to different tunneling mechanisms, the properties of heat generation at bias larger than the superconducting gap are quite different from those at biases smaller than the gap,[4] and the heating can be controlled by the gate voltage, the bias and the system temperature.

In this Letter, we investigate the heat generation in the QD of an N–QD–S system, in which no electric voltage bias is present. We restrict our attention to how the heating is affected by the superconducting gap and lead temperatures.

The model under our consideration can be described by the Hamiltonian (hereafter $e = 1$, $\hbar = 1$)

$$H = H_{\rm L} + H_{\rm R} + H_{\rm D} + H_{\rm T}, \tag{1}$$

where $H_{\rm L}$ and $H_{\rm R}$ represent the left normal-metal and right superconducting lead, respectively, with $H_{\rm L} = \sum_{k,\sigma} \epsilon_{{\rm L},k} c^\dagger_{{\rm L},k\sigma} c_{{\rm L},k\sigma}$ and $H_{\rm R} = \sum_{k,\sigma} \epsilon_{{\rm R},k} c^\dagger_{{\rm R},k\sigma} c_{{\rm R},k\sigma} + \Delta \sum_k (c^\dagger_{{\rm R},k\uparrow} c^\dagger_{{\rm R},-k\downarrow} + c_{{\rm R},-k\downarrow} c_{{\rm R},k\uparrow})$, and $H_{\rm D} = \sum_\sigma \epsilon_{\rm d} d^\dagger_\sigma d_\sigma + \omega_0 a^\dagger a + \lambda(a^\dagger + a)\sum_\sigma d^\dagger_\sigma d_\sigma$ describes the dot state. Hybridization between leads and dot is depicted by $H_{\rm T} = \sum_{\alpha,k\sigma}(t_\alpha c^\dagger_{\alpha,k\sigma} d_\sigma + t^*_\alpha d^\dagger_\sigma c_{\alpha,k\sigma})$. The fermion operators $c^\dagger_{\alpha,k\sigma}$ ($c_{\alpha,k\sigma}$) and $d^\dagger_\sigma$ (d_σ) create (destroy) an electron of vector k and spin σ ($\sigma = \uparrow, \downarrow$) in the α-lead and dot, respectively. The boson operator $a^\dagger$ (a) creates (annihilates) a phonon mode of frequency ω_0, λ is the electron–phonon interaction (EPI) strength, 2Δ is the superconducting gap, and

*Supported by the National Natural Science Foundation of China under Grant No 11164011.
**Corresponding author. Email: 147667897@qq.com

t_α is the electron hopping amplitude between the QD and α-lead.

In the QD, heat can be emitted or absorbed via the EPI. An electron tunnels into the QD and leaves with diminished (or increased) energy. The loss (or gain) energy is transferred to (or comes from) ionic degrees of freedom. The heat generation per unit time at time t can be calculated from the time evolution of the energy operator $E_{\rm ph} = \omega_0 a^\dagger(t)a(t)$: $Q(t) = \langle dE_{\rm ph}/dt\rangle$. By connecting the QD to a large outside thermal bath, we can assume that the phonons in the QD are in the equilibrium Boson distribution $N_{\rm ph} = 1/[\exp(\omega_0/\kappa_{\rm B}T_{\rm ph}) - 1]$, with $T_{\rm ph}$ being the phonon bath temperature.

Following our previous work,[6] we obtain an analytic expression of $Q(t)$ with the help of Green's function formalism and Wick's theorem,

$$\begin{aligned} Q =& \omega_0\lambda^2 \sum_{\sigma,\sigma'}\int\frac{d\omega}{2\pi}\{\widetilde{G}^<_{\sigma\sigma'}(\omega)\widetilde{G}^>_{\sigma'\sigma}(\omega-\omega_0) \\ & - 2N_{\rm ph}{\rm Re}[\widetilde{G}^{\rm r}_{\sigma\sigma'}(\omega)\widetilde{G}^<_{\sigma'\sigma}(\omega-\omega_0) \\ & + \widetilde{G}^<_{\sigma\sigma'}(\omega)\widetilde{G}^{\rm a}_{\sigma'\sigma}(\omega-\omega_0)]\}, \end{aligned} \tag{2}$$

where we have made a canonical transformation $\widetilde{H} = VHV^\dagger$, with the unitary operator $V = \exp[\lambda/\omega_0(a^\dagger - a)\sum_\sigma d^\dagger_\sigma d_\sigma]$, and made an approximation to replace X with its expectation value $\langle X\rangle = \exp[-(\lambda/\omega_0)^2(N_{\rm ph}+1/2)]$,[3,4,6,10,23] the operator $X = \exp[-\lambda/\omega_0(a^\dagger - a)]$ arising from the canonical transformation of the electron operator $\tilde{d}_\sigma = Vd_\sigma V^\dagger = d_\sigma X$, and $\widetilde{G}^{</>/{\rm r}/{\rm a}}_{\sigma\sigma'}(\omega)$ is the Fourier transformation of the standard Keldysh lesser (greater, retarded, advanced) Green function for the QD electron and is governed by the transformed Hamiltonian $\widetilde{H}$. Employing the equation-of-motion technique, we obtain

