Fractional charges and fractional spins for composite fermions in quantum electrodynamics*

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By using the Faddeev–Senjanovic path integral quantization method, we quantize the composite fermions in quantum electrodynamics (QED). In the sense of Dirac's conjecture, we deduce all the constraints and give Dirac's gauge transformations (DGT). According to that the effective action is invariant under the DGT, we obtain the Noether theorem at the quantum level, which shows the fractional charges for the composite fermions in QED. This result is better than the one deduced from the equations of motion for the statistical potentials, because this result contains both odd and even fractional numbers. Furthermore, we deduce the Noether theorem from the invariance of the effective action under the rotational transformations in 2-dimensional (x, y) plane. The result shows that the composite fermions have fractional spins and fractional statistics. These anomalous properties are given by the constraints for the statistical gauge potential.

Keywords: constrained Hamiltonian systems, Faddeev–Senjanovic path integral quantization formalism, Noether theorem

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1. Introduction

Despite the successes of the effective-field-theory (EFT) model of the fractional quantum Hall effect (FQHE),[1] it is still important to discuss the symmetries of the EFT model to understand the physical properties of the fractional charge and the fractional spin. An important step in the direction that the EFT model is used to illustrate the FQHE[1] was taken by Prange and Girvin[1] and by Girvin and MacDonald,[2], who proposed a field-theory model containing a complex scalarfield ϕ coupled to a vector field $(A_0, \boldsymbol{A})$ with a Chern–Simons term. The Chern–Simons term is deduced from the invariance of an action under a statistical gauge transformation,[3] in which the statistical gauge fields can not describe any physical variable. Though the quantum flux Φ_0 introduced into the Chern–Simons term endows the statistical gauge potential to have the physical properties of the U(1) electromagnetic gauge potential, in fact the statistical gauge potential a_μ still can not be substituted by the electromagnetic potential A_μ totally.[4,5] The physical meanings of the statistical gauge potential a_ν were discussed clearly by Zhang *et al.*[4,5] and Jain.[6–9] Zhang *et al.* gave a result that the statistical gauge potential makes the vortex with a unit flux have fractional charge $e/(2p+1)$, where $p = 0, 1, 2, \ldots$. Jain supplied a new particle with anomalous fluxes, which is called the composite fermion, to describe the FQHE. Actually, the two results are equivalent. The former averages a unit charge to several vortexes with a unit flux, and the latter confines two or more fluxes to an electron. As the use of the EFT to describe the FQHE has been discussed widely, the quantum symmetries have been widely discussed in the system with a singular Lagrangian and Chern–Simons term,[10–18] and even with the Maxwell–Chern–Simons term.[19–22] In most papers, the statistical gauge potential was taken as the U(1) electromagnetic gauge potential.

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In this paper, we consider the composite fermions in quantum electrodynamics (QED) (the spinor fields coupled with Maxwell–Chern–Simons fields) in (2+1)-dimensional space. In Section 2, we quantize the composite fermions in QED by using the Faddeev–Senjanovic path integral quantization formalism. In this process, the statistical gauge potential a_μ and the electromagnetic potential A_μ are introduced by gauge conditions respectively, and are discussed separately. In Section 3, in the sense of Dirac's conjecture, we deduce all constraints and separate them into first and second classes. According to the first-class constraints, we give the generator of the gauge transformations, and deduce the gauge transformations. Under the gauge transformations, the invariance of the effective action gives the Noether theorem at the quantum level, which shows that the fractional charge is confined to a vortex with many unit fluxes. This result is better than the one in Ref. [4]. It contains not only odd fractional charges, but also even fractional charges. In Section 4, the Noether theorem at the quantum level is discussed again under the rotational transformations in the 2-dimensional (x, y) plane. There are two terms which contribute fractional spins to the composite fermions in QED.

