Simultaneous identification of unknown time delays and model parameters in uncertain dynamical systems with linear or nonlinear parameterization by autosynchronization*

Gu Wei-Dong(顾卫东), Sun Zhi-Yong(孙志勇)†, Wu Xiao-Ming(吴晓明), and Yu Chang-Bin(于长斌)

Shandong Provincial Key Laboratory of Computer Network, Shandong Computer Science Center, Jinan 250014, *China*

(Received 5 January 2013; revised manuscript received 19 April 2013)

In this paper, we propose a general method to simultaneously identify both unknown time delays and unknown model parameters in delayed dynamical systems based on the autosynchronization technique. The design procedure is presented in detail by constructing a specific Lyapunov function and linearizing the model function with nonlinear parameterization. The obtained result can be directly extended to the identification problem of linearly parameterized dynamical systems. Two typical numerical examples confirming the effectiveness of the identification method are given.

Keywords: autosynchronization, parameter identification, delay identification, nonlinear parameterization

PACS: 02.30.Ks, 05.45.Tp, 05.45.Xt **DOI:** 10.1088/1674-1056/22/9/090203

1. Introduction

Identifying the uncertainties (e.g., unknown parameters) in nonlinear dynamical systems is always a matter of interest in many disciplines such as physics, neuron sciences, and engineering areas. The research on this topic has generated very rich literature on the identification technique for various dynamical models. Some traditional and representative methods include the least-squares fitting algorithm, the evolution strategy, the swarm intelligence algorithm, etc. In recent years, another promising identification method based on the adaptive control concept has been developed.[1] This synchronization-based identification method, termed the autosynchronization technique, was firstly proposed in Refs. [1] and [2] for the parameter identification of uncertain dynamical systems. Later, this method has been extensively studied for the adaptive synchronization design of chaotic systems with unknown parameters.[3,4] In two recent review articles,[5,6] many adaptive synchronization schemes reported in the literature are carefully revisited and commented. As revealed in Refs. [5] and [6] and the references therein, the adaptive control method is effective in achieving the adaptive synchronization in the presence of unknown system parameters. The adaptive synchronization concept and the autosynchronization scheme are conceptually similar in the sense that they employ the same design method and analysis approach (i.e., Lyapunov stability and LaSalle's invariance principle); however, the latter focuses more on the accurate estimations of the unknown parameters.

The basic strategy in the autosynchronization technique is similar to the observer design method, which is an established concept in control theory. The relationship between the autosynchronization method and the observer concept has been outlined and discussed in Refs. [7]–[9]. It is worth noting that the aim of using the autosynchronization method is to not only reconstruct the state information of the dynamical systems (which is the main aim in the observer design problem), but also to calculate the true values of the unknown parameters. In comparison with other identification algorithms mentioned in the beginning of this section, the autosynchronization identification method possesses some exceptional virtues as follows. The design procedure is guided by the stability theory of dynamical systems, hence the stability and convergence analysis of the controllers and estimation updating laws are guaranteed. Furthermore, this method features low computational complexity and low memory storage, which can be adopted as a real-time algorithm.[10] Hence, this technique has received extensive attention for the parameter identification problem in uncertain dynamical systems.[11–18]

Many natural and artificial systems contain some delay components and can be described by the delay dynamical system, and typical examples include the delayed feedback system, coupled dynamical systems with interaction delays, complex networks with coupling delays, etc. In the literature there may be found many results on the stability analysis and control synthesis of time delayed dynamical systems (see e.g., Ref. [19]). However, for the inverse problems of delay identification in time delayed systems, there are not so many results. Recently, some researchers attempted to extend the au-

*Project supported by the National Science and Technology Major Project of the Ministry of Science and Technology of China (Grant No. 2011ZX03005-002), the Shandong Academy of Science Development Fund for Science and Technology, China, and the Pilot Project for Science and Technology in Shandong Academy of Science, China.

†Corresponding author. E-mail: kfmuzik@126.com; sun.zhiyong.cn@gmail.com; sunzhy@sdas.org

 http://iopscience.iop.org/cpb http://cpb.iphy.ac.cn

tosynchronization method for this delay estimation issue. In Ref. [20], Yu and Boccaletti reported on the real-time estimation of interaction delays by using the adaptive synchronization strategy. Later, Ma *et al.*[21] presented a comprehensive study on the delay identification by combining the adaptive control method and the autosynchronization concept. Another recent article worth mentioning is Ref. [22]. The authors of Ref. [22] proposed an estimation approach to identify the communication delay by combining the chaos synchronization strategy and the gradient control method. However, the above mentioned articles almost only consider such cases that either the system parameters are unknown (e.g., Refs. [1], [2], [11]– [14]) or the time delays are unknown (e.g., Refs. [20] and [22]). An exception is Ref. [21], where the authors showed by numerical simulations that the autosynchronization method can be used to estimate both the unknown delays and the unknown parameters. However, that article did not give theoretical result and detailed analysis.

The present paper is devoted to the design problem of an adaptive approach to achieve the synchronization as well as the online estimations for model uncertainties in time delayed dynamical systems. A simple and general method is proposed based on a suitable structure of the Lyapunov function to simultaneously estimate both the unknown parameters and unknown delays. Compared with those previous papers, this paper makes the contributions that are highlighted by the following features. First, we consider a more general model than the ones used in most previous articles (e.g., in Refs. [1], [2], [10]–[14]). The dynamical model considered previously is usually a linear parameterization one. For example, in Refs. [10] and [15] the authors investigated the parameter identification problems in dynamical systems while the parameters are linearly coupled in the systems. Other similar work on adaptive synchronization in the case of linear parameterization can be found in the review article.[5,6] In the present paper, we will generalize the identification problem by considering both the linear parameterization case and the nonlinear parameterization case. In fact, this scheme can also be applied to the adaptive synchronization problem for uncertain chaotic systems which has been discussed in Refs. [3]–[6]. Second, we intend to combine the autosynchronization concept[1,2] and the delay identification method[20,21] for addressing the problem that both the parameters and time delays are unknown which need to be estimated simultaneously. To the best of our knowledge, this issue has not been discussed in the previous articles.