$$\begin{aligned} \widetilde{G}^{\rm r}_{\sigma\sigma'}(\omega) =& \delta_{\sigma\sigma'}\Big\{\omega - \tilde{\epsilon}_{\rm d} + i\widetilde{\varGamma}_{\rm L}/2 + i\gamma(\omega)\widetilde{\varGamma}_{\rm R}/2 \\ & + \frac{(\Delta\gamma(\omega)\widetilde{\varGamma}_{\rm R}/2\omega)^2}{\omega + \tilde{\epsilon}_{\rm d} + i\widetilde{\varGamma}_{\rm L}/2 + i\gamma(\omega)\widetilde{\varGamma}_{\rm R}/2}\Big\}^{-1}, \end{aligned} \tag{3}$$

where $\tilde{\epsilon}_{\rm d} = \epsilon_{\rm d} - \lambda^2/\omega_0$ is the renormalized dot level, $\widetilde{\varGamma}_\alpha = \langle X\rangle^2\varGamma_\alpha = 2\pi\langle X\rangle^2\varSigma_k|t_\alpha|^2\delta(\omega - \epsilon_{\alpha,k})$ is the effective linewidth and is assumed to be independent of energy, $\gamma(\omega) = |\omega|/\sqrt{\omega^2 - \varDelta^2}$ for $|\omega| > \varDelta$, and $\gamma(\omega) = -i\omega/\sqrt{\varDelta^2 - \omega^2}$ for $|\omega| < \varDelta$. The advanced Green function is $\widetilde{G}^{\rm a}_{\sigma\sigma'}(\omega) = \widetilde{G}^{r*}_{\sigma\sigma'}(\omega)$. Since the lesser and greater components $\widetilde{G}^{</>}_{\sigma\sigma'}(\omega)$ are proportional to the term $\widetilde{G}^{\rm r}_{\sigma\sigma'}(\omega) - \widetilde{G}^{\rm a}_{\sigma\sigma'}(\omega)$ and $\widetilde{G}^{\rm r}_{\sigma\sigma'} - \widetilde{G}^{\rm a}_{\sigma\sigma'} = \widetilde{G}^>_{\sigma\sigma'} - \widetilde{G}^<_{\sigma\sigma'}$,[24] Eq. (2) reduces to

$$\begin{aligned} Q =& 2\omega_0\lambda^2\int\frac{d\omega}{2\pi}\{(N_{\rm ph}+1)\widetilde{G}^<_{\uparrow\uparrow}(\omega)\widetilde{G}^>_{\uparrow\uparrow}(\omega-\omega_0) \\ & - N_{\rm ph}\widetilde{G}^>_{\uparrow\uparrow}(\omega)\widetilde{G}^<_{\uparrow\uparrow}(\omega-\omega_0)\}. \end{aligned} \tag{4}$$

In an N–QD–S system, it is convenient to carry out calculation in the Nambu space. The 2×2 matrix Green function of the QD electron in the Nambu space is

$$\widetilde{\boldsymbol{G}}(t,t') = \begin{pmatrix} \langle\langle d_\uparrow(t); d^\dagger_\uparrow(t')\rangle\rangle & \langle\langle d_\uparrow(t); d_\downarrow(t')\rangle\rangle \\ \langle\langle d^\dagger_\downarrow(t); d^\dagger_\uparrow(t')\rangle\rangle & \langle\langle d^\dagger_\downarrow(t); d_\downarrow(t')\rangle\rangle \end{pmatrix}. \tag{5}$$

The lesser matrix Green function is

$$\widetilde{\boldsymbol{G}}^<(t,t') = i\begin{pmatrix} \langle d^\dagger_\uparrow(t')d_\uparrow(t)\rangle & \langle d_\downarrow(t')d_\uparrow(t)\rangle \\ \langle d^\dagger_\uparrow(t')d^\dagger_\downarrow(t)\rangle & \langle d_\downarrow(t')d^\dagger_\downarrow(t)\rangle \end{pmatrix}. \tag{6}$$