2. Quantization of composite fermions in QED

We consider the composite fermions in QED in (2+1)-dimensional space, which has the Lagrangian density

$$\begin{aligned}\mathscr{L} = {} & \bar{\psi}[\mathrm{i}\gamma^\mu(\partial_\mu - eA_\mu - ea_\mu) - m]\psi \\ & - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{e\pi}{2\theta\Phi_0}\varepsilon^{\mu\nu\rho}a_\mu\partial_\nu a_\rho,\end{aligned} \tag{1}$$

where $\psi = \psi(x)$ and $\bar{\psi}(x) = \psi^+(x)\gamma^0$ are singular elements in Grassmann's algebra, e is the unit charge, Φ_0 is the unit quantum flux, θ is the gauge parameter for the Chern–Simons fields, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ are the electromagnetic tensors, A_μ are the electromagnetic potentials, a_μ are the statistical gauge potentials, γ_μ are the Dirac matrices ($\gamma^0 = \sigma^3$, $\gamma^1 = \mathrm{i}\sigma^1$, $\gamma^2 = \mathrm{i}\sigma^2$, σ is the Pauli matrix), and $\varepsilon^{\mu\nu\rho}$ is the Levi–Civita three-order antisymmetry tensor. We also have set $c = \hbar = 1$. On the right-hand side of Eq. (1), the first term describes the spinor fields in the statistical gauge fields and the electromagnetic fields, the second term is the free electromagnetic fields, and the third is the Chern–Simons gauge fields.

In the Faddeev–Senjanovic path integral quantization formalism, the canonical momenta conjugated to the field variables $\bar{\psi}, \psi, A_\mu, a_\nu$ are

$$\begin{aligned}&\pi_{\bar{\psi}} = \frac{\delta\mathscr{L}}{\delta\dot{\bar{\psi}}} = 0, \quad \pi_\psi = \frac{\delta\mathscr{L}}{\delta\dot{\psi}} = \mathrm{i}\bar{\psi}\gamma^0, \\ &\pi_{A_i} = \frac{\delta\mathscr{L}}{\delta\dot{A}_i} = -F^{0i}, \\ &\pi_{a_i} = \frac{\delta\mathscr{L}}{\delta\dot{a}_i} = \frac{e\pi}{2\theta\Phi_0}\varepsilon^{ij}a_j, \quad \pi_{A_0} = \frac{\delta\mathscr{L}}{\delta\dot{A}_0} = 0,\end{aligned} \tag{2}$$

respectively, where $\varepsilon^{0ij} = \varepsilon^{ij}$. Using the Legendre transformation, according to Eq. (1), we can give the canonical Hamiltonian density as

$$\begin{aligned}\mathscr{H}_{\mathrm{C}} = {} & \pi_{\bar{\psi}}\dot{\bar{\psi}} + \pi_\psi\dot{\psi} + \pi_{A_\mu}\dot{A}_\mu + \pi_{a_\mu}\dot{a}_\mu - \mathscr{L} \\ = {} & \frac{1}{2}\pi_{A_i}^2 + \frac{1}{4}F_{ij}F^{ij} - A_0\partial_i\pi^{A_i} - \mathrm{i}\bar{\psi}\gamma^i\partial_i\psi + m\bar{\psi}\psi \\ & - \frac{e\pi}{\theta\Phi_0}\varepsilon^{ij}a_0\partial_i a_j + e\bar{\psi}\gamma^\mu\psi A_\mu + e\bar{\psi}\gamma^\mu\psi a_\mu.\end{aligned} \tag{3}$$

It is easy to check that the primary constraints in Eq. (2) are

$$\phi_1^0 = \pi_{\bar{\psi}} \approx 0, \tag{4}$$

$$\phi_2^0 = \pi_\psi - \mathrm{i}\bar{\psi}\gamma^0 \approx 0, \tag{5}$$

$$\phi_3^0 = \pi_{a_i} - \frac{e\pi}{2\theta\Phi_0}\varepsilon^{ij}a_j \approx 0, \tag{6}$$

$$\phi_4^0 = \pi_{A_0} \approx 0. \tag{7}$$

As for the constrained system, in the sense of Dirac's conjecture, the canonical Hamiltonian should be replaced by the total Hamiltonian including all primary constraints. Thus, for system (1), the total Hamiltonian density is