The remaining part of this paper is organized as follows. The general dynamical model with nonlinear parameterization, together with the problem formulation, is introduced in Section 2. The main result for solving the simultaneous identification problem is presented in Section 3. By choosing an elaborate structure of the Lyapunov functional candidate, the convergences of the synchronization process as well as the identification process are proved in this section. Besides, we further discuss the concept of identifiability and, accordingly, some special cases where the identification would be failed are also analyzed. In Section 4, two examples are presented to illustrate the identification theory. Finally, some conclusive remarks are given in Section 5.

2. Model preliminaries and problem formulation

We consider an n-dimensional time delayed dynamical system described by the following general differential equation:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{F}(\boldsymbol{x}(t), \boldsymbol{x}(t-\boldsymbol{\tau}), \boldsymbol{p}), \tag{1}$$

where $\boldsymbol{x}(t) = [x_1(t), x_2(t), \cdots, x_n(t)]^{\mathrm{T}} \in \mathbf{R}^n$ are the state variables. The vector field

$$\boldsymbol{F}(\boldsymbol{x}(t), \boldsymbol{x}(t-\boldsymbol{\tau}), \boldsymbol{p}) = \{F_i(\boldsymbol{x}(t), \boldsymbol{x}(t-\tau_i), p_i)\}_{i=1,2,\cdots,n},$$

which describes the interactions among state variables, can be expressed by the following particular form:

$$\begin{aligned} &\boldsymbol{F}_i(\boldsymbol{x}(t), \boldsymbol{x}(t-\tau_i), p_i) \\ &= c_i(\boldsymbol{x}(t)) + \sum_{j=1}^{m_i} f_{ij}(\boldsymbol{x}(t), \boldsymbol{x}(t-\tau_{ij}), p_{ij}), \end{aligned} \tag{2}$$

where $p_i \in R^{m_i}$ and $\tau_i \in R_+^{m_i}$ are the unknown parameter vector and unknown delay vector, respectively. The function fields $c_i(\cdot)$ and $f_{ij}(\cdot)$ are assumed to be some real valued functions which possess, at least locally, Lipchitz properties. Model (1) includes many famous dynamical and chaotic systems. Representative examples include the time delayed Lotka–Volterra model, the Lorenz chaotic system with time delays, the Mackey–Glass model, the time delayed Logistic model, etc. It is worth noting that this model is also more general than the ones studied in several previous articles.[10–12,14,15,20,21] Two extensions, unknown delays and nonlinear parameterizations, are considered in Eqs. (1) and (2) of this paper. The main topic in this paper is to simultaneously identify these uncertainties, which is more challenging than the identification problems studied in previous articles.

Before presenting the main result, we make the following assumptions.

Assumption I The function $c_i(\boldsymbol{x}(t))$ follows the Lipchitz condition with the nonnegative constant L_i^c, i.e.,

$$||c_i(\boldsymbol{y}(t)) - c_i(\boldsymbol{x}(t))|| \le L_i^c ||\boldsymbol{y}(t) - \boldsymbol{x}(t)||. \tag{3a}$$

In addition, the function vector $f_{ij}(\cdot)$ obeys the following Lipchitz-like condition with a nonnegative constant M_{ij}:

$$\left\| f_{ij}(\boldsymbol{y}(t), \boldsymbol{y}(t-\tau_{ij}), p_{ij}) - f_{ij}(\boldsymbol{x}(t), \boldsymbol{x}(t-\tau_{ij}), p_{ij}) \right\|^2$$

$$\leq M_{ij}\left(||\boldsymbol{y}(t)-\boldsymbol{x}(t)||^2+||\boldsymbol{y}(t-\tau_{ij})-\boldsymbol{x}(t-\tau_{ij})||^2\right), \quad (3b)$$

which can be derived by the Lipchitz property of the function field. Note that this assumption has also been used in several previous articles including Refs. [23] and [24].

Problem Assume that we have access to the state vector $\boldsymbol{x}(t)$ which represents the real data generated from the drive system (1). The problem is to design an identification scheme, including a response system with simple coupling terms, suitable feedback controllers and feasible dynamical estimators, for simultaneously estimating the unknown parameters and unknown time delays existed in Eq. (1).

3. Simultaneous identification scheme via autosynchronization

The autosynchronization technique requires a response system (or called the observer system) to record and monitor the evolution of the original dynamical system. The aim for constructing the response system is to not only synchronize with the original system but also identify the uncertainties in the synchronization process. Referring to Eq. (1), we construct the following response system with coupling terms:

$$\dot{\boldsymbol{y}}(t)=\boldsymbol{F}(\boldsymbol{y}(t),\boldsymbol{y}(t-\hat{\tau}),\hat{\boldsymbol{p}})+\boldsymbol{u}(t), \quad (4)$$

where $\boldsymbol{u}(t)$ is the coupling controller to be designed for achieving the synchronization. Similarly,