As the voltage dropped on the superconducting lead is zero, we can obtain the lesser self-energy $\widetilde{\boldsymbol{\varSigma}}^<$,

$$\begin{aligned} \widetilde{\boldsymbol{\varSigma}}^<(\omega) =& i\widetilde{\varGamma}_{\rm R}f_{\rm R}(\omega)\frac{|\omega|\theta(|\omega|-\varDelta)}{\sqrt{\omega^2-\varDelta^2}}\begin{pmatrix} 1 & -\varDelta/\omega \\ -\varDelta/\omega & 1 \end{pmatrix} \\ & + i\widetilde{\varGamma}_{\rm L}\begin{pmatrix} f_{\rm L}(\omega+eV_{\rm L}) & 0 \\ 0 & f_{\rm L}(\omega-eV_{\rm L}) \end{pmatrix}, \end{aligned} \tag{7}$$

where $f_\alpha(x) = [\exp(x/\kappa_{\rm B}T_\alpha)+1]^{-1}$ is the Fermi distribution function, and $V_{\rm L}$ is the voltage dropped on the left normal lead. Then, using the Keldysh equation $\widetilde{\boldsymbol{G}}^< = \widetilde{\boldsymbol{G}}^{\rm r}\widetilde{\boldsymbol{\varSigma}}^<\widetilde{\boldsymbol{G}}^{\rm a}$,[24] we can obtain the matrix element $\widetilde{\boldsymbol{G}}^<_{11}(\omega)$, i.e., $\widetilde{G}^<_{\uparrow\uparrow}(\omega)$,

$$\begin{aligned} \widetilde{\boldsymbol{G}}^<_{11}(\omega) =& i\widetilde{\varGamma}_{\rm L}[|\widetilde{\boldsymbol{G}}^{\rm r}_{11}(\omega)|^2 f_{\rm L}(\omega+eV_{\rm L}) + |\widetilde{\boldsymbol{G}}^{\rm r}_{12}(\omega)|^2 \\ & \times f_{\rm L}(\omega-eV_{\rm L})] + i\widetilde{\varGamma}_{\rm R}f_{\rm R}(\omega)\frac{|\omega|\theta(|\omega|-\varDelta)}{\sqrt{\omega^2-\varDelta^2}} \\ & \times\Big\{|\widetilde{\boldsymbol{G}}^{\rm r}_{11}(\omega)|^2 + |\widetilde{\boldsymbol{G}}^{\rm r}_{12}(\omega)|^2 \\ & - \frac{2\varDelta}{\omega}{\rm Re}[\widetilde{\boldsymbol{G}}^{\rm r}_{11}(\omega)\widetilde{\boldsymbol{G}}^{r*}_{12}(\omega)]\Big\}, \end{aligned} \tag{8}$$

where $\widetilde{\boldsymbol{G}}^{\rm r}_{11}(\omega)$ is given by Eq. (3) and $\widetilde{\boldsymbol{G}}^{\rm r}_{12}(\omega) = \widetilde{\boldsymbol{G}}^{\rm r}_{11}(\omega)\gamma(\omega)\widetilde{\varGamma}_{\rm R}\varDelta i/(2\omega)\times[\omega + \tilde{\epsilon}_{\rm d} + \frac{i}{2}\widetilde{\varGamma}_{\rm L} + \frac{i}{2}\gamma(\omega)\widetilde{\varGamma}_{\rm R}]^{-1}$. Replacing the fermi distribution function $f_\alpha(x)$ in Eq. (8) with $f_\alpha(x) - 1$, we obtain straightforwardly the greater Green function $\widetilde{\boldsymbol{G}}^>_{11}(\omega)$.[24] Putting the $\widetilde{\boldsymbol{G}}^{</>}_{11}(\omega)$, i.e., $\widetilde{G}^{</>}_{\uparrow\uparrow}(\omega)$ into Eq. (4) we obtain the heat generation Q in the QD.

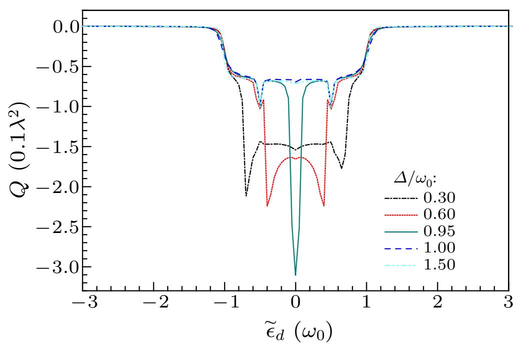

Fig. 1. (Color online) Heat generation Q in the QD as a function of the renormalized QD level $\tilde{\epsilon}_{\rm d}$ with superconducting gap $\varDelta$ varying from $0.3\omega_0$ to $1.5\omega_0$. The other parameters for calculation are, in units of ω_0, $T_{\rm L,R} = 0.001$, $T_{\rm ph} = 1.5$, $\widetilde{\varGamma} = 0.1$ and $\lambda = 0.6$.

