Chin. Phys. B Vol. 26, No. 1 (2017) 017201

Direct spin–phonon coupling of spin-flip relaxation in quantum dots*

Ji-Wen Yin(尹辑文)[1,†], Wei-Ping Li(李伟萍)[2], Hong-Juan Li(李红娟)[1], and Yi-Fu Yu(于毅夫)[1]

[1]Department of Physics and Electronic Informational Engineering, Chifeng University, Chifeng 024000, China
[2]Department of Basic Courses, Tianjin Sino-German University of Applied Sciences, Tianjin 300350, China

(Received 18 September 2016; revised manuscript received 12 October 2016; published online 30 November 2016)

Within the frame of the Pavlov–Firsov spin–phonon coupling model, we study the spin-flip assisted by the acoustical phonon scattering between the first-excited state and the ground state in quantum dots. We analyze the behaviors of the spin relaxation rates as a function of an external magnetic field and lateral radius of quantum dot. The different trends of the relaxation rates depending on the magnetic field and lateral radius are obtained, which may serve as a channel to distinguish the relaxation processes and thus control the spin state effectively.

Keywords: spin relaxation, spin–phonon coupling, quantum dot

PACS: 73.21.La, 72.10.Di, 72.25.Rb **DOI:** 10.1088/1674-1056/26/1/017201

1. Introduction

The electron spin relaxation in quantum dots (QDs) is of central importance for the applications on the spin polarization and quantum computation devices.[1,2] In the semiconductor QD, extensive theoretical works have researched the main phonon mediated spin-flip relaxation mechanism, such as the admixture processes due to spin–orbit coupling,[2–5] phonon coupling due to interface motion (ripple mechanism),[6] the direct spin–phonon coupling mechanism,[10] the fluctuating magnetic field due to the fluctuating electron density or the modulation of the hyperfine coupling with nuclei by lattice vibration,[2] and so on. Among them, what remains in discussion is which mechanism is the dominant one and needs more endeavor in detail. The direct spin–phonon coupling mechanism is an intrinsic one that cannot be eliminated in principle, so it can provide the most fundamental upper bound on the lifetime of the electron spin state in QDs when the spin–orbit coupling does not produce a significant admixture between spin states. This situation can be reached in symmetrical samples in the absence of an electric field and in materials with negligible Dresselhaus contribution. In particular, recent experiments[7–9,11,12] suggest that the spin–phonon coupling dominates the spin relaxation in GaAs QDs in the strong magnetic field and some nonpolar or monolayer materials, such as silicene and graphene.[13,14]

In the present paper, we study the spin relaxation between the first-excited and ground states based on the spin–phonon interaction model proposed by Pavlov and Firsov,[15,16] which describes the transitions with spin reversal of the conduction band electrons due to the scattering by the longitudinal lattice vibrations, and has the advantage of being adaptable to the study of the other scattering mechanism by various phonon modes. Romano *et al.*[17] have used this Hamiltonian to investigate the spin-flip process of a single electron between the Zeeman sublevels in the same orbital state by considering the deformation potential mechanism as the dominant electron–phonon coupling. They give the regions in which the spin relaxation rates can be practically suppressed by choosing the suitable magnetic field and lateral QD size. We investigate the spin relaxation rates for the $\psi_{0,1,\uparrow} \rightarrow \psi_{0,0,\downarrow}$ and $\psi_{0,1,\downarrow} \rightarrow \psi_{0,0,\uparrow}$ processes as a function of the magnetic field and lateral radius. We find that the magnetic field dependence of the relaxation rates for the two cases are very different. This may serve as a channel to distinguish the relaxation processes in experiments, and thus control the spin state effectively. We hope that our results can stimulate and highlight related experimental work to identify this conclusion.

2. Theoretical model

We consider a single electron quantum dot with different parabolic confinement potentials in lateral and vertical directions $V(\rho,z) = (1/2)m^*\omega_0^2\rho^2 + (1/2)m^*\omega_1^2 z^2$, $\rho^2 = x^2 + y^2$ denotes the polar coordinate. The uniform magnetic field $\boldsymbol{B}$ is applied in the vertical direction. The Hamiltonian is given by

$$H_0 = \frac{P^2}{2m^*} + V(\rho,z) + H_{\mathrm{B}}, \tag{1}$$

in which $P = -\mathrm{i}\hbar\boldsymbol{\nabla} + (e/c)\boldsymbol{A}$ with $\boldsymbol{A} = (B/2)(-y,x,0)$ stands for the electron momentum operator, m^* is the electron effective mass, and $H_{\mathrm{B}} = (g\mu_{\mathrm{B}}B\sigma_z)/2$ is the Zeeman energy with σ representing the Pauli matrices.