$$\boldsymbol{F}(\boldsymbol{y}(t),\boldsymbol{y}(t-\hat{\tau}),\hat{\boldsymbol{p}})=\{F_i(\boldsymbol{y}(t),\boldsymbol{y}(t-\hat{\tau}_i),\hat{p}_i)\}_{i=1,2,\cdots,n},$$

where $\hat{\tau}_i$ and $\hat{p}_i$ are time-varying estimations for the unknown delay τ_i and parameters p_i respectively. Define the synchronization error (or called the state estimation error) as

$$\boldsymbol{e}(t)=\boldsymbol{y}(t)-\boldsymbol{x}(t)=(e_1(t),\ e_2(t),\ \ldots,\ e_n(t)),$$

then the error dynamics can be obtained as

$$\begin{aligned}\dot{e}_i(t) &= \dot{y}_i(t)-\dot{x}_i(t)=c_i(\boldsymbol{y}(t))-c_i(\boldsymbol{x}(t))+u_i(t)\\ &\quad+\sum_{j=1}^{m_i}[f_{ij}(\boldsymbol{y}(t),\boldsymbol{y}(t-\hat{\tau}_{ij}),\hat{p}_{ij})\\ &\quad-f_{ij}(\boldsymbol{y}(t),\boldsymbol{y}(t-\tau_{ij}),p_{ij})]\\ &\quad+\sum_{j=1}^{m_i}[f_{ij}(\boldsymbol{y}(t),\boldsymbol{y}(t-\tau_{ij}),p_{ij})\\ &\quad-f_{ij}(\boldsymbol{x}(t),\boldsymbol{x}(t-\tau_{ij}),p_{ij})]. \quad (5)\end{aligned}$$

According to the adaptive feedback method,[25] the adaptive controllers $\boldsymbol{u}(t)=(u_1(t),u_2(t),\cdots,u_n(t))^{\mathrm{T}}\in R^n$ are designed by the following simple linear feedback form:

$$\begin{cases}\boldsymbol{u}(t)=\boldsymbol{k}\boldsymbol{e}(t),\\ \boldsymbol{k}=\mathrm{diag}(k_1,k_2,\cdots,k_n), \quad \dot{k}_i=-\delta_i e_i^2(t).\end{cases} \quad (6)$$

It is known that if $\hat{\tau}_i=\tau_i$ and $\hat{p}_i=p_i$, then the above controllers are effective for achieving the synchronization between systems (1) and (4). In fact, this is an established result which has been extensively studied in detail in several articles (see e.g., Refs. [12], [14], [18], and [25]). However, the uncertainties in the parameters and delays would destroy the desired synchronization. A careful design for the updating laws is desirable in order to eliminate the errors caused by the differences between the estimated parameters or delays and their true values in the synchronization process. This will be obtained by the autosynchronization method and a careful chosen Lyapunov function. Here, we present the main result of this section.

Theorem 1 Suppose that Assumption I holds, then the adaptive synchronization between system (1) and response system (2) can be achieved by the above designed controller (6), the following parameter updating law and delay estimation law:

$$\dot{\hat{p}}_{ij}=-\alpha_{ij}e_i(t)\frac{\partial f_{ij}(\boldsymbol{y}(t),\boldsymbol{y}(t-\hat{\tau}_{ij}),\hat{p}_{ij})}{\partial \hat{p}_{ij}}, \quad (7)$$

$$\dot{\hat{\tau}}_{ij}=-\beta_{ij}e_i(t)\frac{\partial f_{ij}(\boldsymbol{y}(t),\boldsymbol{y}(t-\hat{\tau}_{ij}),\hat{p}_{ij})}{\partial \hat{\tau}_{ij}}, \quad (8)$$

where $1\leq i,\ j\leq N$ and α_{ij}, β_{ij} are positive constants. Furthermore, if the identifiability condition (which will be given later) holds, then the unknown parameters and time delays can be simultaneously identified.

Proof Construct the following Lyapunov function candidate:

$$\begin{aligned}V &= \sum_{i=1}^{n}\frac{1}{2}e_i^2(t)+\sum_{i=1}^{n}\frac{1}{2\delta_i}(k_i+k^*)^2+\sum_{i=1}^{n}\sum_{j=1}^{m_i}\frac{1}{2\alpha_{ij}}(\hat{p}_{ij}-p_{ij})^2\\ &\quad+\sum_{i=1}^{n}\sum_{j=1}^{m_i}\frac{1}{2\beta_{ij}}(\hat{\tau}_{ij}-\tau_{ij})^2+\sum_{i=1}^{n}\sum_{j=1}^{m_i}\int_{t-\tau_{ij}}^{t}\frac{M_{ij}}{2}e_i^2(t)\mathrm{d}s, \quad (9)\end{aligned}$$

where k^* is a sufficiently large positive constant.

Differentiating the Lyapunov function candidate (9) along the error trajectory (5) with respective to time t yields

$$\begin{aligned}\dot{V}(t) &= \sum_{i=1}^{n}e_i(t)\dot{e}_i(t)+\sum_{i=1}^{n}\frac{1}{\delta_i}(k_i+k^*)\dot{k}_i\\ &\quad+\sum_{i=1}^{n}\sum_{j=1}^{m_i}\frac{1}{\alpha_{ij}}(\hat{p}_{ij}-p_{ij})\dot{\hat{p}}_{ij}\\ &\quad+\sum_{i=1}^{n}\sum_{j=1}^{m_i}\frac{1}{\beta_{ij}}(\hat{\tau}_{ij}-\tau_{ij})\dot{\hat{\tau}}_{ij}+\sum_{i=1}^{n}\sum_{j=1}^{m_i}\frac{M_{ij}}{2}e_i^2(t)\\ &\quad-\sum_{i=1}^{n}\sum_{j=1}^{m_i}\frac{M_{ij}}{2}e_i^2(t-\tau_{ij}). \quad (10)\end{aligned}$$