We now numerically investigate the heat generation Q for the case of zero electric bias voltage and symmetric barriers, $\Gamma_{\rm L} = \Gamma_{\rm R} = \Gamma$. Set $\omega_0 = 1$ as the energy unit and the chemical potential $\mu_{\rm L} = \mu_{\rm R} = \mu = 0$ as the reference energy. Here $T_{\rm ph}$, $T_{\rm L}$ and $T_{\rm R}$ are the temperatures of the phonons, the left normal metal and the right superconductor, respectively. Due to the EPI, the energy exchange can take place between the phonon and electron subsystems as an electron tunnels through the QD.

Because of small heat capacity and inefficient heat conduction, the Joule heat in a nanojunction can cause a sharp rise in temperature of the junction. We first discuss the case of $T_{\rm L,R} \ll \omega_0$. In Fig. 1, we plot the heat generation as a function of the QD level $\tilde{\epsilon}_{\rm d}$ with superconducting gap Δ varying from below ω_0 to above ω_0. The temperatures are chosen as $T_{\rm ph} = 1.5$ and $T_{\rm L,R} = 0.001$. Because of $\mu_{\rm L} = \mu_{\rm R}$ and $T_{\rm L} = T_{\rm R}$, the electric current through the QD vanishes. However, the heat generation can be nonzero. In Fig. 1, Q exhibits negative values as the dot level locates between $-\omega_0$ and ω_0, i.e., $-\omega_0 < \tilde{\epsilon}_{\rm d} < \omega_0$. As $\tilde{\epsilon}_{\rm d}$ lies far above ω_0, heat in QD can hardly be absorbed or emitted, $Q = 0$. When the level moves down to around $\tilde{\epsilon}_{\rm d} = \omega_0$, Q decreases sharply. That is, heat is absorbed. This is because the opening of a tunnel channel, in which an electron of energy $\tilde{\epsilon}_{\rm d} - \omega_0$ from the normal metal, slightly below μ, tunnels into the QD and jumps to $\tilde{\epsilon}_{\rm d}$ by absorbing one phonon, and then tunnels out with gain energy. In the case of $\Delta \geq \omega_0$, this phonon-assisted tunneling process is contributed by the normal metal alone. When Δ is smaller than ω_0, Q displays further sharp decrease as $\tilde{\epsilon}_{\rm d}$ goes down to around $\tilde{\epsilon}_{\rm d} = -\Delta+\omega_0$, e.g., at $\tilde{\epsilon}_{\rm d} = 0.4$ for $\Delta = 0.6$. We attribute this to opening up of a new channel, in which an electron slightly below $-\Delta$ from the superconductor enters the QD and leaves via the level $\tilde{\epsilon}_{\rm d}$ by absorbing one phonon. The density of states of the superconducting lead is $N_{\rm S}(\omega) = N_{\rm N}|\omega|/\sqrt{\omega^2-\Delta^2}$ for $|\omega| > \Delta$ and $N_{\rm S}(\omega) = 0$ for $|\omega| \leq \Delta$,[23,25] with $N_{\rm N}$ being the density of states in the bulk normal metal. As the level moves from $\tilde{\epsilon}_{\rm d} = -\Delta+\omega_0$ down to $\tilde{\epsilon}_{\rm d} = 0$, the magnitude of Q decreases after it reaches its peak value, see the curves of $\Delta = 0.30$ and of $\Delta = 0.60$, because of the decrease of $N_{\rm S}(\tilde{\epsilon}_{\rm d} - \omega_0)$, where $\omega = \tilde{\epsilon}_{\rm d} - \omega_0$ is the energy of electrons incident from the superconductor in the new channel. For the same reason as explained above, Q shows another negative peak at around $\tilde{\epsilon}_{\rm d} = \Delta - \omega_0$. With increasing Δ, the two peaks in a curve approach each other and merge into a single one at Δ slightly smaller than ω_0. At the same time, the magnitude of the peak rises with widening Δ. This is mainly because the weight of resonant peak and phonon side peaks of the spectral function of QD electron changes with moving the QD level.[26] One can see another two small negative peaks arising at $\tilde{\epsilon}_{\rm d} = \pm 0.5$. This results from the process of phonon-assisted Andreev reflections, in which an electron (for $\tilde{\epsilon}_{\rm d} = -0.5$) or a hole (for $\tilde{\epsilon}_{\rm d} = 0.5$) from the normal metal tunnels to the QD level and is Andreev reflected, at the interface with the superconductor, as a hole or an electron back into QD, and then by absorbing one phonon the reflected hole or electron can jump to the level $\tilde{\epsilon}_{\rm d}$. Now, we can see that at $\Delta < \omega_0$ the superconducting lead makes a large contribution to the cooling effect of QD. At some Δ slightly smaller than ω_0, the maximum magnitude of negative Q appears, which is several times larger than the saturated value of Q contributed by the normal metal alone.