By solving

$$H_0\psi_{n,l,\sigma} = E_{n,l,\sigma}\psi_{n,l,\sigma}, \tag{2}$$

*Project supported by the National Natural Science Foundation of China (Grant No. 11264001) and the Natural Science Foundation of Inner Mongolia, China (Grant No. 2012MS0116).

†Corresponding author. E-mail: ji-wenyin@163.com

one can determine the eigenenergy and the eigenfunction of the Hamiltonian H_0 as follows:

$$\psi_{n,l,\sigma} = \sqrt{n!/\pi l_\rho^2 (n+|l|)!}\zeta^{|l|}\mathrm{e}^{-\zeta^2/2}\mathrm{e}^{\mathrm{i}l\theta}\mathrm{L}_n^{|l|}(\zeta^2)\chi_\sigma, \quad (3)$$

$$E_{n,l,\sigma} = \hbar\Omega(2n+|l|+1) - \hbar\omega_{\mathrm{B}} l + \sigma E_{\mathrm{B}}, \quad (4)$$

where $n = 0,1,2,\dots$ and $l = 0,\pm1,\pm2,\dots$ are quantum numbers, $\Omega = \sqrt{\omega_0^2+\omega_{\mathrm{B}}^2}$ and $\omega_{\mathrm{B}} = eB/(2m^*)$, $\zeta = \rho/l_\rho$ is scaled radius with $l_\rho = \sqrt{\hbar/m^*\Omega}$, $E_{\mathrm{B}} = g\mu_{\mathrm{B}}B\sigma_z/2$ is the Zeeman splitting energy, $\sigma =\uparrow,\downarrow$ refers to the spin polarization along the vertical direction, and $\mathrm{L}_n^{|l|}$ is the generalized Laguerre polynomial.

The spin–orbit interaction mediates a coupling between the spin and the electron orbital bath. Based on the effective mass theory, the spin–phonon interaction Hamitonian in the presence of an external magnetic field, which describes the transitions with spin reversal of the conduction band electrons due to scattering with lattice vibrations, can be expressed as[17,18]

$$\begin{aligned}\Re(q) = & \sum_q \frac{D(\boldsymbol{q})F(q_z)}{\sqrt{\rho V\omega_q/\hbar}} \\ & \times\left[\mathrm{e}^{\mathrm{i}q_\parallel\cdot r_k} b_q \begin{Bmatrix} 0 & \hat{\boldsymbol{n}}^-\times\hat{\boldsymbol{e}}_q \\ \hat{\boldsymbol{n}}^+\times\hat{\boldsymbol{e}}_q & 0 \end{Bmatrix}\right] \\ & \times\left[\left(\frac{\boldsymbol{p}}{\hbar}+\frac{e\boldsymbol{A}}{\hbar}+\boldsymbol{q}\right)+\mathrm{H.c.}\right], \end{aligned} \quad (5)$$

where $b_q(b_q^\dagger)$ is the annihilation (creation) operator of the acoustic phonon with wave vector $q = (q_\parallel, q_z)$; $\hat{\boldsymbol{n}}^\pm = \hat{\boldsymbol{x}}\pm\mathrm{i}\hat{\boldsymbol{y}}$, where $\hat{\boldsymbol{x}}$ and $\hat{\boldsymbol{y}}$ are unitary vectors along the x and y axes; $\hat{\boldsymbol{e}}_q$ is the unit vector in the direction of the phonon polarization; ρ is the mass density; and V is the system volume. We assume the linear dispersion $\omega_q = vq$, v being the average sound velocity. The form factor[19,20] $F(q_z) = \int \mathrm{d}z \exp(\mathrm{i}q_z z) \mid \phi(z) \mid^2$ equals unity for $\mid q_z \mid \ll 1/R_\perp$. The electron phonon scattering due to the deformation potential coupling described by the parameter $D(q)$ is defined as

$$D(q) = \frac{\xi_{\mathrm{d}}}{E_{\mathrm{g}}}\frac{\hbar^2 q^2}{2m^*}\left(1-\frac{m^*}{m_0}\right) f\left(\frac{E_{\mathrm{g}}}{\varDelta}\right), \quad (6)$$

where ξ_{d} denotes the deformation potential constant, E_{g} is the semiconductor band gap, $\varDelta$ is the spin–orbit splitting, and $f(\varLambda) = (1+2\varLambda)/[(1+\frac{3}{2}\varLambda)(1+\varLambda)]$.[17,18]