$$\begin{aligned}\mathscr{H}_{\mathrm{T}} = {} & \mathscr{H}_{\mathrm{C}} + \lambda_1\phi_1^0 + \lambda_2\phi_2^0 + \lambda_3\phi_3^0 + \lambda_4\phi_4^0 \\ = {} & \frac{1}{2}\pi_{A_i}^2 + \frac{1}{4}F_{ij}F^{ij} - A_0\partial_i\pi_{A_i} - \mathrm{i}\bar{\psi}\gamma^i\partial_i\psi + m\bar{\psi}\psi \\ & - \frac{e\pi}{\theta\Phi_0}\varepsilon^{ij}a_0\partial_i a_j + e\bar{\psi}\gamma^\mu\psi A_\mu + e\bar{\psi}\gamma^\mu\psi a_\mu \\ & + \lambda_1\phi_1^0 + \lambda_2\phi_2^0 + \lambda_3\phi_3^0 + \lambda_4\phi_4^0,\end{aligned} \tag{8}$$

where $\lambda_i(x)$ $(i = 1, 2, 3, 4)$ are Lagrange multipliers with respect to the primary constraints ϕ_i^0. The total Hamiltonian can be integrated by using the total Hamiltonian density as

$$H_{\mathrm{T}} = \int_S \mathrm{d}^2x\mathscr{H}_{\mathrm{T}}. \tag{9}$$

Based on the total Hamiltonian, the stationarity of the primary constraint ϕ_1^0, $\{\phi_1^0, H_{\mathrm{T}}\} \approx 0$, leads to the equation

$$\mathrm{i}\lambda_2\gamma^0 = e\gamma^\mu\psi A_\mu + e\gamma^\mu\psi a_\mu - (\mathrm{i}\gamma^i\partial_i - m)\psi. \tag{10}$$

Obviously, Lagrange multiplier λ_2 can be determined from the above equation.

The stationarity of constraint ϕ_2^0, $\{\phi_2^0, H_{\mathrm{T}}\} \approx 0$, gives

$$\mathrm{i}\gamma^0\lambda_1 = \bar{\psi}(\mathrm{i}\gamma^i\partial_i - m) - e\bar{\psi}\gamma^\mu A_\mu - e\bar{\psi}\gamma^\mu a_\mu. \quad (11)$$

From Eq. (11), Lagrange multiplier λ_1 can be solved as well.

The stationarity of the primary constraint ϕ_3^0, $\{\phi_3^0, H_{\mathrm{T}}\} \approx 0$, defines Lagrangian multiplier λ_3 by the following equation:

$$\lambda_3 \frac{e\pi}{2\theta\varPhi_0} = -e\bar{\psi}\gamma^i\psi. \quad (12)$$

However, the stationarity of constraint (7) gives birth to a new constraint

$$\phi_1^1 = -\{\phi_4^0, H_{\mathrm{T}}\} = \partial_i\pi_{A_i} + e\bar{\psi}\gamma^0\psi \approx 0. \quad (13)$$

Furthermore, its stationarity can not give any new constraint.

In this constrained system, there are four primary constraints $(\phi_1^0, \phi_2^0, \phi_3^0, \phi_4^0)$ and a secondary constraint ϕ_1^1. In the sense of Dirac's conjecture, the total Hamiltonian density $\mathscr{H}_{\mathrm{T}}$ should be replaced by the extended Hamiltonian density containing the secondary constraint.[23] But there are some different points on this topic. It is argued that the total Hamiltonian should not be substituted by the extended Hamiltonian[24–33] under certain conditions. The main cause is that the secondary constraints might lead to the broken invariance of the canonical action under the Dirac's gauge transformation (DGT). In other words, the secondary constraints might lead to the loss of the equivalence between the canonical equations and the Euler–Lagrangian equations. And this point has been proved in different constrained systems. However we do not think so. This problem can be discussed from the principle of the least action. In system (1), the extended Hamiltonian is equivalent to the total Hamiltonian entirely, because the secondary constraint does not give a variance to the canonical action.