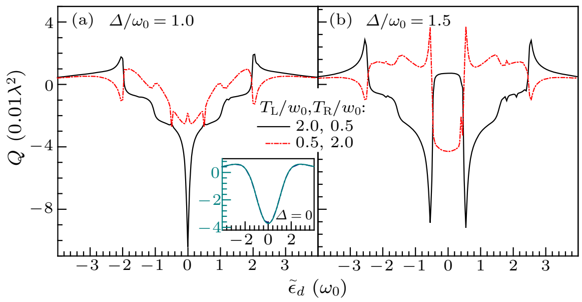

Fig. 2. (Color online) Heat generation Q versus the QD level $\tilde{\epsilon}_{\rm d}$ with (a) $\Delta = 1.0$ and (b) $\Delta = 1.5$. The other parameters are, in units of ω_0, $\widetilde{\Gamma} = 0.1$, $\lambda = 0.6$, $T_{\rm ph} = 1.5$, $T_{\rm L,R} = 2.0, 0.5$ (black solid) and $T_{\rm L,R} = 0.5, 2.0$ (red dash-dotted). When Δ reduced from 1.0 to 0, curves in (a) become totally overlapped as shown in the inset.

Now, we turn to the case that the α lead is at elevated temperature T_α. Because of T_α, electrons in leads are excited. Heat generation in the QD is dominated by tunneling processes of electrons in the N-QD-S system. Here $T_{\rm L}$ and $T_{\rm R}$ influence electron distribution in leads and then, undoubtedly, affect Q. Figure 2 shows the $Q \sim \tilde{\epsilon}_{\rm d}$ curves at $T_{\rm L,R} = 2.0, 0.5$ and $T_{\rm L,R} = 0.5, 2.0$. At elevated lead temperature, Q can be positive, as shown in Fig. 2(a), which is given at $\Delta = \omega_0$. When the lead temperatures are $T_{\rm L} = 2.0$ and $T_{\rm R} = 0.5$, small positive peaks arise at $\tilde{\epsilon}_{\rm d}$, slightly larger than $\Delta + \omega_0$, i.e., $2\omega_0$, or smaller than $-\Delta-\omega_0$. They result from conventional phonon-assisted tunneling processes, in which an electron or a hole from the normal metal enters QD via the level $\tilde{\epsilon}_{\rm d}$, and tunnels out into the superconductor after emitting one phonon in QD. As $\tilde{\epsilon}_{\rm d}$ lies between $-\Delta - \omega_0$ and $\Delta + \omega_0$, the phonon-absorbing process dominates in the QD, $Q < 0$. An electron of energy $\tilde{\epsilon}_{\rm d} - \omega_0$ (for $\Delta < \tilde{\epsilon}_{\rm d} < \Delta+\omega_0$) or $\tilde{\epsilon}_{\rm d}$ (for $0 < \tilde{\epsilon}_{\rm d} < \Delta$) enters the QD and tunnels out into the superconductor by absorbing one phonon. The density of empty states in the superconductor, $[1 - f_{\rm R}(\omega)]N_{\rm S}(\omega)$, increases sharply when ω goes down close to Δ. At the same time, the density of electrons in the normal metal, $N_{\rm N} f_{\rm L}(\omega)$, rises with reducing ω. Therefore, with $\tilde{\epsilon}_{\rm d}$ moving from $\Delta + \omega_0$ down to 0, the magnitude of negative Q increases and exhibits two steep steps at around Δ and $\Delta - \omega_0$, and finally reaches its peak value at $\tilde{\epsilon}_{\rm d} = 0$. Interchanging $T_{\rm L}$ and $T_{\rm R}$, we can see in Fig. 2(a) that Q behaves quite differently, even though it shows the same property at $\Delta = 0$, see the inset. In the curve of $T_{\rm L,R} = 0.5, 2.0$ in Fig. 2(a), at around $\tilde{\epsilon}_{\rm d} = 2\omega_0$, Q exhibits a small negative peak, which is caused by such a process that an electron of energy $\tilde{\epsilon}_{\rm d} - \omega_0$ from the superconductor tunnels into the QD and by absorbing one phonon tunnels out into the normal metal via the level $\tilde{\epsilon}_{\rm d}$. Increased temperature at the superconductor leads to more electronic states slightly below $-\varDelta$ unoccupied and above $\varDelta$ occupied. At the same time, decreased temperature causes less electrons excited to above $\tilde{\epsilon}_{\rm d} = 0$ in the normal metal. As a result of these, at $-\varDelta - \omega_0 < \tilde{\epsilon}_{\rm d} < \varDelta + \omega_0$ the chance for phonon-absorbing tunneling process through the QD reduces and that for phonon-emitting process increases. At $\varDelta < \tilde{\epsilon}_{\rm d} < \varDelta + \omega_0$ the phonon-emitting process dominates. Although it is the phonon-absorbing process that prevails at $0 < \tilde{\epsilon}_{\rm d} < \varDelta$, the magnitude of negative Q is quite small, see the curve of $T_{\rm L,R} = 0.5, 2.0$ in Fig. 2(a). Apparently, with this temperature arrangement the QD cannot be efficiently cooled. Figure 2(b) gives Q at $\varDelta = 1.5$. Similar to what appears in Fig. 2(a), the behavior of Q changes rapidly as the lead temperatures interchange and the largest negative Q appears in the curve of $T_{\rm L,R} = 2.0, 0.5$. We obtain that at $-0.5\omega_0 < \tilde{\epsilon}_{\rm d} < 0.5\omega_0$, Q shows a wide negative (positive) peak in the curve of $T_{\rm L,R} = 0.5, 2.0$ (of $T_{\rm L,R} = 2.0, 0.5$). This is contributed mainly by the normal metal.