Based on the Fermi golden rule, the spin-flip relaxation rate assisted by one phonon scattering between an initial electron state ψ_i and a final state ψ_f is determined by

$$W = \frac{2\pi}{\hbar}\sum_q \mid \langle\psi_i|\Re(q)|\psi_i\rangle \mid^2 \delta(E_f - E_i - \hbar\omega_q). \quad (7)$$

We mainly consider the spin-flip processes between the first-excited and ground states. After complexly calculations, the expression of the spin relaxation rate can be written as

$$\begin{aligned} W = & \frac{D^2(q)X^2q^3}{2\pi\hbar\rho c^2}\int_0^\pi \sin^6\theta \bigg\{(6-q^2\sin^2\theta R_0^2) \\ & \times \mathrm{I}_0\left(\frac{q^2\sin^2\theta R_0^2}{8}\right) + (q^2\sin^2\theta R_0^2 - 2) \\ & \times \mathrm{I}_1\left(\frac{q^2\sin^2\theta R_0^2}{8}\right)\bigg\}^2 \\ & \times \exp\left(-\frac{q^2\sin^2\theta R_0^2 + 2q^2\cos^2\theta R_\perp^2}{4}\right)\mathrm{d}\theta. \end{aligned} \quad (8)$$

The parameter $X = eBR_0^2/\hbar - 1$ and $eBR_0^2/\hbar + 1$ for the $\psi_{0,1,\uparrow} \to \psi_{0,0,\downarrow}$ and $\psi_{0,1,\downarrow} \to \psi_{0,0,\uparrow}$ processes, respectively. Here, I_0 (I_1) is the zero (first) order Bessel function, $R_0 = \sqrt{\hbar/(m^*\omega_0)}$ is the lateral radius, $R_\perp = \sqrt{\hbar/(m^*\omega_1)}$ denotes the height of QD in the vertical direction. We assume that the radius is much longer than the height, that is $\omega_0 \gg \omega_1$. The dynamics along the vertical direction is restricted to the lowest subband, corresponding to the ground-state wave function $\phi(z) = 1/(\sqrt{R_\perp}\pi^{1/4})\exp(-z^2/2R_\perp^2)$.[18–20] Throughout the paper, the ratio $R_\perp/R_0 = 0.1$ is fixed.

3. Numerical results

We only consider the contribution of the electron-longitudinal acoustic (LA) phonon coupling via the deformation potential mechanism to the relaxation rates, which plays the predominant role when the energy separation between the orbital states is in a several meV regime. The calculation of the relaxation rates is performed at temperature $T \sim 0$ K. The temperature dependence of the relaxation rate for one-phonon emission is determined from $W = W_0(n_q+1)$, where n_q is the Bose–Einstein distribution for the LA phonon and W_0 is the rate at $T = 0$ K. In the present paper, we consider the temperature regime $T \le 10$ K. Therefore, the approximation $n_1 + 1 \approx 1$ and $W \approx W_0$ is satisfied. In numerical calculation, we adopt the parameters in GaAs material:[21] $m^* = 0.067m_0$, $\rho = 5.32\times10^3$ kg/m^3, $v_{\mathrm{LA}} = 4.72\times10^3$ m/s, $g = 0.44$, $\xi_{\mathrm{d}} = 13.5$ eV, $E_{\mathrm{g}} = 1.5$ eV, and $\Delta = 0.346$ eV.