As usual, the first-class constraints are denoted by $\varLambda$, and the second-class by θ. In system (1), the two second-class constraints are given by $\theta_1 = \pi_{\bar{\psi}} \approx 0$ and $\theta_2 = \pi_\psi - \mathrm{i}\bar{\psi}\gamma^0 \approx 0$, and the three first-class constraints are expressed as $\varLambda_1 = \pi_{A_0} \approx 0$, $\varLambda_2 = \pi_{a_i} - (e\pi/2\theta\varPhi_0)\varepsilon^{ij}a_j \approx 0$, and $\varLambda_3 = \phi_1^1 - \mathrm{i}e(-\phi_1^0\bar{\psi} + \phi_2^0\psi) = \partial_i\pi_{A_i} - \mathrm{i}e(\pi_\psi\psi - \pi_{\bar{\psi}}\bar{\psi}) \approx 0$. It is easy to check that

$$\{\varLambda_1, \varLambda_2\} \approx 0, \quad \{\varLambda_2, \varLambda_3\} \approx 0, \quad \{\varLambda_1, \varLambda_3\} \approx 0, \quad (14)$$

$$\{\varLambda_1, \theta_1\} \approx 0, \quad \{\varLambda_1, \theta_2\} \approx 0, \quad (15)$$

$$\{\varLambda_2, \theta_1\} \approx 0, \quad \{\varLambda_2, \theta_2\} \approx 0, \quad (16)$$

$$\{\varLambda_3, \theta_1\} = -\mathrm{i}e\theta_1 \approx 0, \quad \{\varLambda_3, \theta_2\} = -\mathrm{i}e\theta_2 \approx 0, \quad (17)$$

$$\{\theta_1, \theta_2\} = \mathrm{i}\gamma^0. \quad (18)$$

Hence, constraints $\varLambda_1$, $\varLambda_2$, and $\varLambda_3$ are first-class, while constraints θ_1 and θ_2 are second-class. In the Faddeev–Senjanovic path integral quantization formalism, for each first-class constraint, we must choose a gauge condition. Simply, we consider the radiation gauge and the Coulomb gauge

$$\varOmega_1 = A_0 \approx 0, \quad (19)$$

$$\varOmega_2 = \partial_i a_i \approx 0, \quad (20)$$

$$\varOmega_3 = \partial_i A_i \approx 0 \quad (21)$$

as the gauge conditions with respect to the first-class constraints $\varLambda_1$, $\varLambda_2$, and $\varLambda_3$.

Based on the previous discussion, in the phase space, the generating functional of the Green function for singular Lagrangian (1) can be given as

$$\begin{aligned} Z[J] = &\int \mathscr{D}\varphi^\alpha \mathscr{D}\pi_\alpha \det|\{\varLambda_a, \varOmega^b\}| \prod_{a=1}^{3} \delta(\varLambda_a)\delta(\varOmega_a) \\ &\times \prod_{l=1}^{2} \delta(\theta_l) \det|\{\theta_i, \theta_j\}|^{1/2} \\ &\times \exp\left[\mathrm{i}\int \mathrm{d}^3x(\pi_\alpha\varphi^\alpha - \mathscr{H}_{\mathrm{C}} + J_\alpha\varphi^\alpha)\right], \quad (22) \end{aligned}$$

where $\pi_\alpha = (\pi_{\bar{\psi}}, \pi_\psi, \pi_{A_\mu}, \pi_{a_\mu})$, $\varphi^\alpha = (\bar{\psi}, \psi, A_\mu, a_\mu)$, and J_α are exterior source variables with respect to φ^α. It is easy to check that $\det|\{\varLambda_a, \varOmega_b\}|$ and $\det|\{\theta_i, \theta_j\}|$ do not depend on any canonical variables ϕ^α or their canonical momenta π_α. Thus we can simplify the above generating functional (22) as