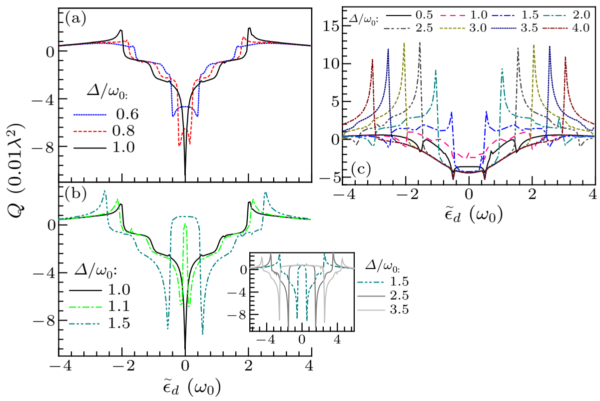

Fig. 3. (Color online) Heat generation Q versus the QD level $\tilde{\epsilon}_{\rm d}$ for varying superconducting gap, with $T_{\rm L,R} = 2.0, 0.5$ in (a), (b) and $T_{\rm L,R} = 0.5, 2.0$ in (c). The other parameters are the same as in Fig. 2.

In Fig. 3, we present Q properties for varying superconductor gap magnitude at elevated lead temperatures. In Figs. 3(a) and 3(b), lead temperatures are taken to be $T_{\rm L} = 2.0$ and $T_{\rm R} = 0.5$. One can see in Fig. 3(a) that Q displays similar behaviors as in Fig. 1, two negative peaks arise at around $\tilde{\epsilon}_{\rm d} = \mp\varDelta \pm \omega_0$. They rise and approach each other with enlarging $\varDelta$ until merging into a single one. However, due to the distribution function $f_\alpha(\omega) = [\exp(\omega/\kappa_B T_\alpha) + 1]^{-1}$, here the maximum magnitude of Q appears at $\varDelta = \omega_0$, not some $\varDelta$ slightly smaller than ω_0, and is several times smaller than the one shown in Fig. 1. Increasing $\varDelta$ to above ω_0, we can see in Fig. 3(b) that this peak splits into two again and the peak magnitude at around $\tilde{\epsilon}_{\rm d} = \mp\varDelta \pm \omega_0$ reduces suddenly. The large negative peak at $\tilde{\epsilon}_{\rm d} = 0$ shifts into a positive peak, since contribution from the superconductor to phonon-absorbing process vanishes, and Q is caused completely by coupling between the normal metal and QD. With further enlarging of $\varDelta$, the magnitude of peak at $\tilde{\epsilon}_{\rm d} = \mp\varDelta \pm \omega_0$ increases and then goes down again after reaching a maximum value at a moderate $\varDelta$, see the inset in Fig. 3(b). However, the small positive Q peak at $\tilde{\epsilon}_{\rm d} = \pm(\varDelta + \omega_0)$, which is several times smaller than the negative one, changes slightly with varying $\varDelta$. As the lead temperatures interchange, see Fig. 3(c), the magnitude of positive Q peak can be several times larger than the negative one, which locates at $\tilde{\epsilon}_{\rm d} = \pm 0.5\omega_0$ and is caused by the phonon-assisted Andreev reflections as explained above for Fig. 1.

Now, we can see that the heat generation is sensitive to the lead temperature and the superconductor gap magnitude. The QD can be cooled much more efficiently when the superconductor is colder.