Figure 1 plots the magnetic field dependence of the spin relaxation rate between the ground state and first-excited state for different radii of QD. In the process of $\psi_{0,1,\uparrow} \to \psi_{0,0,\downarrow}$ (Fig. 1(a)), we can see that the rates decrease until a minimum and then increase. Hence, the spin relaxations show the cusplike structures, which means that the maximum of the relaxation time can be obtained by modulating the magnetic field. This fact is very beneficial to controlling the coherence of the spin qubit in quantum information. Moreover, the minimums of the relaxation rates are moved to the small value of the magnetic field and the second turn points appear with the increase of the QD radius, which are caused by the interplay effects between the variation of the QD radius and magnetic field confinement. We emphasize that the cusplike structure of the relaxation rates in the present model are contrary to that in Refs. [19], [20], and [22] based on the spin admixture mechanism due to the Dresselhaus and Rashba spin–orbital coupling. In fact, the same structures are obtained and show the oscillatory behavior with magnetic field for the process between the Zeeman sublevels of the ground orbital state in InSb QD in Ref. [12]. In the present processes, we considered that the oscillatory behavior does not appear. However, the cusplike structures disappear in the process $\psi_{0,1,\downarrow} \rightarrow \psi_{0,0,\uparrow}$ (Fig. 1(b)). In Fig. 1(b), one can see that not only is the variational magnitude of the relaxation rates smaller than that of the process shown in Fig. 1(a), but also the magnetic field and radius dependence of the rates differ from that in Fig. 1(a). From Eq. (9), we know that the factors X for two relaxation processes are different. This factor is related to the magnetic field and radius of QD directly, and thus results in the discrepancy between them. Several experiments have verified that the spin–phonon coupling mechanism dominates the spin relaxation in GaAs QDs under a strong magnetic field.[11,12] Therefore, this discrepancy may be useful to distinguishing the spin states between the Zeeman levels of the ground and first-excited states, and to control and readout the spin state in experiments effectively.

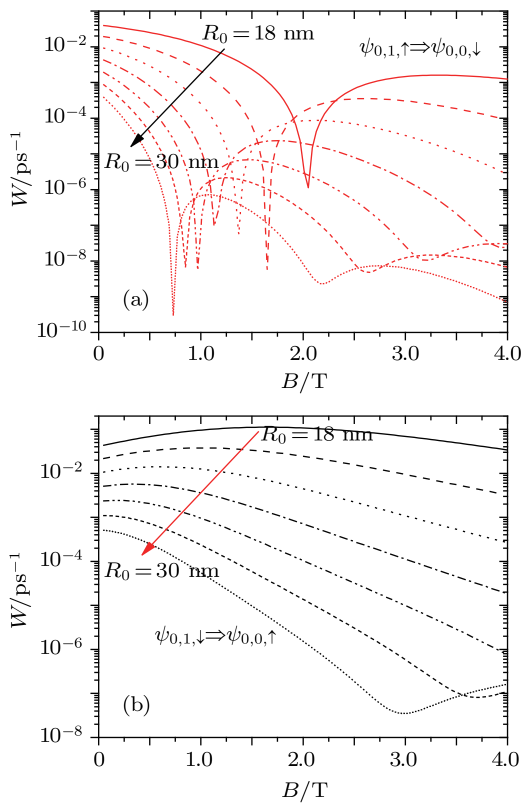

Fig. 1. (color online) The spin relaxation rate as a function of the magnetic field for different radii of QD, (a) for the process $\psi_{0,1,\uparrow} \rightarrow \psi_{0,0,\downarrow}$ and (b) for the process $\psi_{0,1,\downarrow} \rightarrow \psi_{0,0,\uparrow}$, respectively.

The spin-flip relaxation rates as functions of the magnetic field and radius are contour plotted in Figs. 2(a) and 2(b) for the process $\psi_{0,1,\uparrow} \rightarrow \psi_{0,0,\downarrow}$ and $\psi_{0,1,\downarrow} \rightarrow \psi_{0,0,\uparrow}$, respectively. From them, we can see that the relaxation rates can be varied in several orders of magnitude by modulating the radius of QD and strength of external magnetic field, so one can obtain the desirable relaxation time for the coherent states of spin qubit in quantum computation and information. From the comparison between two cases, we also clearly see that the variational magnitude of the relaxation rate for the process in Fig. 2(a) is larger than that in Fig. 2(b). In addition, the cusplike structures of the process (Fig. 2(a)) show the slight oscillatory behaviors arising from the interplays between the magnetic field and radius confinement, not for the process in Fig. 2(b).