$$\begin{aligned} Z[J, K, U] = &\int \mathscr{D}\varphi^\alpha \mathscr{D}\pi_\alpha \mathscr{D}\lambda_f \mathscr{D}\bar{C}_a \mathscr{D}C_b \\ &\times \exp\left[\mathrm{i}I_{\mathrm{eff}}^{\mathrm{P}} + \mathrm{i}\int \mathrm{d}^3x(J_\alpha\varphi^\alpha \right. \\ &\left. + K^\alpha\pi_\alpha + U\lambda)\right], \quad (23) \end{aligned}$$

where

$$I_{\mathrm{eff}}^{\mathrm{P}} = \int \mathrm{d}^3x \mathscr{L}_{\mathrm{eff}}^{\mathrm{P}} = \int \mathrm{d}^3x(\mathscr{L}^{\mathrm{P}} + \mathscr{L}_{\mathrm{m}}), \quad (24)$$

$$\mathscr{L}^{\mathrm{P}} = \pi_\alpha\dot{\varphi}^\alpha - \mathscr{H}_{\mathrm{C}}, \quad (25)$$

$$\mathscr{L}_{\mathrm{m}} = \lambda_a\varLambda_a + \lambda_b\varOmega_b + \lambda_l\theta_l. \quad (26)$$

Here we introduce exterior sources J_α with respect to canonical variables φ^α, K^α with respect to canonical momenta π_α, and U with respect to Lagrange multipliers $\lambda = (\lambda_a, \lambda_b, \lambda_l)$.

3. Fractional charges for composite fermions in QED

According to Dirac's conjecture that all first-class constraints are generators of gauge transformations, the generator of the gauge transformations can be expressed as[34,35]

$$\begin{aligned} G &= \int \mathrm{d}^2x[\omega_1(x)\Lambda_1 + \omega_2(x)\Lambda_2 + \omega_3(x)\Lambda_3] \\ &= \int \mathrm{d}^2x\bigg\{\omega_1(x)\pi_{A_0} + \omega_2(x)\left(\pi_{a_i} - \frac{e\pi}{2\theta\Phi_0}\varepsilon^{ij}a_j\right) \\ &\quad + \omega_3(x)[\partial_i\pi_{A_i} - \mathrm{i}e(\pi_\psi\psi - \pi_{\bar\psi}\bar\psi)]\bigg\}. \end{aligned} \tag{27}$$

From the master equation $\partial G/\partial t + [G, H_{\mathrm{T}}] \approx 0$,[36,37] we can obtain

$$\begin{aligned} &\delta\lambda_1 = \mathrm{i}e\bar\psi\dot\omega_3, \quad \delta\lambda_2 = -\mathrm{i}e\psi\dot\omega_3, \\ &\delta\lambda_3 = \dot\omega_2, \quad \delta\lambda_4 = \dot\omega_1, \quad \dot\omega_3 - \omega_1 = 0. \end{aligned} \tag{28}$$

It is obvious that $\omega_1 = \dot\omega_3$. Generator (27) can be simplified as

$$G = \int \mathrm{d}^2x(\dot\omega\Lambda_1 + \omega_2\Lambda_2 + \omega\Lambda_3), \tag{29}$$

where $\omega_3 = \omega$. According to $\delta F = [F, G]$ (F is an arbitrary function depending on canonical variables and momenta), from generator (29), we can deduce the gauge transformations in the sense of Dirac's conjecture as

$$\begin{aligned} &\delta\bar\psi = \mathrm{i}\omega e\bar\psi, \quad \delta\psi = -\mathrm{i}\omega e\psi, \\ &\delta A_0 = \dot\omega, \quad \delta A_i = \omega\partial_i\delta(x-y), \\ &\delta a_i = \omega_2, \quad \delta\pi_{\bar\psi} = -\mathrm{i}\omega e\pi_{\bar\psi}, \\ &\delta\pi_\psi = \mathrm{i}\omega e\pi_\psi, \quad \delta\pi_{a_i} = -\frac{\omega_2 e\pi}{2\theta\Phi_0}\varepsilon^{ji}, \\ &\delta\lambda_1 = \mathrm{i}e\bar\psi\dot\omega, \quad \delta\lambda_2 = -\mathrm{i}e\psi\dot\omega, \\ &\delta\lambda_3 = \dot\omega_2, \quad \delta\lambda_4 = \ddot\omega, \end{aligned} \tag{30}$$