Inserting the expression of the error system into the first term of Eq. (10) and carrying out some simple calculations, we obtain

$$\sum_{i=1}^{n}e_i(t)\dot{e}_i(t) = \sum_{i=1}^{n}e_i(t)[c_i(\boldsymbol{y}(t))-c_i(\boldsymbol{x}(t))+k_ie_i(t)]$$

$$
\begin{aligned}
&+\sum_{i=1}^{n}\sum_{j=1}^{m_i} e_i(t)[f_{ij}(\boldsymbol{y}(t),\boldsymbol{y}(t-\tau_{ij}),p_{ij})\\
&-f_{ij}(\boldsymbol{x}(t),\boldsymbol{x}(t-\tau_{ij}),p_{ij})]\\
&+\sum_{i=1}^{n}\sum_{j=1}^{m_i} e_i(t)[f_{ij}(\boldsymbol{y}(t),\boldsymbol{y}(t-\hat{\tau}_{ij}),\hat{p}_{ij})\\
&-f_{ij}(\boldsymbol{y}(t),\boldsymbol{y}(t-\tau_{ij}),p_{ij})]. \qquad (11)
\end{aligned}
$$

According to the Lipchitz condition introduced in Assumption 1, one has

$$
\sum_{i=1}^{n} e_i(t)[c_i(\boldsymbol{y}(t))-c_i(\boldsymbol{x}(t))] \le \sum_{i=1}^{n} l_i e_i^2(t) \qquad (12)
$$

and

$$
\begin{aligned}
&\sum_{j=1}^{m_i} e_i(t)[f_{ij}(\boldsymbol{y}(t),\boldsymbol{y}(t-\tau_{ij}),p_{ij})\\
&-f_{ij}(\boldsymbol{x}(t),\boldsymbol{x}(t-\tau_{ij}),p_{ij})]\\
\le{}& \sum_{j=1}^{m_i}\frac{1}{2}e_i^2(t)+\frac{1}{2}\sum_{j=1}^{m_i}||f_{ij}(\boldsymbol{y}(t),\boldsymbol{y}(t-\tau_{ij}),p_{ij})\\
&-f_{ij}(\boldsymbol{x}(t),\boldsymbol{x}(t-\tau_{ij}),p_{ij})||^2\\
\le{}& \frac{m_i}{2}e_i^2(t)+\sum_{j=1}^{m_i}\frac{M_{ij}}{2}e_i^2(t)+\sum_{j=1}^{m_i}\frac{M_{ij}}{2}e_i^2(t-\tau_{ij}). \qquad (13)
\end{aligned}
$$

Next, we examine the third term of Eq. (10). This term can be regarded as a multivariable function of $\hat{\tau}_{ij}$ and $\hat{p}_{ij}$. Employing the Taylor series expansion for multivariable functions, we have

$$
\begin{aligned}
&f_{ij}(\boldsymbol{y}(t),\boldsymbol{y}(t-\hat{\tau}_{ij}),\hat{p}_{ij})-f_{ij}(\boldsymbol{y}(t),\boldsymbol{y}(t-\tau_{ij}),p_{ij})\\
={}&\frac{\partial f_{ij}(\boldsymbol{y}(t),\boldsymbol{y}(t-\hat{\tau}_{ij}),\hat{p}_{ij})}{\partial \hat{p}_{ij}}(\hat{p}_{ij}-p_{ij})\\
&+\frac{\partial f_{ij}(\boldsymbol{y}(t),\boldsymbol{y}(t-\hat{\tau}_{ij}),\hat{p}_{ij})}{\partial \hat{\tau}_{ij}}(\hat{\tau}_{ij}-\tau_{ij})+O(\boldsymbol{y},\tilde{\tau}_{ij},\tilde{p}_{ij}). \qquad (14)
\end{aligned}
$$

For choosing $\hat{p}_{ij}$ and $\hat{\tau}_{ij}$ in a region close to the true values of p_{ij} and τ_{ij}, one can reasonably assume that the higher order term $O(\boldsymbol{y},\tilde{\tau}_{ij},\tilde{p}_{ij})$ is bounded quantity or has a bounded increasing rate. By comparing the expressions of Eq. (14) with Eq. (10), in order to cancel the unnecessary terms in Eq. (10), one can design suitable updating laws to estimate the parameters and delays that appear nonlinearly in the function field. This is also the reason behind the design method for the updating laws which take the form as Eqs. (7) and (8). Hence, by considering Eqs. (11)–(14) as well as the updating laws (7) and (8) and by inserting them into Eq. (10), we can obtain

$$
\begin{aligned}
\dot{V}(t) \le{}& \sum_{i=1}^{n}\left(L_i^c+\frac{m_i}{2}+\sum_{j=1}^{m_i}M_{ij}-k^*\right)e_i^2(t)\\
&+\sum_{i,j}^{n,m_i}O(\boldsymbol{y},\tilde{\tau}_{ij},\tilde{p}_{ij})e_i(t). \qquad (15)
\end{aligned}
$$