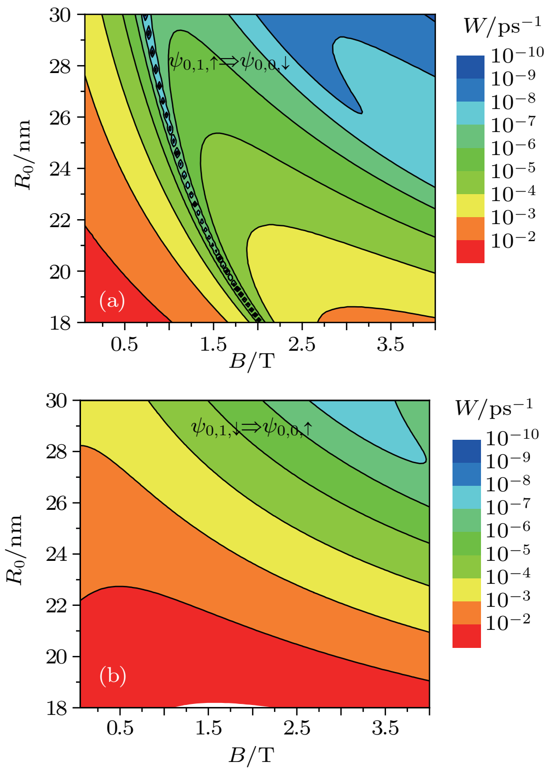

Fig. 2. (color online) Contour plot of the spin relaxation rates as a function of magnetic field and radius for processes (a) $\psi_{0,1,\uparrow} \rightarrow \psi_{0,0,\downarrow}$ and (b) $\psi_{0,1,\downarrow} \rightarrow \psi_{0,0,\uparrow}$, respectively.

4. Summary

In conclusion, we study the spin relaxation rates for the transitions between Zeeman sublevels of the first-excited and ground state in quantum dots based on the Pavlov–Firsov spin–phonon coupling mechanism. We find that the magnetic field and radius dependence of the relaxation rates for the two cases are very different, which may serve as an effective channel to distinguish the relaxation processes in experiment, and thus control the spin coherent state for the application of spin qubit.

References

[1] Hanson R, Kouwenhoven L P, Petta J R, Tarucha S and Vandersypen L M K 2007 *Rev. Mod. Phys.* **79** 1217

[2] Khaetskii A V and Nazarov Y V 2000 *Phys. Rev. B* **61** 12639

[3] Khaetskii A V and Nazarov Y V 2001 *Phys. Rev. B* **64** 125316

[4] Wang Q W and Hong L 2012 *Acta Phys. Sin.* **61** 017107 (in Chinese)
[5] Kang N L and Choib S D 2014 *Chin. Phys. B* **23** 087104
[6] Woods L M, Reinecke T L and Lyanda-Geller Y 2002 *Phys. Rev. B* **66** 161318
[7] Hanson R, Witkamp B, Vandersypen L M K, Willems L H Beveren V, Elzerman J M and Kouwenhoven L P 2003 *Phys. Rev. Lett.* **91** 196802
[8] Heiss D, Schaeck S, Huebl H, Bichler M, Abstreiter G and Finley J J 2007 *Phys. Rev. B* **76** 241306
[9] Jan F and Loss D 2009 *Science* **324** 1277
[10] Calero C, Chudnovsky E M and Garanin D A 2005 *Phys. Rev. Lett.* **95** 166603
[11] Elzerman J M, Hanson R, Willems L H, Witkamp B, Vandersypen L M K and Kouwenhoven L P 2004 *Nature* **430** 431
[12] Amasha S, MacLean K, Iuliana P R, Zumbuhl D M, Kastner M A, Hanson M P and Gossard A C 2008 *Phys. Rev. Lett.* **100** 046803
[13] Struck P R and Burkard G 2010 *Phys. Rev. B* **82** 125401
[14] Droth M and Burkard G 2013 *Phys. Rev. B* **87** 205432
[15] Pavlov S T and Firsov V A 1966 *Sov. Phys. Solid State* **7** 2131
[16] Pavlov S T and Firsov V A 1967 *Sov. Phys. Solid State* **9** 1394
[17] Romano C L, Marques G E, Sanz L and Alcalde A M 2008 *Phys. Rev. B* **77** 033301
[18] Wang Z W and Li S S 2012 *Solid. State. Commun.* **152** 1098
[19] Bulaev D V and Loss D 2005 *Phys. Rev. Lett.* **95** 076805
[20] Bulaev D V and Loss D 2005 *Phys. Rev. B* **71** 205324
[21] Madelung O 1982 *Semiconductors: Physics of Group IV Elements and III–V Compounds* (Berlin: Springer) p. 172
[22] Prabhakar S, Melnik R and Bonilla L L 2013 *Phys. Rev. Lett.* **87** 235202