the other variable variances are zeros, that is $\delta a_0 = \delta\pi_{A_0} = \delta\pi_{A_0} = \delta\pi_{A_i} = \delta\pi_{a_0} = 0$. For simplicity, we will call transformations (30) as Dirac's gauge transformations (DGT) in the following. Under DGT (30), according to the Noether theorem at the quantum level, we obtain two conserved charges as

$$Q_1 = \int_S \mathrm{d}^2x\left[\frac{e\pi}{2\theta\Phi_0}\varepsilon^{ij}a_j\right] = \text{const.}, \tag{31}$$

and

$$Q_2 = \int_S \mathrm{d}^2x[-e\bar\psi\gamma^0\psi - F^{0i}\delta_i(x-y)] = \text{const.}, \tag{32}$$

where the effective factor θ is[2]

$$\theta = (2p+1)\pi, \quad (p = 0, 1, 2, \ldots), \tag{33}$$

and $\bar\psi\gamma^0\psi$ is the density of fermions.

From Eqs. (31) and (33), we can obtain

$$Q_1 = -\frac{ne}{2(2p+1)}\frac{1}{n\Phi_0}\int \mathrm{d}^2x\varepsilon^{ij}a_j, \tag{34}$$

which implies that each vortex with a unit quantum flux can have $ne/2(2p+1)$ fractional charge.[4]

From the equation of motion derived from Eq. (23)–(26), we can obtain an equation for a_0 as

$$j^0 \equiv \frac{\delta S}{\delta a_0} = \frac{e\pi}{\theta\Phi_0}\varepsilon^{ij}\partial_i a_j = \frac{e}{(2p+1)}\frac{b}{\Phi_0}, \tag{35}$$

where

$$b = \varepsilon^{ij}\partial_i a_j. \tag{36}$$

So the total charge carried by the vortex with n unit quantum fluxes is

$$Q_{\mathrm{v}} = \int \mathrm{d}^2xj^0 = \pm\frac{e}{(2p+1)}\frac{\int \mathrm{d}^2xb}{\Phi_0}. \tag{37}$$

If the introduced gauge field b is assumed to have the fundamental properties of the electromagnetic field, we can obtain fractional charge

$$Q_{\mathrm{v}} = \pm\frac{ne}{(2p+1)} \tag{38}$$

for the vortex with a unit flux.[4]

The results in Eqs. (34) and (38) both give odd fractional charge for the vortex with a unit flux, however Eq. (34) also contains an even fractional charge, which is better than Eq. (38). In other words, the Noehter theorem at the quantum level shows more physical properties than the equation of motion for the constrained system. According to the previous discussion, we know that when unit flux Φ_0 is introduced into the Chern–Simons term, the statistical gauge potential a_μ has the properties of electromagnetic potential A_μ.

4. Fractional spins for composite fermions in QED

Under the rotational transformations in the 2-dimensional plane, the fractional spins and the fractional statistics have been deduced for the composite bosons.[18] In this section, we will discuss the conservation law of angular momenta for the composite fermions in QED under the rotational transformations in the (x, y) plane.

It is well known that the Lorentz transformations are

$$\begin{cases}\Delta x_\mu = \varepsilon_{\mu\nu} x_\nu, \\ \Delta A_\gamma = \dfrac{1}{2}\varepsilon_{\mu\nu} S^{\mu\nu}_{\gamma s} A_s, \\ \Delta \psi_\mu = \dfrac{1}{2}\varepsilon_{\mu\nu} T^{\mu\nu}_{\gamma s}\psi_s,\end{cases} \tag{39}$$