As stated above, if the higher order term in the above inequality is bounded and the constants L_i^c, m_i, and M_{ij} are all assumed to be bounded values, then one can choose large enough k^* to ensure the negative semi-positiveness of $\dot{V}(t)$. This indicates that $V(t)$ is monotonically decreasing. Since $V(0)$ is bounded, $V(t)$ is also bounded. This further implies that $e_i(t)$, k_i, $\hat{p}_{ij}$, and $\hat{\tau}_{ij}$ are also bounded and accordingly, $\dot{e}_i(t)$ is bounded from Eq. (5). Hence, we have the existence of $\lim_{t\to\infty}\boldsymbol{k}(t)$ because of the monotonicity and boundedness of each component of the control gain variable $k_i(t)$ designed in Eq. (6). Hence, Integrating the expression of control gain (6), we have

$$
k_i(0)-k_i(\infty)=\delta_i\int_0^{\infty}e_i^2(s)\,\mathrm{d}s<+\infty, \qquad (16)
$$

which implies that the square of the error $e_i(t)=y_i(t)-x_i(t)$ is integrable. Thus, the uniform boundedness, along with the integrability and continuity of $e_i(t)$, implies that $e_i(t)\to 0$ as $t\to\infty$ for $i=1,2,\ldots,n$, which means that the synchronization will be achieved.

The above analysis confirms the achievement of synchronization between drive system (1) and response system (4) by using the adaptive controller (6) and updating laws (7) and (8). Then, for $t\to\infty$ and according to Eq. (5), one has

$$
f_{ij}(\boldsymbol{x}(t),\boldsymbol{x}(t-\hat{\tau}_{ij}),\hat{p}_{ij})-f_{ij}(\boldsymbol{x}(t),\boldsymbol{x}(t-\tau_{ij}),p_{ij})=0. \quad (17)
$$

The above formula cannot readily guarantee the identification of the unknown time delays and system parameters. In order to determine the correct estimations of the uncertainties, we introduce the following essential identifiability definition:

Definition Function $\boldsymbol{F}(\boldsymbol{x},\boldsymbol{\tau},\boldsymbol{p})$ where $\boldsymbol{\tau}\in\Omega^1\subset R^+$, $\boldsymbol{p}\in\Omega^2\subset R$ is said to be identifiable over the parameter region Ω^2 and the delay region Ω^1 if there exists neither $p_1,p_2\in\Omega^2\subset R$ with $p_1\neq p_2$, nor $\tau_1,\tau_2\in\Omega^1\subset R^+$ with $\tau_1\neq\tau_2$ such that $\lim_{\substack{\tau\to\tau_1\\ p\to p_1}}\boldsymbol{F}(\boldsymbol{x},\boldsymbol{\tau},\boldsymbol{p})=\lim_{\substack{\tau\to\tau_2\\ p\to p_2}}\boldsymbol{F}(\boldsymbol{x},\boldsymbol{\tau},\boldsymbol{p})$.

This definition implies that identifiability follows if equation (17) has the unique solution over the parameter region and the delay region. Before determining whether the unknown parameters and delays have been estimated to be their true values, one should carefully check the identifiability property of the function. This is a non-trivial task which should be examined by the specific structures of model equations. In fact, in the parameter identification problem of the dynamical model with linear parameterization, an established condition is the linear independence condition (see Refs. [12], [15], [23], and [26]). However, for the dynamical system (1) with nonlinear parameterization and time delays, this problem is more complicated which depends on the model equations themselves. Some in-depth analysis on the topic of identifiability can be found in e.g., Refs. [27] and [28] and we will not address this issue in detail in this paper.

Nevertheless, we can exclude some special cases where the identifiability may not hold. For the parameter updating law (7), the driving function is derived as $\partial f_{ij}(\boldsymbol{y}(t),\boldsymbol{y}(t-$

$\hat{\tau}_{ij}),\hat{p}_{ij})/\partial\hat{p}_{ij}$, which should provide sufficiently rich information for the true estimations of the unknown parameter. If the driving function converges to a stable state in a short time, the finite-time persistent time (see Ref. [26] for detailed discussion on this concept) would not be guaranteed which may result in parameter identification failures. We consider the delay estimation law (8). Suppose that model (1) possesses some periodic states with the specific period T, then the unknown time delay vector will be generally not identifiable in the vector field. In fact, the unknown time delays $\hat{\tau}_{ij}$ would be estimated to be $\tau_{ij}+\kappa T$ where κ is a non-negative integer constant. Furthermore, if the dynamical system converges to a stable state, then the estimation for τ_{ij} would fail as the estimators $\hat{\tau}_{ij}(t)$ could be any constant. In the simulation, we will give some specific analysis by concrete models.

According to the main result in this section, the following corollaries can be readily obtained to deal with some special cases.

Corollary 1 Consider the dynamical model with linear parameterization, which is described by

$$\dot{x}_i(t)=c_i(\boldsymbol{x}(t))+\sum_{j=1}^{m_i}p_{ij}f_{ij}(\boldsymbol{x}(t),\boldsymbol{x}(t-\tau_{ij})). \tag{18}$$

The response system is similar to Eq. (4) and the feedback laws are the same as Eq. (6). According to the above design procedure, the updating laws for unknown parameters and delays are modified as

$$\begin{cases}\dot{\hat{p}}_{ij}=-\alpha_{ij}e_i(t)f_{ij}(\boldsymbol{y}(t),\boldsymbol{y}(t-\hat{\tau}_{ij})),\\ \dot{\hat{\tau}}_{ij}=-\beta_{ij}e_i(t)\hat{p}_{ij}\dfrac{\partial f_{ij}(\boldsymbol{y}(t),\boldsymbol{y}(t-\hat{\tau}_{ij}))}{\partial\hat{\tau}_{ij}}.\end{cases} \tag{19}$$

The similar analyses on the stability proof and identifiability can be obtained by modifying the steps in Theorem 1.