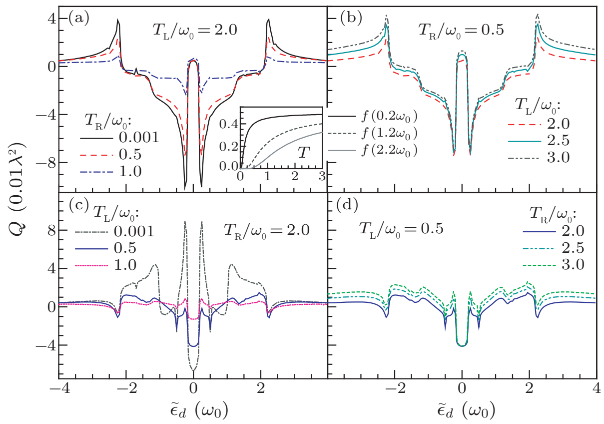

Fig. 4. (Color online) The heat generation Q in the QD versus the QD level $\tilde{\epsilon}_{\rm d}$ for varying lead temperatures. The parameters are, in units of ω_0, $\widetilde{\varGamma} = 0.1$, $\lambda = 0.6$, $T_{\rm ph} = 1.5$, and $\varDelta = 1.2$. The inset in (a) is the fermi function $f(\omega) = 1/[\exp(\omega/T) + 1]$ versus the lead temperature T for three specific energy values $\omega = 0.2$, 1.2 and 2.2.

Figure 4 shows the heat generation Q versus the QD level $\tilde{\epsilon}_{\rm d}$ for varying lead temperatures and with $\varDelta = 1.2\omega_0$, $T_{\rm ph} = 1.5\omega_0$. In Figs. 4(a) and 4(b), the lead temperatures are taken to be $T_{\rm R} < T_{\rm ph} < T_{\rm L}$. For fixed $T_{\rm L} = 2.0$, both the negative peaks at around $\tilde{\epsilon}_{\rm d} = \mp\varDelta \pm \omega_0$, i.e., $\tilde{\epsilon}_{\rm d} = \pm 0.2\omega_0$ and positive peaks at $\tilde{\epsilon}_{\rm d} = \pm(\varDelta + \omega_0)$, reduce rapidly as $T_{\rm R}$ increases from $0.001\omega_0$ to ω_0, see Fig. 4(a). However, the peak at $\tilde{\epsilon}_{\rm d} = 0$ keeps constant. These properties can be easily understood. The peak at around $\tilde{\epsilon}_{\rm d} = 0.2\omega_0$ (or $2.2\omega_0$) is induced by such a process in which an electron of energy $\tilde{\epsilon}_{\rm d}$ from the normal metal tunnels into QD and absorbs (or emits) one phonon before it tunnels out and enters an empty state slightly above $\varDelta$ in the superconductor. Increasing $T_{\rm R}$ from $0.001\omega_0$ to ω_0 causes a significant rise of probability of an electronic state slightly above $\varDelta$ in the superconductor to be occupied, see the curve of $f(1.2)$ in the inset. Therefore, the chance for phonon-assisted tunneling process depicted above lessens, and as a result the peak at

$\tilde{\epsilon}_{\rm d} = 0.2\omega_0$ (or $2.2\omega_0$) lowers. For the same reason, peaks at $\tilde{\epsilon}_{\rm d} = -0.2\omega_0$ and $-2.2\omega_0$ decrease with rising $T_{\rm R}$. Because of the contribution from the normal metal alone, the peak value at $\tilde{\epsilon}_{\rm d} = 0$ is constant. Increasing $T_{\rm L}$ from $2\omega_0$ to $3\omega_0$ for fixed $T_{\rm R} = 0.5\omega_0$, we see, in Fig. 4(b), the negative peaks at $\tilde{\epsilon}_{\rm d} = \pm 0.2\omega_0$ keep unchanged, despite the peaks at $\tilde{\epsilon}_{\rm d} = \pm(\varDelta + \omega_0)$ exhibiting a visible rise. This is because with increasing this temperature, the fermi function $f(0.2)$ changes slightly but $f(2.2)$ goes up obviously, see the inset. In Figs. 4(c) and 4(d), we set $T_{\rm R} > T_{\rm ph} > T_{\rm L}$. For the same reason behind Figs. 4(a) and 4(b), Q peaks change heavily as $T_{\rm L}$ rises from $0.001\omega_0$ to ω_0 for fixed $T_{\rm R} = 2.0\omega_0$ as shown in Fig. 4(c), but display a slight change as $T_{\rm R}$ climbs from $2.0\omega_0$ to $3.0\omega_0$ for fixed $T_{\rm L} = 0.5\omega_0$, see Fig. 4(d). The peak at $\tilde{\epsilon}_{\rm d} = 0$ in Fig. 4(d) keeps constant because of the same reason behind the peak at $\tilde{\epsilon}_{\rm d} = 0$ in Fig. 4(a), being contributed by the normal metal alone.