where $\varepsilon_{\mu\nu}$ is an antisymmetric matrix, A_μ are (3+1)-dimensional vector fields, ψ_μ are spinor fields, $S^{\mu\nu}_{\gamma s} = \delta_{\gamma\mu}\delta_{s\nu} - \delta_{\gamma\nu}\delta_{s\mu}$, and $T^{\mu\nu}_{\gamma s} = (1/4)(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)_{\gamma s} = \mathrm{i}/\sigma_{\mu\nu}$. According to the Noether theorem at the quantum level, we obtain the conserved charge as

$$Q = \int \mathrm{d}^2 x[\pi(\Delta\phi - \phi_{,i}\Delta x_i) - H_{\text{eff}}\Delta x_0] = \text{const.}, \tag{40}$$

where $\phi_{,i} = \partial\phi/\partial x_i$. When rotational transformations (39) are pressed in the 2-dimensional plane, Δx_0 is zero. Then the above conserved charge (40) can be simplified as

$$Q = \int \mathrm{d}^2 x[\pi(\Delta\phi - \phi_{,i}\Delta x_i)] = \text{const.} \tag{41}$$

Substituting Eqs. (2) and (39) into Eq. (41), we obtain

$$\begin{aligned}&\int \mathrm{d}^2 x\left[\mathrm{i}\bar{\psi}^\mu\gamma^0\left(\frac{1}{2}T^{\mu\nu}_{\gamma s}\psi^\nu - \psi^\mu_{,i}x_i\right)\right] \\ &- \int \mathrm{d}^2 x\left[F^{0i}\left(\frac{1}{2}S^{\mu\nu}_{\gamma s}A_\nu - \partial_i A_\mu x_i\right)\right] \\ &+ \int \mathrm{d}^2 x\left[\frac{e\pi}{2\theta\Phi_0}\varepsilon^{ij}a_j\left(\frac{1}{2}S^{\mu\nu}_{\gamma s}a_\nu - \partial_i a_\nu x_i\right)\right] \\ &= \text{const.}\end{aligned} \tag{42}$$

The above equation can be expanded as

$$\begin{aligned}&\int \mathrm{d}^2 x\left[\frac{\mathrm{i}}{8}\bar{\psi}^\gamma\gamma^0(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)_{\gamma s}\psi^s\right] \\ &- \int \mathrm{d}^2 x(\mathrm{i}\bar{\psi}\gamma^0\psi_{,i}x_i) - \int \mathrm{d}^2 x\left(\frac{1}{2}F^{0i}S^{\mu\nu}_{\gamma s}A_\nu\right) \\ &+ \int \mathrm{d}^2 x(F^{0i}\partial_i A_\mu x_i) + \int \mathrm{d}^2 x\left(\frac{e\pi}{4\theta\Phi_0}\varepsilon^{ij}a_j S^{\mu\nu}_{\gamma s}a_\nu\right) \\ &- \int \mathrm{d}^2 x\left(\frac{e\pi}{2\theta\Phi_0}\varepsilon^{ij}a_j\partial_i a_\mu x_i\right) = \text{const.}\end{aligned} \tag{43}$$

In the left-hand side of Eq. (43), the first term is the total angular momentum of the spinor fields, the second term is the orbital angular momentum of the spinor fields, the third and the forth terms are the orbital angular momenta of the electromagnetic fields, and the fifth and the sixth terms are the sources for the appearance of the fractional spins. The two terms are generated by the Chern–Simons gauge fields entirely. They are produced by the interaction between the orbital motion of the fluxes attached to the fermions through the statistical gauge transformations. These results offer illustrations of the appearance of the fractional spins for the composite fermions in QED. Furthermore, the fractional spins of the composite fermions in QED are endowed only by the introduction of the statistical gauge potential, that is the Chern–Simons gauge fields, and the QED term makes no difference. The statistical gauge fields provide two contributions to the fractional spins of the composite fermions in QED. But for the bosons, there is only a normal spin term.[38,39]

Following the discussion in Ref. [18], we can obtain

$$\int \mathrm{d}^2 x\frac{e\pi}{2\theta\Phi_0}\varepsilon^{ij}a_j\partial_i a_{j'}x_{j'} = \frac{\pi}{2\theta}e\Phi_0. \tag{44}$$