Corollary 2 Furthermore, if the time delays τ_{ij} are assumed to be known, then the above result further degenerates into

$$\dot{\hat{p}}_{ij}=-\alpha_{ij}e_i(t)f_{ij}(\boldsymbol{y}(t),\boldsymbol{y}(t-\tau_{ij})). \tag{20}$$

One difference is that the above estimators can be used for global identification of the unknown parameters. In fact, these are the results studied in several articles including Refs. [12], [14], [15], [18], and [25]. In other words, we have presented a more general model with a more general result in this paper.

4. Illustrative examples and simulations

In the following, we will present two typical examples together with intuitive simulation results to show the effectiveness and application of the above method.

Example I Consider the following hyperchaotic Lorenz system with multiple time delays:

$$\begin{cases}\dot{x}_1(t)=a[x_2(t)-x_1(t)]+cx_4(t),\\ \dot{x}_2(t)=rx_1(t)-x_2(t-\tau_1)-x_1(t)x_3(t),\\ \dot{x}_3(t)=-bx_3(t)+x_1(t)x_2(t),\\ \dot{x}_4(t)=-x_1(t-\tau_2)-2x_4(t).\end{cases} \tag{21}$$

The true values for the parameters and time delays are $a=2$, $c=1.5$, $r=28$, $\tau_1=0.4$, and $\tau_2=0.6$. Via the concept of autosynchronization, we firstly construct the following response system:

$$\begin{cases}\dot{y}_1(t)=\hat{a}[y_2(t)-y_1(t)]+\hat{c}y_4(t)+u_1(t),\\ \dot{y}_2(t)=\hat{r}y_1(t)-y_2(t-\hat{\tau}_1)-y_1(t)y_3(t)+u_2(t),\\ \dot{y}_3(t)=-\hat{b}y_3(t)+y_1(t)y_2(t)+u_3(t),\\ \dot{y}_4(t)=-y_1(t-\hat{\tau}_2)-2y_4(t)+u_4(t),\end{cases} \tag{22}$$

where $\hat{a}$, $\hat{b}$, $\hat{c}$, $\hat{r}$, $\hat{\tau}_1$, and $\hat{\tau}_2$ are time varying estimations for the unknown parameters and delays, respectively. The feedback controller is designed as $u_i(t)=k_i(t)\,[y_i(t)-x_i(t)]$ with adaptive control gains $k_i(t)=-\delta_i e_i^2(t)$. According to Eq. (7), the estimators for identifying unknown parameters are designed as

$$\begin{cases}\dot{\hat{a}}(t)=-\alpha_1\,[y_2(t)-y_1(t)]\,e_1(t),\\ \dot{\hat{b}}(t)=\alpha_2 y_3(t)e_3(t),\\ \dot{\hat{c}}(t)=-\alpha_3 y_4(t)e_1(t),\\ \dot{\hat{r}}(t)=-\alpha_4 y_1(t)e_2(t).\end{cases} \tag{23}$$

The estimators for identifying unknown time delays are designed as

$$\begin{cases}\dot{\hat{\tau}}_1(t)=-\beta_1 e_2(t)\dot{y}_2(t-\hat{\tau}_1)\\ \qquad=-\beta_1 e_2(t)[\hat{r}y_1(t-\hat{\tau}_1)-y_2(t-2\hat{\tau}_1)\\ \qquad\quad-y_1(t-\hat{\tau}_1)y_3(t-\hat{\tau}_1)+u_2(t-\hat{\tau}_1)],\\ \dot{\hat{\tau}}_2(t)=-\beta_2 e_4(t)\dot{y}_1(t-\hat{\tau}_2)\\ \qquad=-\beta_2 e_4(t)[\hat{a}(y_2(t-\hat{\tau}_2)-y_1(t-\hat{\tau}_2))\\ \qquad\quad+\hat{c}y_4(t-\hat{\tau}_2)+u_1(t-\hat{\tau}_2)].\end{cases} \tag{24}$$

In Eq. (23), it is obvious that all the driving functions are persistently excited and linearly independent. Furthermore, notice that the state evolutions in the hyperchaotic system (12) are sufficiently complex, which exclude the special cases discussed in Section 3. Then the unknown parameters and delays are theoretically identifiable. In the simulations, we choose $\delta_i=50$, $\alpha_i=20$ for $i=1-4$, and $\beta_1=\beta_2=1$. The initial conditions are set to be

$$\begin{cases}[x_1(s),x_2(s),x_3(s),x_4(s)]^{\mathrm{T}}=[1,3,3,3]^{\mathrm{T}},\\ [y_1(s),y_2(s),y_3(s),y_4(s)]^{\mathrm{T}}=[2,2,2,2]^{\mathrm{T}},\\ k_i(s)=1,\hat{\tau}_1(s)=\hat{\tau}_2(s)=0.3,\\ \left[\hat{a}(s),\hat{b}(s),\hat{c}(s),\hat{r}(s)\right]^{\mathrm{T}}=[1,1,1,20]^{\mathrm{T}},\end{cases} \tag{25}$$

for $\forall s=[-\max(\tau_1,\tau_2),0]$. The time evolutions of the synchronization errors and adaptive gains are presented in Fig. 1,