Now, we can see that the temperature of the cold lead affects Q heavily, while the hotter one influences it only slightly.

In summary, we have investigated the heat generation Q in a QD, coupled to a normal metal and a superconductor, without electric bias voltage. The QD has a single electronic energy level that is coupled to a phonon bath of frequency ω_0. We restrict our attention to finding how Q is affected by the lead temperatures $T_{\rm L,R}$ and the superconductor gap magnitude $\varDelta$. It is found that Q is largely sensitive to $T_{\rm L,R}$ and $\varDelta$. At $T_{\rm L,R} \ll \omega_0$, the superconductor affects Q only at $\varDelta < \omega_0$. Because of the superconductor, Q versus the QD level position exhibits two negative peaks. They rise and approach each other with increasing $\varDelta$ and finally merge into a single one. At some $\varDelta$ slightly smaller than ω_0 the maximum magnitude of negative Q appears, which is several times larger than the saturate value of negative Q contributed by the normal metal alone. However, at elevated lead temperature, the contribution to Q from the superconductor appears at $\varDelta$, ranging from less than to much larger than ω_0. However, the peak value of negative Q is several times smaller than in the case of $T_{\rm L,R} \ll \omega_0$. Interchanging lead temperatures $T_{\rm L}$ and $T_{\rm R}$ leads to quite different Q behaviors, even though this makes no difference for the system of a QD coupled to two normal metals. The QD can be cooled much more efficiently when the superconductor is colder. The temperature of the colder lead shows great effect on Q, while that of the hotter one influences it only slightly.

References

[1] Brüggemann J, Weiss S, Nalbach P and Thorwart M 2014 *Phys. Rev. Lett.* **113** 076602
[2] Wang Q, Xie H Q, Jiao H J and Nie Y H 2013 *Europhys. Lett.* **101** 47008
[3] Chen Q, Tang L M, Chen K Q and Zhao H K 2013 *J. Appl. Phys.* **114** 084301
[4] Chen Q and Deng Y H 2011 *Commun. Theor. Phys.* **56** 517
[5] Chi F, Zheng J, Liu Y S and Guo Y 2012 *Appl. Phys. Lett.* **100** 233106
[6] Zhou L L, Li S S, Wei J N and Wang S Q 2011 *Phys. Rev.* B **83** 195303
[7] Jonas F and Michael G 2010 *Phys. Rev.* B **81** 075311
[8] Huang Z et al 2006 *Nano Lett.* **6** 1240
Huang Z et al 2007 *Nat. Nanotechnol.* **2** 698
[9] Chen Y C et al 2003 *Nano Lett.* **3** 1691
Chen Y C et al 2005 *Nano Lett.* **5** 621
[10] Sun Q F and Xie X C 2007 *Phys. Rev.* B **75** 155306
Liu J et al 2009 *Phys. Rev.* B **79** 161309
[11] LüJ T and Wang J S 2007 *Phys. Rev.* B **76** 165418
[12] Segal D and Nitzan A 2002 *J. Chem. Phys.* **117** 3915
[13] Montgomery M J, Todorov T N and Sutton A P 2002 *J. Phys.: Condens. Matter* **14** 5377
[14] Pecchia A, Romano G and Carlo A D 2007 *Phys. Rev.* B **75** 035401
[15] Galperin M, Zitzan A and Ratner M 2007 *Phys. Rev.* B **75** 155312
[16] Horsfield A P, Bowler D R, Bowler A J, Fiser A J, Todorov T N and Montgomery M J 2004 *J. Phys.: Condens. Matter* **16** 3609
[17] Horsfield A P, Bowler D R, Ness H, Sánchez C G, Todorov T N and Fiser A J 2006 *Rep. Prog. Phys.* **69** 1195
[18] Wang J S, Wang J and Lü J T 2008 *Eur. Phys. J.* B **62** 381
[19] Muller C J, van Ruitenbeek J M and de Jongh J L 1992 *Phys. Rev. Lett.* **69** 140
[20] van den Brom H E, Yanson A I and van Ruitenbeek J M 1998 *Physica* B **252** 69
[21] Agraït N, Untiedt C, Rubio-Bollinger G and Vieira S 2002 *Phys. Rev. Lett.* **88** 216803
[22] Smit R H M, Untiedt C and van Ruitenbeek J M 2004 *Nanotechnology* **15** S472
[23] Mahan G D 2000 *Many-Particle Physics* 3rd edn (New York: Plenum Press)
[24] Haug H and Jauho A P 1998 *Quantum Kinetics in Transport and Optics of Semiconductor* (Berlin: Springer-verlag)
[25] Whan C B and Orlando T P 1996 *Phys. Rev.* B **54** R5255
[26] Chen Z Z, Lü R and Zhu B F 2005 *Phys. Rev.* B **71** 165324