We can define an operator $\hat{A}$ for the anomalous spin term (44) as

$$\hat{A} = \frac{\pi}{2\theta}e\hat{\Phi}_0, \tag{45}$$

and another operator $\hat{S}$ for the composite fermions in QED as

$$\hat{S} = \frac{e\pi}{4\theta\Phi_0}\int \mathrm{d}^2 x\varepsilon^{ij}a_k\hat{S}^{kl}_{ij}a_l. \tag{46}$$

Assuming a one-particle state with 1/2 spin denoted as $|1/2, 1\rangle_{\text{any}}$, we then take the above two terms to act on the state and obtain

$$\mathrm{e}^{\mathrm{i}\alpha\hat{S} - \mathrm{i}\beta\hat{A}}\left|\frac{1}{2}, 1\right\rangle_{\text{any}} = \mathrm{e}^{\mathrm{i}\alpha\frac{\pi}{4\theta} - \mathrm{i}\beta\frac{\pi}{2\theta}}\left|\frac{1}{2}, 1\right\rangle_{\text{any}}, \tag{47}$$

where α is the spin-rotation parameter, and β is the rotation parameter. Since $\theta = (2p + 1)\pi$ $(p = 0, 1, 2, \ldots)$, we can obtain the eigenvalues of the two operators $\hat{S}$ and $\hat{A}$ as

$$s = \frac{1}{4(2p + 1)}, \tag{48}$$

and

$$a = \frac{1}{2(2p + 1)}, \tag{49}$$

respectively. For the composite fermions in QED with the Chern–Simons term, their anomalous spins can be calculated as

$$\begin{aligned}s_{\mathrm{T}} &= \pm\frac{1}{2} + \frac{1}{4(2p + 1)} - \frac{1}{2(2p + 1)} \\ &= \pm\frac{1}{2} - \frac{1}{4(2p + 1)}.\end{aligned} \tag{50}$$

Under the rotational transformations in the 2-dimensional space, the invariance of the effective action supplies the fractional spins for the composite fermions in QED. In this way, for an arbitrary state, we can give corresponding anomalous results. Obviously, as parameters α, β and coefficient θ in the Chern–Simons term take special values, we obtain factor $\mathrm{e}^{\mathrm{i}\alpha s-\mathrm{i}\beta a}=1,-1$, and the others. If it is 1, the system is consisted of bosons; if it is -1, the system is consisted of fermions; and if it is a fractional number, the system is consisted of anyons.[40,41] This result shows that the quantum fluxes attached to particles can change the statistics of the particles. The result of Eq. (50) does not agree with the one given in Ref. [18]. When the vortex with a unit flux is defined as a particle with fractional charge, the fractional spins (48) brought by the interactions between the spins and the statistical gauge potentials can vanish from Eq. (50), and then Eq. (49) is replaced by $n/4(2p+1)$. This result proves to a certain extent that the inherent constraints define some symmetries. Furthermore, the subsidiary constraints introduced into a constrained system can change the system's symmetries.

5. Conclusion

In conclusion, we have considered the composite fermions in QED, and quantized them by using the Faddeev–Senjanovic path integral quantization method. We have separately considered the statistical gauge potential and the electromagnetic potential, and introduced gauge conditions for them, respectively. By using the Dirac–Bergmann method, we have deduced all constraints, and given the DGT. Under the DGT, the invariance of the effective action has given the Noether theorem at the quantum level, which has endowed fractional charge to the vortex with a unit flux. The result is better than that in Ref. [4]. The Noether theorem has also been reconsidered under the rotational transformations in the (x,y) plane. A new result has been given that if we want to have a result agreeing with that in Ref. [18], the fractional charge brought by the inherent constraints should be regarded as the inherent property of the vortex with a unit flux. It has proved to a certain extent that the constraints can define the symmetries of the constrained system. So subsidiary constraints can be used to change the symmetries of a constrained system. The constrained systems with subsidiary constraints have been discussed recently.[42,43] What the Mei symmetries[44] are in this system needs to be studied further.

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