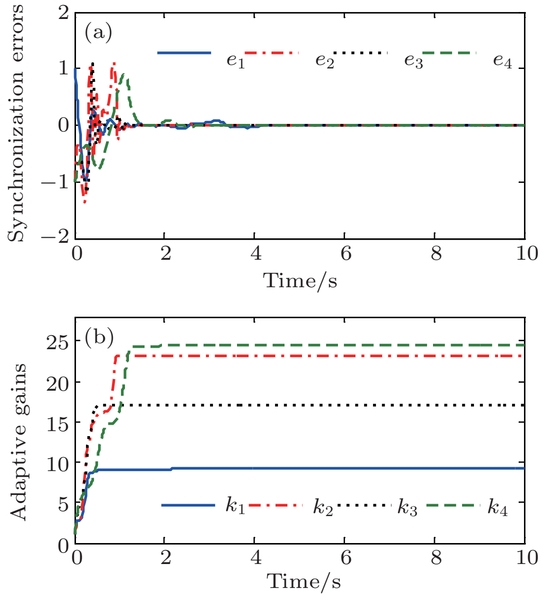

Fig. 1. (color online) Synchronization processes and the evolutions of the adaptive gains for time delayed hyperchaotic Lorenz systems, showing (a) time evolutions of the synchronization errors and (b) time evolutions of the adaptive gains.

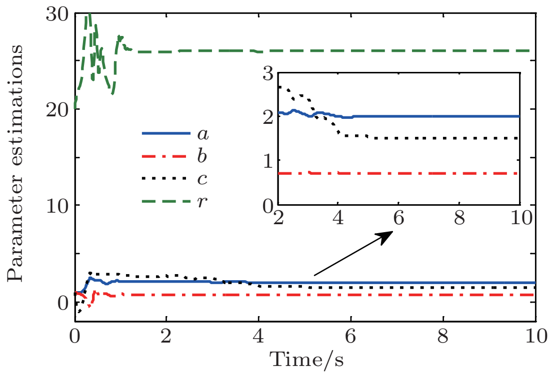

Fig. 2. (color online) Parameter identifications in time delayed hyperchaotic Lorenz system. The inset presents the enlarged time evolutions of the parameter estimators.

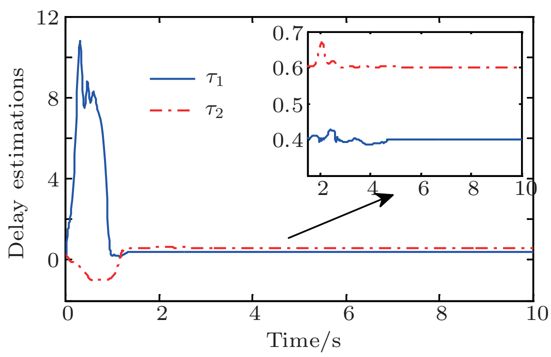

Fig. 3. (color online) Delay identifications in time delayed hyperchaotic Lorenz system. The inset presents the enlarged time evolutions of the delay estimators.

which shows that in a short time the synchronization errors tend to zero and the adaptive gains have been online adjusted to certain bounded constant values. The estimations of the unknown parameters and time delays are depicted in Figs. 2 and 3, respectively, which show that the uncertain information about the parameters and delays has been correctly identified simultaneously.

Example II In Example I, the unknown time delays and parameters are not coupled together but separated in different functions. Now, we give another example in which the unknown parameters and the time delays are nonlinearly coupled in one function field. The time delayed Logistic oscillator, which is described below, is used in this example

$$\dot{x}(t) = -x(t) + px(t-\tau)\,[1-x(t-\tau)]. \tag{26}$$

We assume that the system parameter p and the model delay τ are both unknown and need identification. Construct the response model as

$$\dot{y}(t) = -y(t) + \hat{p}y(t-\hat{\tau})\,[1-y(t-\hat{\tau})] + ke(t), \tag{27}$$

where $e(t) = y(t) - x(t)$ denotes the synchronization error, the last term $ke(t)$ is the control input for the synchronization process with the adaptive control gain as $\dot{k} = -\delta e(t)^2$. According to Eqs. (7) and (8), the estimations $\hat{p}$ and $\hat{\tau}$ are adjusted with the following dynamical equations:

$$\begin{cases} \dot{\hat{p}} = -\alpha y(t-\hat{\tau})\,[1-y(t-\hat{\tau})]\,e(t) \\ \dot{\hat{\tau}} = -\beta e(t)\,[-\hat{p}\dot{y}(t-\hat{\tau}) + 2\hat{p}y(t-\hat{\tau})\dot{y}(t-\hat{\tau})]. \end{cases} \tag{28}$$

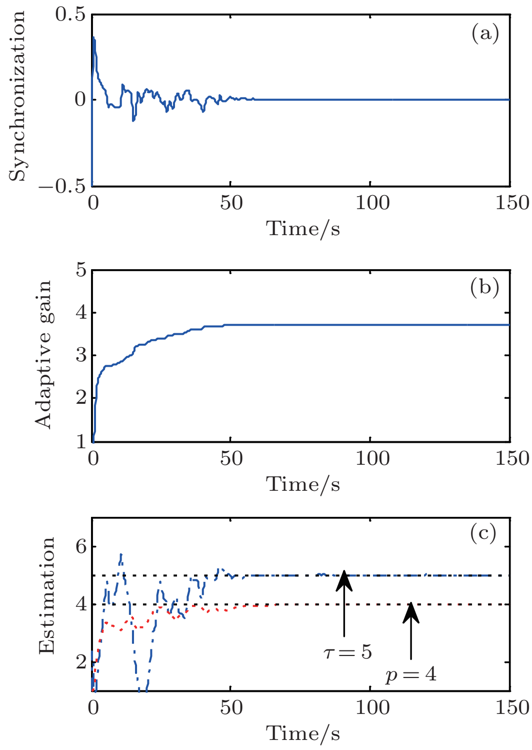

Fig. 4. (color online) Simultaneous identifications of unknown parameter and time delay in Logistic oscillator via autosynchronization method, showing (a) the synchronization processes, (b) time evolutions of the adaptive gain, and (c) time evolutions of the unknown parameter and time delay.

The driving function $y(t-\hat{\tau})\,[1-y(t-\hat{\tau})]$ for parameter estimate $\hat{p}$ does not converge to zero and the time evolution of the model possesses neither stationary state nor periodic state. Hence, it follows from the definition of the identifiability concept that the unknown delays and parameters in Eq. (26) can be locally identified via the updating laws (28). To confirm this, we perform the corresponding numerical simulation. In the simulation, the true values for the parameter and delay are $p = 4$ and $\tau = 5$. The gain parameters are set to be $\delta = \alpha = \beta = 15$. The initial conditions for the drive Logistic model (26) and its response version (27) are $x(s) = 0.5$ and $y(s) = 1$. The initial conditions for adaptive feedback gain, parameter estimation and delay estimation are taken to be $k(s) = 0$, $\hat{p}(s) = \hat{\tau}(s) = 3$ for $\forall s = [-\tau, 0]$. The results are shown in Fig. 4. The synchronization between drive system and response system is achieved as shown in Fig. 4(a). In addition, the time delay and system parameter are correctly and simultaneously identified to be their true values in the synchronization process.

5. Conclusions

The main motivation of this paper is to propose a systematical design method of estimating unknown delays and parameters for time-delayed dynamical systems. The unknown time delays are regarded as special parameters that are nonlinearly parameterized in the dynamical system. The model under investigation is quite general, which includes the linear parameterization case and the nonlinear parameterization case. Hence, the results in this paper are more general than those in some published articles. It is demonstrated that via both theoretical analysis and typical numerical examples, the unknown parameters and time delays can be simultaneously identified by using the autosynchronization technique. In fact, the identification method proposed in this paper can also be used to solve the synchronization problem for dynamical or chaotic systems with both unknown parameters and unknown delays. Some recent articles have reported on the synchronization design for uncertain time delayed chaotic systems.[29–32] However, it is assumed in these papers that the model delays (or the coupling delays) are known in advance to facilitate the controller design. By using the result of the delay identification method proposed in this paper, this assumption can also be well eliminated.

References

[1] Parlitz U 1996 *Phys. Rev. Lett.* **76** 1232
[2] Parlitz U, Junge L and Kocarev L 1996 *Phys. Rev. E* **54** 6253
[3] Al-Sawalha M M and Noorani M 2010 *Commun. Nonlinear Sci. Numer. Simul.* **15** 1036
[4] Sun Z, Si G, Min F and Zhang Y 2012 *Nonlinear Dyn.* **68** 471
[5] Adloo H and Roopaei M 2011 *Nonlinear Dyn.* **65** 141
[6] Sun Z, Zhu W, Si G, Ge Y and Zhang Y 2013 *Nonlinear Dyn.* **72** 729
[7] Nijmeijer H and Mareels I M Y 1997 *IEEE Trans. Circuit Syst. I* **44** 882
[8] Abarbanel H D I, Creveling D R, Farsian R and Kostuk M 2009 *SIAM J. Appl. Dyn. Syst.* **8** 1341
[9] Loria A, Panteley E and Zavala-Río A 2009 *IEEE Trans. Circ. Syst. I* **56** 2703
[10] Ma H and Lin W 2009 *Phys. Lett. A* **374** 161
[11] Ghosh D 2008 *Europhys. Lett.* **84** 40012
[12] Lin W and Ma H F 2007 *Phys. Rev. E* **75** 066212
[13] Liu Y, Tang W and Kocarev L 2008 *Int. J. Bifur. Chaos* **18** 2415
[14] Wei H and Li L 2010 *Chaos* **20** 023112
[15] Yu D and Parlitz U 2008 *Phys. Rev. E* **77** 66221
[16] Wang X Y and Ren X L 2011 *Chin. Phys. Lett.* **28** 050502
[17] Al-sawalha M M and Noorani M 2011 *Chin. Phys. Lett.* **28** 110507
[18] Huang D 2005 *Phys. Rev. E* **71** 037203
[19] Atay F M 2010 *Complex Time-Delay Systems: Theory and Applications* (Berlin: Springer-Verlag)
[20] Yu D and Boccaletti S 2009 *Phys. Rev. E* **80** 036203
[21] Ma H, Xu B, Lin W and Feng J 2010 *Phys. Rev. E* **82** 066210
[22] Sorrentino F and DeLellis P 2012 *Chaos Solitons Fractals* **45** 35
[23] Liu H, Lu J A, Lü J and Hill D J 2009 *Automatica* **45** 1799
[24] Che Y, Li R, Han C, Wang J, Cui S, Deng B and Wei X 2012 *Eur. Phys. J. B* **85** 1
[25] Huang D 2006 *Phys. Rev. E* **73** 066204
[26] Peng H, Li L, Yang Y and Sun F 2011 *Phys. Rev. E* **83** 036202
[27] Jacquez J A and Greif P 1985 *Math. Biosci.* **77** 201
[28] Dedieu H and Ogorzalek M 1997 *IEEE Trans. Circuit Syst. I* **44** 948
[29] Zhang Y, Lu S and Wang Y H 2009 *Chin. Phys. Lett.* **26** 090501
[30] Tang M 2012 *Chin. Phys. B* **21** 020509
[31] Shi X and Wang Z 2012 *Nonlinear Dyn.* **69** 2147
[32] Pourdehi S, Karimaghaee P and Karimipour D 2011 *Phys. Lett. A* **375** 1769